This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

+ *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.
+ *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.
+ *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.
+ *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at http://books.google.com/

RIES
59 9

[illegible]

20 June [illegible]

[illegible]

INTERNATIONAL LIBRARY OF TECHNOLOGY

A SERIES OF TEXTBOOKS FOR PERSONS ENGAGED IN THE ENGINEERING PROFESSIONS AND TRADES OR FOR THOSE WHO DESIRE INFORMATION CONCERNING THEM. FULLY ILLUSTRATED AND CONTAINING NUMEROUS PRACTICAL EXAMPLES AND THEIR SOLUTIONS

NEW YORK
PUBLIC
LIBRARY

ASTRONOMY
DRAINAGE
SEWERAGE
WATER SUPPLY AND DISTRIBUTION
IRRIGATION

SCRANTON
INTERNATIONAL TEXTBOOK COMPANY

35

THE NEW YORK
PUBLIC LIBRARY
599042
ASTOR, LENOX AND
TILDEN FOUNDATIONS.
R 1913 L

Copyright, 1897, 1898, 1899, by THE COLLIERY ENGINEER COMPANY.
Copyright, 1903, by INTERNATIONAL TEXTBOOK COMPANY.

Drainage: Copyright, 1897, by THE COLLIERY ENGINEER COMPANY.
Sewerage: Copyright, 1897, by THE COLLIERY ENGINEER COMPANY.
Water Supply and Distribution: Copyright, 1898, by THE COLLIERY ENGINEER COMPANY.
Irrigation: Copyright, 1898, by THE COLLIERY ENGINEER COMPANY.

All rights reserved.

1786a

BURR PRINTING HOUSE,
FRANKFORT AND JACOB STREETS,
NEW YORK.

PREFACE

The International Library of Technology is the outgrowth of a large and increasing demand that has arisen for the Reference Libraries of the International Correspondence Schools on the part of those who are not students of the Schools. As the volumes composing this Library are all printed from the same plates used in printing the Reference Libraries above mentioned, a few words are necessary regarding the scope and purpose of the instruction imparted to the students of—and the class of students taught by—these Schools, in order to afford a clear understanding of their salient and unique features.

The only requirement for admission to any of the courses offered by the International Correspondence Schools is that the applicant shall be able to read the English language and to write it sufficiently well to make his written answers to the questions asked him intelligible. Each course is complete in itself, and no textbooks are required other than those prepared by the Schools for the particular course selected. The students themselves are from every class, trade, and profession and from every country; they are, almost without exception, busily engaged in some vocation, and can spare but little time for study, and that usually outside of their regular working hours. The information desired is such as can be immediately applied in practice, so that the student may be enabled to exchange his present vocation for a more congenial one or to rise to a higher level in the one he now pursues. Furthermore, he

wishes to obtain a good working knowledge of the subjects treated in the shortest time and in the most direct manner possible.

In meeting these requirements, we have produced a set of books that in many respects, and particularly in the general plan followed, are absolutely unique. In the majority of subjects treated the knowledge of mathematics required is limited to the simplest principles of arithmetic and mensuration, and in no case is any greater knowledge of mathematics needed than the simplest elementary principles of algebra, geometry, and trigonometry, with a thorough, practical acquaintance with the use of the logarithmic table. To effect this result, derivations of rules and formulas are omitted, but thorough and complete instructions are given regarding how, when, and under what circumstances any particular rule, formula, or process should be applied; and whenever possible one or more examples, such as would be likely to arise in actual practice —together with their solutions—are given to illustrate and explain its application.

In preparing these textbooks, it has been our constant endeavor to view the matter from the student's standpoint, and to try and anticipate everything that would cause him trouble. The utmost pains have been taken to avoid and correct any and all ambiguous expressions—both those due to faulty rhetoric and those due to insufficiency of statement or explanation. As the best way to make a statement, explanation, or description clear is to give a picture or a diagram in connection with it, illustrations have been used almost without limit. The illustrations have in all cases been adapted to the requirements of the text, and projections and sections or outline, partially shaded, or full-shaded perspectives have been used, according to which will best produce the desired results. Half-tones have been used rather sparingly, except in those cases where the general effect is desired rather than the actual details.

It is obvious that books prepared along the lines mentioned must not only be clear and concise beyond anything

heretofore attempted, but they must also possess unequaled value for reference purposes. They not only give the maximum of information in a minimum space, but this information is so ingeniously arranged and correlated, and the indexes are so full and complete, that it can at once be made available to the reader. The numerous examples and explanatory remarks, together with the absence of long demonstrations and abstruse mathematical calculations, are of great assistance in helping one to select the proper formula, method, or process and in teaching him how and when it should be used.

This volume is the third of a series of three devoted to civil-engineering topics. The subjects treated are astronomy, drainage, sewerage, water supply and distribution, and irrigation. The paper on Astronomy contains information of direct value to every surveyor. The papers on Drainage and Sewerage treat the subjects from the standpoint of the municipal engineer. The papers on Water Supply and Distribution were written by a well-known hydraulic engineering expert and author, and contain the results of a long, varied, and successful experience in this important branch of civil engineering; the paper on Irrigation was likewise prepared by him.

The method of numbering the pages, cuts, articles, etc. is such that each subject or part, when the subject is divided into two or more parts, is complete in itself; hence, in order to make the index intelligible, it was necessary to give each subject or part a number. This number is placed at the top of each page, on the headline, opposite the page number; and to distinguish it from the page number it is preceded by the printer's section mark (§). Consequently, a reference such as § 37, page 26, will be readily found by looking along the inside edges of the headlines until § 37 is found, and then through § 37 until page 26 is found.

INTERNATIONAL TEXTBOOK COMPANY.

CONTENTS

DESCRIPTIVE ASTRONOMY.

INTRODUCTION.

GENERAL ASTRONOMY.

SUBJECT MATTER.

1. Astronomy is the science that treats of the heavenly bodies. It investigates their real and apparent motions, and determines the laws which control those motions. It measures their dimensions, distances, and masses. It considers their nature and physical constitution. It deals with their relations to each other, and the effects which they produce upon each other by their mutual action. The science that covers this wide range is called **General Astronomy**; it is one of the most interesting, and at the same time most complex of modern sciences.

DIVISIONS OF THE SCIENCE.

2. It is convenient to divide the science of astronomy into three branches.

3. The first branch is usually called **Descriptive Astronomy.** It consists of an orderly statement of astronomical facts ascertained by systematic observation, and of the astronomical principles theoretically derived from those facts.

4. The second branch may be called **Gravitational Astronomy.** It is the application of dynamical principles to account for the motions of the heavenly bodies.

5. The third branch is **Physical Astronomy,** which treats of the physical condition, chemical constitution, and temperature of the heavenly bodies.

6. In this Paper we shall deal exclusively with Descriptive Astronomy. In order to present the facts in the simplest possible manner, it is necessary to explain a few fundamental properties of the sphere which are of continual application in astronomy.

THE SPHERE.

DEFINITIONS.

7. In geometry a sphere is defined as a solid bounded by a uniformly curved surface every point of which is equidistant from a point within, called the center. The surface of such a solid is properly called a **spherical surface;** but for the sake of brevity a spherical surface is usually called a **sphere,** just as the word circle is often used instead of the longer word circumference. When a spherical surface is called a sphere, the solid itself is called a **globe.**

8. A **radius** of a sphere is a straight line drawn from the center to the surface. A straight line passing through the center and terminated at both ends by the surface is called a **diameter** of the sphere.

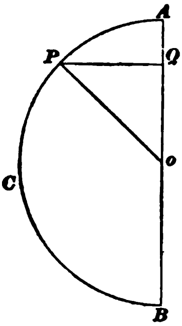


FIG. 1.

9. *A sphere may be generated by the revolution of a semicircle about its diameter.*

For, if the semicircle $A\ C\ B$, Fig. 1, is turned about its diameter $A\ B$, its center o remains fixed, and any point P on the semicircle is at a constant distance $o\ P$ from the center o. Consequently, during the revolution, every point on the semicircle lies on a sphere whose center is o and whose radius is $o\ P$.

10. Let Q (Fig. 1) be a fixed point in the diameter $A\ B$, and let $Q\ P$ be drawn perpendicular to $A\ B$. Then during

the revolution of the semicircle, $Q\ P$ lies always in the same plane, and the point P describes a circle whose center is Q.

Hence, *every plane section of a sphere is a circle.*

11. A section of a sphere made by a plane passing through the center is called a **great circle.** A section made by a plane which does not pass through the center is called a **small circle.** Thus, $A\ B\ A'\ B'$ and $B\ P\ B'\ P'$, Fig. 2, are great circles, because their planes pass through C, the center of the sphere; while $a\ b\ a'\ b'$ and $c\ c'$ are small circles, because their planes do not pass through C.

A great circle divides the sphere into two equal parts called **hemispheres.**

12. A straight line through the center of a great or small circle, and perpendicular to its plane, is called the **axis** of the circle. The points where the axis of a circle meets the sphere are called the **poles** of the circle. Thus, $P\ P'$ (Fig. 2) is the axis of the great circle $A\ B\ A'\ B'$ and of the small circle $a\ b\ a'\ b'$; the points P and P' are the poles of these circles.

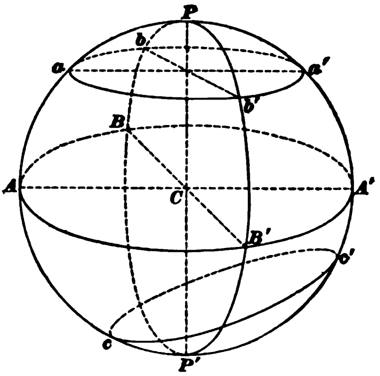


FIG. 2.

The axis of any circle is a diameter of the sphere; in other words, the axis of a circle must pass through the center of the sphere.

13. If any great circle of the sphere is taken as a **primary,** or **fundamental,** circle the great circles passing through its poles are called its **secondaries.** Thus, if $A\ B\ A'\ B'$ (Fig. 2) is taken as a primary circle, then $P\ B\ P'\ B'$, a great circle passing the poles $P\ P'$, is one of its secondaries.

It is evident that the plane of a great circle is perpendicular to the plane of each of its secondaries; hence, it follows that *if one circle is a secondary to another, the latter is also*

a secondary to the former. Thus, the circle $A\ B\ A'\ B'$ (Fig. 2) is a common secondary to the two circles $A\ P\ A'\ P'$ and $B\ P\ B'\ P'$.

14. The angle between two great circles is called a **spherical angle,** and is equal to the angle between their planes. The angle between two planes is measured by the angle between two lines drawn, one in each plane, perpendicular to the line in which the planes intersect; and evidently the angle between two great circles can be measured in the same way. Thus, the angle between the great circles $A\ P\ A'\ P'$ and $B\ P\ B'\ P'$ (Fig. 2) is measured by the angle $A'\ C\ B$. Now the angle $A'\ C\ B$ is measured by its intercepted arc $A'\ B$. Hence, the angle between the circles $A\ P\ A'\ P'$ and $B\ P\ B'\ P'$ is measured by the arc $A'\ B$ which they intercept on their common secondary $A\ B\ A'\ B'$.

Whence we conclude that *the angle between two great circles is measured by the arc which they intercept on their common secondary.*

15. The **angular distance between two points on a sphere** is measured by the arc of the great circle joining them, or by the angle which they subtend at the center of the sphere.

16. A **spherical triangle** is a portion of a sphere, bounded by three arcs of great circles.

17. **Parallel circles** of a sphere are those whose planes are parallel.

PROPERTIES OF SPHERICAL CIRCLES.

18. *Every point on a circle of a sphere is at a constant angular distance from either of the poles of the circle.*

For, during the revolution of the generating semicircle (Fig. 1), the arc $A\ P$ remains constant, and A is the pole of the circle described by the point P.

19. The constant angular distance of a point on a circle of a sphere from its adjacent pole is called the **angular radius** of the circle.

The angular radius of a small circle is less than a quadrant, and the angular radius of a great circle is a quadrant, or 90°.

20. The shortest distance, measured on the surface, between two points on a sphere is the arc of the great circle joining the two points.

21. Through two points on a sphere, which are not the extremities of a diameter of the sphere, one and only one great circle can be described.

22. Through two points on a sphere, which are not the extremities of a diameter of the sphere, an infinite number of small circles can be described.

23. In plane geometry the straight line joining two points is the shortest distance between those points, and through two given points one and only one straight line can be drawn.

Thus the great circle in spherical geometry possesses properties analogous to those of the straight line in plane geometry. A secondary to a great circle corresponds to a perpendicular to a straight line.

POSITION OF A POINT ON A SPHERE.

24. Let $A\,O\,A'$, Fig. 3, be a fixed great circle whose axis is $P\,P'$, and let O be a fixed point on this circle.

If we conceive the sphere to be generated by the revolution of a semicircle about its diameter $P\,P'$, it is evident that during a complete revolution the generating semicircle passes once and only once through every point on the sphere. Let $P\,X\,B\,P'$ be the position of the generating semicircle in which it passes through the point X. This position of the

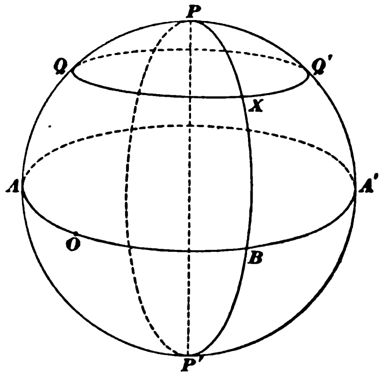


FIG. 3.

generating semicircle may be fixed by measuring the arc $O B$. Let $Q X Q'$ be a plane parallel to the plane $A O A'$; then evidently the plane $Q X Q'$ and the semicircle $P X P'$ can intersect only in the one point X. Hence, the position of the point X can be fixed by fixing the position of the plane $Q X Q'$ and the position of the semicircle $P X P'$. We have seen that the position of the semicircle $P X P'$ is fixed by measuring the arc $O B$. The position of the plane $Q X Q'$ can be fixed by measuring how far it is above or below the plane $A O A'$; manifestly if we measure the arc $B X$, it will fix the height of the plane $Q X Q'$ above the plane $A O A'$.

Thus, *the position of the point X may be fixed by measuring the arcs O B and B X.*

Suppose that X and X' are two points on a sphere, the position of X being fixed by the arcs $O B$ and $B X$, while the position of X' is fixed by the arcs $O B'$ and $B' X'$. Then, if $O B$ is equal to $O B'$, the points X and X' lie on the same circle passing through P and P'; that is, on the same secondary to $A O A'$. If $B X$ is equal to $B' X'$, the points X and X' are on the same small circle parallel to $A O A'$.

The position of a point, then, is fixed by specifying : (1) which of the secondaries to a fixed circle $A O A'$ it lies upon and which of the halves of that secondaries it lies upon; and (2) which of the parallels to $A O A'$ it lies upon.

The method here described is employed for fixing the position of a place on the earth's surface, and for fixing the position of a celestial body.

25. The **complement** of an angle or of an arc is the remainder obtained by subtracting it from 90°. Thus, in Fig. 3, the arc $X P$ is the complement of the arc $B X$.

It is evident that the position of the point X may be fixed by measuring the arcs $O B$ and $P X$, for this is equivalent to measuring $O B$ and $B X$.

26. The **supplement** of an angle or of an arc is the remainder obtained by subtracting it from 180°. Thus, in Fig. 3, the arc $P' X$ is the supplement of the arc $X P$.

THE TERRESTRIAL SPHERE.

27. The figure of the earth is approximately spherical, or globular; and except in special investigations we may regard the earth as a globe.

28. Every day the sun appears to move across the sky; rising every morning in the one side, and setting every evening in the other side. The part of the earth towards the sunrise is called the **east,** and the part towards the sunset is called the **west.**

29. Real and Apparent Motion.—If one is sitting in a moving train and looking out upon the landscape, the trees and other objects appear to be rushing past in the direction opposite to that in which the train is moving. If one is sitting in one of two trains which are standing upon parallel tracks, and one of the trains begins to move very slowly and smoothly, it is almost impossible to determine which of the trains is in motion. These illustrations are sufficient to show that when one observes the apparent motion of any body, this apparent motion may not be the real motion of that body.

30. Relative Motion.—Suppose that A (Fig. 4) is a fixed point on a horizontal table, and that a small body B is moving along the table with a velocity represented by $B\,P$. It is evident that the motion of B relative to A will not be changed by any motion that may be given to the table. Let the table be moved along the floor with a velocity equal and opposite to the velocity of B; this will impart to A a motion along the line $A\,Q$ parallel to $B\,P$. The effect of moving the table is to bring the body B to rest, while the point A has a velocity imparted to it equal and opposite to the velocity $B\,P$. Thus, as far

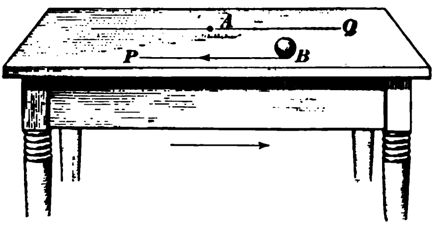


FIG. 4.

as the relative motion of A and B is concerned, it is indifferent whether A is at rest and B moving with a velocity represented by BP, or B is at rest and A moving in the line AQ with a velocity equal and opposite to BP.

This is the reason why a passenger in one of two trains on parallel tracks can not determine which of them is moving, for their relative motion will be the same whether the one, or the other, or both, move. To determine which is moving, we must compare them with objects external to both.

31. The ancients believed that the apparent daily motion of the sun from east to west was a real motion. It is now known that this apparent motion of the sun is really due to a daily rotation of the earth about one of its diameters; since the sun's apparent motion is from east to west, the earth's rotation, in accordance with the principle of Art. **30,** must be from west to east.

32. The diameter about which the earth performs its daily rotation is called the earth's **axis,** and its extremities are called the earth's **poles.**

If one stands with his right hand towards the east and his left hand towards the west, the pole of the earth towards which his face is directed is called the **north pole,** and the opposite pole is called the **south pole.**

If the face of a watch is turned northwards, the apparent motion of the sun is in the direction in which the hands of the watch turn; and the real rotation of the earth is opposite to the direction in which the hands of the watch turn, that is, **counter-clockwise.** With few exceptions, the rotations and revolutions of the heavenly bodies are counter-clockwise.

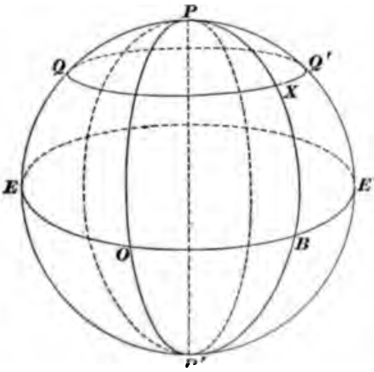


FIG. 5.

33. The great circle EBE' (Fig. 5), whose plane is perpendicular to PP', the

[illegible]

[illegible] selected [illegible] near [illegible] meridian.

34. [illegible] in Art. **24.** [illegible] equal [illegible]

35. [illegible] the prime meridian [illegible] place [illegible] Fig. [illegible] passing [illegible]

Longitude is measured [illegible] through [illegible] prime meridian [illegible] E [illegible] this is [illegible] T [illegible]

36. [illegible] **latitude** [illegible]

37. All places that lie on the same small circle parallel to the equator have the same latitude; hence, small circles parallel to the equator are called **parallels of latitude.**

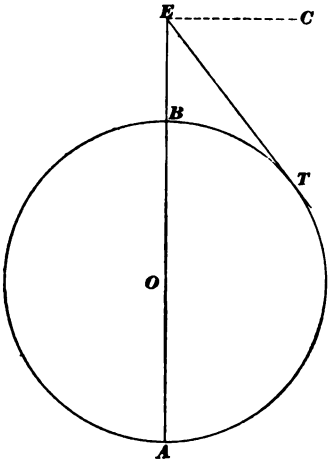


FIG. 6.

38. Let E, Fig. 6, be the position of an observer's eye at a point above the earth's surface, and let $A\ T\ B$ be a section of the earth through its center O and the point E, and let $E\ T$ be the tangent from E to the circle $A\ T\ B$. Then the portion of the earth's surface visible to the observer whose eye is at E is bounded by a small circle whose angular radius is $B\ T$. This small circle is called the **visible horizon** of the observer whose eye is at E.

If $E\ C$ is drawn perpendicular to $O\ E$, then the angle $C\ E\ T$ is called the **dip** of the visible horizon.

THE CELESTIAL SPHERE.

39. To an observer of the heavens at night, the celestial bodies appear to be bright points attached to the inner surface of a vast hollow spherical dome, whose center is at the observer's eye.

A little reflection, however, is sufficient to establish the fact that the heavenly bodies are not all equidistant from the observer's eye, and are not attached to any surface, spherical or otherwise. Indeed, we have no direct means of estimating the distances of these bodies; all we can directly observe is their relative directions. Most astronomical instruments determine merely the relative directions of the heavenly bodies. It is very important, therefore, to have a convenient mode of representing these relative directions.

40. We may imagine a vast spherical surface to be described enclosing all the heavenly bodies, and having its center at the observer's eye. This imaginary surface is called the **celestial sphere,** and circles are imagined to be drawn on it as parallels of latitude and meridians of longitude are drawn on the terrestrial sphere. By reference to these circles of the celestial sphere, we can describe the positions and motions of the heavenly bodies.

The celestial sphere may be taken as the apparent vault on which the heavenly bodies appear to lie; but it must be borne in mind that these bodies do not lie on a sphere; and all we can do by means of the celestial sphere is to fix their relative directions.

41. Let O, Fig. 7, be the position of the observer's eye, and consequently the center of his celestial sphere A' B' C' D'. Let A, B, C, and D be any heavenly bodies. Imagine the lines $O\ A$, $O\ B$, $O\ C$, and $O\ D$ drawn and produced to meet the celestial sphere in the points A', B', C', and D'. The apparent positions of the bodies A, B, C, and D depend only on their directions, and are independent of their distances from O. Therefore the positions of A, B, C, and D as they appear to the observer at O are correctly represented by the points A', B', C', and D' on the celestial sphere.

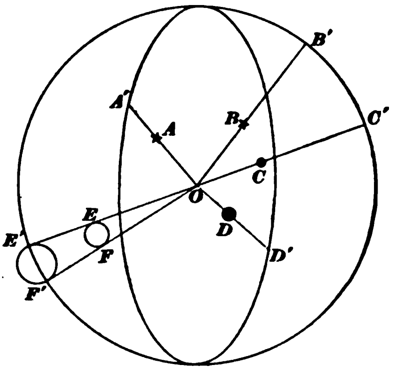


Fig. 7.

42. Angular Distance.—Let B' C' (Fig. 7) be a great circle of the celestial sphere. Then the arc B' C' is measured by the angle B' O C', or B O C. Hence, the arc B' C' of the celestial sphere, or the angle B O C, is called the **angular distance** between the bodies B and C, and is usually expressed in degrees, minutes, and seconds. The

angular distance must not be confused with the actual linear distance $B\ C$. If we know the angular distance $B\ O\ C$, we would require also to know the distances $O\ B$ and $O\ C$ before we could determine the linear distance $B\ C$.

43. Angular Magnitude.—If $E\ F$ is the diameter of a distant globe, such as the sun or moon, the angle $E\ O\ F$ is called its **angular diameter.** This angular diameter is measured by the arc $E'\ F'$ of the celestial sphere.

44. Relation Between Distance and Apparent Size.—Let $A\ B$, Fig. 8, be the radius of a globe, and let o

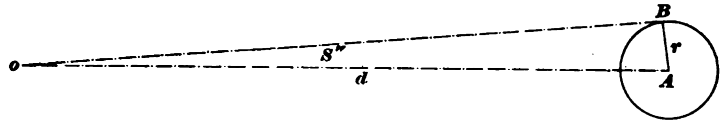


FIG. 8.

be the position of the observer's eye. Then, the angle $A\ o\ B$ is the angular semidiameter of the globe as seen from o. If the angle $A\ o\ B$ contains S seconds, we have (*Geometry and Trigonometry*, Art. **754**)

$$\sin A\ o\ B = \sin S'' = \frac{r}{d}. \qquad (a)$$

Now, if the angle $A\ o\ B$ is very small, it can be proved that

$$\sin S'' = \frac{S}{206,265} \text{ (very nearly)}.$$

Whence, $$S = 206,265 \frac{r}{d}. \qquad (b)$$

Hence, *the angular semidiameter of any body varies directly as its linear semidiameter* (r), *and inversely as its distance* (d).

45. Center of the Celestial Sphere.—If we make a dot with a pencil to represent the center of a circle drawn on paper, that dot is not a true mathematical point, but has some size; yet the magnitude of the dot is so small compared to the magnitude of the circle, that we may, for all practical purposes, regard any mathematical point covered by the

dot as the center of the circle. In like manner the diameter of the earth is utterly insignificant in comparison with the distances of most of the heavenly bodies; and therefore the size of the earth is insignificant in comparison with the size of the celestial sphere which encloses all the heavenly bodies. Hence, any point in the earth may be regarded as the center of the celestial sphere. For many purposes it is convenient to consider the center of the celestial sphere to be at the center of the earth, as in Fig. 9.

It is instructive to look at this matter from another point of view. If we observe the relative positions of several tall trees or tall chimneys which are two or three miles off, and then move a few feet from our first post of observation and observe the relative positions of the trees or chimneys again, we shall be unable to detect any change in their relative positions. Similarly most of the heavenly bodies are so far distant that no change in their relative positions can be detected when their positions are observed from different parts of the earth.

Now the use of the celestial sphere is simply to fix the relative positions of the heavenly bodies, as seen from its center. Hence, for those heavenly bodies whose distances are so great that their apparent directions are the same at all points of the earth, it is a matter of indifference which point of the earth is taken as center of the celestial sphere.

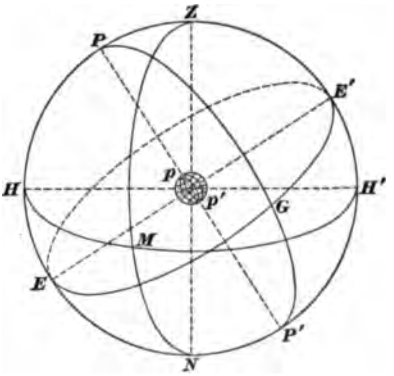


FIG. 9.

46. It must be remembered, however, that the sun, moon, and several other heavenly bodies are nearer to the earth than the great multitude of bodies of which we have been speaking. We shall subsequently see that the relative directions of these nearer bodies depend upon the point from which the observations are made.

47. The **axis of the celestial sphere** PP' is the prolongation of the axis $p\ p'$ of the terrestrial sphere, as shown in Fig. 9.

The points P and P' where the axis intersects the celestial sphere are called the **celestial poles.** The point Z of the celestial sphere which is directly over the observer's head is called the observer's **zenith;** the point N which is directly opposite to the zenith is called the **nadir.**

✓ **48. Diurnal Motion of the Heavens.**—The heavenly bodies appear to rise in the east and to set in the west. By attentive observation it is found that most of the heavenly bodies move uniformly, and make a complete revolution in the same period; this period is about 23 hours and 56 minutes of our ordinary clock time, and is called a **sidereal day.** The sidereal day is divided into 24 sidereal hours, the sidereal hour into 60 sidereal minutes, and the sidereal minute into 60 sidereal seconds.

Each of these celestial bodies always describes the same small circle, and these small circles all have their centers at the same fixed point of the celestial sphere. Since this apparent motion is due to the rotation of the earth on its axis, the fixed center of these small circles is the pole of the celestial sphere. This apparent motion is called the **diurnal motion** of the heavens.

49. Those heavenly bodies which appear to move uniformly in small circles about the celestial pole, and which preserve their relative positions unchanged, are called **fixed stars,** or simply **stars.** It is not to be supposed, however, that the stars are absolutely fixed, but simply that they are so far away that any motion they may have can not be detected by ordinary observations.

50. There are certain heavenly bodies which are seen to be continually changing their position relatively to the other heavenly bodies. The ancients called such bodies **wanderers,** and included among them the sun and the moon.

It is now known that the apparent motion of the sun is due to the real motion of the earth, which performs a circuit about the sun in a year. Therefore the sun is now ranked as a fixed star.

51. A body, like the earth, which performs a circuit about the sun is called a **planet.** A smaller body, like the moon, which revolves about a planet is called a **satellite** of the planet. The sun, planets, and satellites constitute what is called the **solar system.**

52. When viewed through a telescope, a planet shows a circular disk, like that presented to the naked eye by the moon. The fixed stars, except the sun, are so far away that even in the most powerful telescopes they present no disk, but appear merely as a twinkling point. A fixed star appears brighter, but not larger, when viewed through a telescope than when viewed with the naked eye.

53. When one body revolves about another, the path of the revolving body is called its **orbit.** The line joining the center of the revolving body to the center of the body about which it revolves is called the **radius vector.**

54. The time occupied by a revolving body in making a complete revolution is called its **periodic time.**

55. In the apparent diurnal motion of the heavens, the nearer a star is to the pole, the smaller is the circle described by the star. If there were a star exactly at the pole, this star would have no apparent daily motion ; there is, however, no star exactly at either of the celestial poles. Fortunately, the position of the north pole of the heavens is very conveniently indicated by the **pole star** (Polaris), which is now only about $1\frac{1}{4}°$ from the north pole.

POSITION OF A CELESTIAL BODY.

56. As explained in Art. **24,** the position of a point on a sphere is fixed by measuring an arc of a fixed great circle and an arc of a secondary to that fixed great circle. Any fixed great circle and its secondary constitute a **system** of

circles of the sphere; and any such system can be used to define the position of a point on the sphere.

In every system of circles there is a set of small circles parallel to the primary circle of the system; these small circles are called **parallels.**

57. For the purpose of locating a point on the celestial sphere there are three systems of circles in common use; each of these circles is named after its primary circle. These three systems are: (I) the Horizon System, (II) the Equinoctial System, (III) the Ecliptic System.

TABLE.

CIRCLES AND POINTS OF THE CELESTIAL SPHERE.

	Horizon System.	Equinoctial System.	Ecliptic System.
Primary.	Rational Horizon.	Equinoctial, or Celestial Equator.	Ecliptic.
Secondaries.	Verticals.	Hour-Circles.	Circles of Celestial Longitude.
Secondaries Having Special Names.	Prime Vertical. The Meridian.	Equinoctial Colure. Solstitial Colure.	
Points.	Zenith. Nadir.	Celestial Poles. Equinoctial Points.	Poles of the Ecliptic. Equinoctial Points. Solstitial Points.
Measurements.	{ Altitude. } { Zenith Distance. } { Azimuth. } { Amplitude. }	{ Right Ascension. } { Hour-Angle. } { Declination. } { Polar Distance. }	Celestial Longitude. Celestial Latitude.
Parallels.		Declination Parallels.	Parallels of Celestial Latitude.

58. The accompanying table shows, for each of the three systems: (1) the primary circle, (2) the secondary circles, (3) fixed secondaries which have special names, (4) the points belonging to the system which have received special names, (5) the measurements by which the position of a point is determined in that system, (6) the parallels.

The definitions and explanations of the terms are given in the succeeding articles.

In any system only two measurements are required to fix the position of a point; when more than two measurements for any system are given in the table, those connected by braces { } are alternative measurements.

THE HORIZON SYSTEM.

59. The primary circle of this system is the **rational horizon,** which is the great circle of the celestial sphere

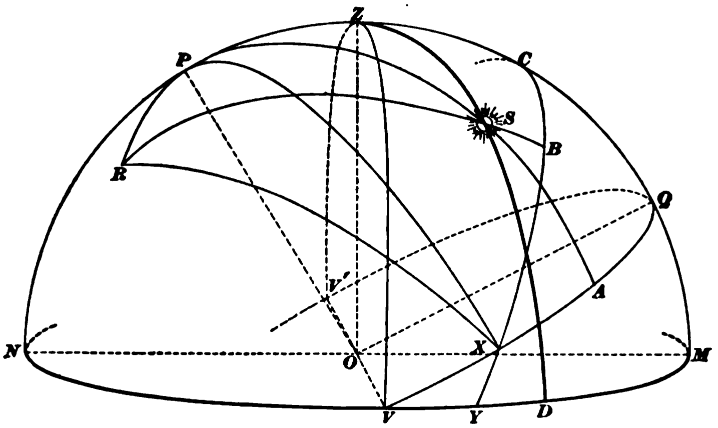


FIG. 10.

that separates the visible from the invisible heavens. The zenith and nadir are the poles of the rational horizon. In Fig. 10, *P* is the celestial pole, *Z* is the zenith, and *N V M* is the rational horizon.

60. Secondaries to the rational horizon are called **verticals.** Since verticals are secondaries to the rational

horizon, they must all pass through the zenith and nadir, which are the poles of the rational horizon.

61. Any great circle passing through the celestial poles is called a **celestial meridian.** Of these meridians the one $N P Z M$ which passes through the zenith is called the **meridian;** it is evidently a vertical and a secondary to the horizon, because it passes through the zenith. The meridian passes through the **north** and the **south point** of the horizon. If P is the north pole, N is the north point, and M is the south point.

62. The **prime vertical** is the vertical circle at right angles to the celestial meridian; hence, the prime vertical passes through the **east** and the **west** point of the horizon. In Fig. 10, $V' Z V$ is the prime vertical.

63. The **altitude** of a celestial body is its angular distance from the horizon, and is measured along the vertical passing through the body. Thus, the arc $D S$ is the altitude of the body S. (Fig. 10.)

64. The **zenith distance** of a celestial body is the distance from the body to the zenith, and is measured on the vertical passing through the body. The zenith distance is the complement of the altitude. Thus, $S Z$ is the zenith distance of the body S, and is the complement of the altitude $S D$.

65. The **azimuth** of a celestial body is the arc of the horizon intercepted between the north or south point and the vertical through the body. Thus, $M D$ (Fig. 10) is the azimuth of the body S. In expressing the azimuth of a body, it is necessary to state whether it is measured from the north or south point, and whether it is measured east or west. For example, if the azimuth of a body is 35° measured from the north towards the east, it would be written as N 35° E.

66. The **amplitude** of a celestial body is the complement of its azimuth, and is measured along the horizon north

or south from the prime vertical. Thus, in Fig. 10, $V D$ is the amplitude of the body S.

67. In the horizon system the position of a celestial body is determined when its altitude (or its zenith distance) and its azimuth (or its amplitude) are known.

THE EQUINOCTIAL SYSTEM.

68. The primary circle of this system is the **celestial equator,** which is the great circle in which the plane of the earth's equator intersects the celestial sphere. The celestial equator is also called the **equinoctial circle,** or simply the **equinoctial,** because, when the sun is in the plane of the equator, the days and nights are of equal length all over the earth.

The poles of the equator coincide with the poles of the celestial sphere.

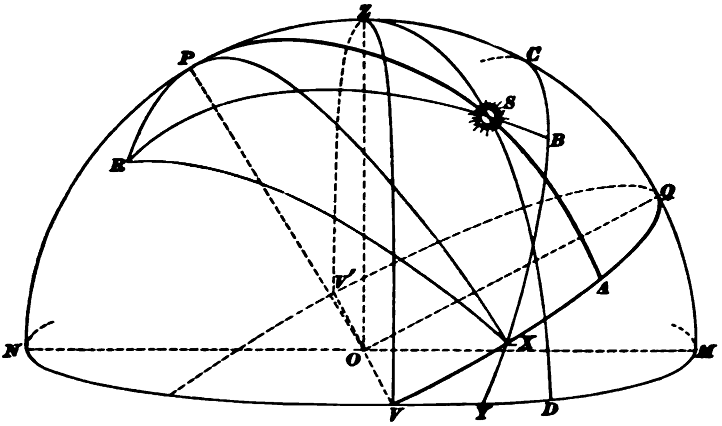


FIG. 11.

In Fig. 11, P is the pole of the celestial sphere, and $V X Q V'$ is the celestial equator, or equinoctial.

69. The meridian $N P Z M$ is a secondary to the equator, because it passes through P, the pole of the equator. By Art. **61,** the meridian is also a secondary to the horizon.

Thus, the meridian is a common secondary to the equator and horizon. From Art. **14** we know that the angle between two great circles is measured by the arc which they intercept on their common secondary. Therefore the angle between the equator and the horizon is measured by the arc QM.

Again, by Art. **19**, the angular radius of a great circle is 90°; whence,

$$PQ = 90°, \text{ or } PZ + ZQ = 90°,$$

and $$ZM = 90°, \text{ or } ZQ + QM = 90°.$$

Therefore, $$PZ + ZQ = ZQ + QM.$$

Hence, $$PZ = QM.$$

That is, *the inclination of the horizon to the equator is equal to the distance of the zenith from the pole.*

70. Since the celestial meridians (Art. **61**) pass through the celestial poles, they are secondaries to the celestial equator. It is also evident that the planes of the celestial meridians coincide with the planes of the terrestrial meridians.

71. Let PSA be the meridian passing through a star S; then as the star, in its apparent diurnal motion, describes uniformly a small circle about the pole, the point A will move uniformly round the equator. The point A makes a complete circuit of the equator in 24 sidereal hours. Using sidereal hours, minutes, and seconds, we find that

A moves 360° in 24 hours.

Whence, A moves $\frac{1}{24}$ of 360° in 1 hour.

Or, A moves along the equator at the rate of 15° in every hour. Hence, the length of any arc of the equator is equivalent to a certain period of time, the relation between the arc and the time being

15° of arc = 1 hour of time,
15′ of arc = 1 minute of time,
or 15″ of arc = 1 second of time.

For this reason celestial meridians are also called **hour-circles**, and this is the more usual name.

72. The angle at the pole between the meridian and the hour-circle passing through a star is called the **hour-angle** of the star.

In Fig. 11, if P is the north pole, V is the west point and V' is the east point. Hence, the star S, in its diurnal motion, has already passed the meridian between P and Q; as the star moves from the meridian to the position S, the hour-circle $P S A$ sweeps out the hour-angle $Q P A$. The angle $Q P A$ is the angle between the two great circles $P Z Q$ and $P S A$, and is measured by the arc $Q A$. (Art. **14.**) Therefore, the time that has elapsed since the star was in the meridian is measured by the hour-angle $Q P A$, or by the arc $Q A$ converted into time at the rate of 15° to the hour.

73. Suppose we have a star globe rotating about an axis parallel to the axis of the celestial sphere and making a complete rotation in the period in which the stars make a complete diurnal revolution about the earth. The relative positions of the heavenly bodies can be represented on this globe by marking points on its surface whose directions from its center correspond to the directions of the heavenly bodies. Night after night and year after year it is found that the positions of the stars are represented by the same points on this globe.

During the day let the position of the sun be marked on the globe. By repeating this process day after day we can map out the sun's path among the stars.

It is found that the sun moves from *west to east* and returns to the same position among the stars at the end of a year. The trace of the sun's path on the celestial sphere is found to be a great circle; this great circle is called the **ecliptic.** In Fig. 11, $X B C$ is the ecliptic and R is its pole.

74. The motion of the sun which we have just described is apparent only, and is due to the real motion of the earth in its orbit about the sun.

Let S (Fig. 12) represent the sun and E the earth. Let $A\ B\ C$ be the path which the sun appears to describe about the earth. In this figure the north pole is supposed to be above the plane of the paper. When the sun is at S, it appears to be moving in the direction $S\ T$. Hence, the sun appears to describe its path in the direction $A\ B\ C$, that is, counter-clockwise. This apparent motion of the sun being due to the real motion of the earth, when the sun is at S and the earth at E, the earth must be moving in the direction $E\ U$; hence, the earth describes its orbit in the direction $D\ E\ F$, that is, counter-clockwise. Thus, the real motion of the earth about the sun takes place in the same direction as the apparent motion of the sun about the earth.

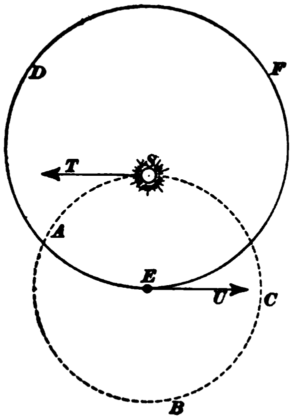


FIG. 12.

75. The points where the ecliptic intersects the celestial equator are called the **equinoctial points** or the **equinoxes.** The point where the sun crosses the celestial equator in passing from the southern to the northern hemisphere is called the **vernal** or spring equinox; the point where the sun crosses the equator in passing from the northern to the southern hemisphere is called the **autumnal** equinox. In Fig. 11, X is the vernal equinox; the autumnal equinox is the point diametrically opposite to X and is not shown in the figure.

The equinoxes are, strictly speaking, not the points, but the times when the sun is at the equinoctial points. The vernal equinox occurs about the 21st of March, and the autumnal equinox occurs about the 21st of September.

76. The points of the ecliptic which are 90° distant from the equinoxes are called the solstitial points, or the

solstices, because at those points the sun stands, or stops moving northwards or southwards from the equator.

The solstices are more correctly defined not as points, but as the times when the sun is at the solstitial points. The summer solstice occurs about the 21st of June, and the winter solstice about the 21st of December.

77. The **equinoctial colure** is the meridian passing through the equinoxes. The **solstitial colure** is the meridian passing through the solstices; the solstitial colure is a common secondary to the ecliptic and equator.

78. The **right ascension** of a celestial body is the arc of the celestial equator measured **eastwards** from the vernal equinox to the hour-circle passing through the body. Thus, in Fig. 11, XA is the right ascension of the body S.

79. The **declination** of a heavenly body is its angular distance north or south from the celestial equator, and is measured by the arc of the hour-circle passing through the object and intercepted between it and the equator. The declination of a heavenly body is *north* if the body is north of the equator, and the declination is *south* if the body is south of the equator.

80. The **polar distance** of a heavenly body is its distance from the nearer pole, and is measured by the arc of the hour-circle intercepted between the pole and the body. The polar distance is, therefore, the complement of the declination.

81. **Parallels of declination** are small circles parallel to the celestial equator.

82. In this system the position of a star is determined by its hour-angle and polar distance, or by its right ascension and declination.

The right ascension of a point on the celestial sphere corresponds exactly to the longitude of a place on the terrestrial sphere. Right ascension is reckoned from the

vernal equinox, just as terrestrial longitude is reckoned from the Greenwich meridian. The declination of a point on the celestial sphere corresponds to the latitude of a place on the terrestrial sphere, and declination parallels correspond to parallels of latitude.

Unfortunately the older astronomers did not use the equinoctial system, and they employed the terms celestial latitude and celestial longitude in connection with the ecliptic system. Consequently, when modern astronomers began to use the equinoctial system, they found the words celestial latitude and celestial longitude already monopolized; and therefore they had to adopt the new terms declination and right ascension.

THE ECLIPTIC SYSTEM.

83. The primary circle of the ecliptic system is the ecliptic (Art. **73**). The angle between the ecliptic and the celestial equator is called the **obliquity of the ecliptic,** and is about 23° 27½′. The plane of the ecliptic coincides with the plane of the earth's orbit, and the plane of the celestial equator is the same as the plane of the terrestrial equator; hence, evidently the obliquity of the ecliptic is equal to the inclination of the earth's orbit to the earth's equator.

84. The angle between two great circles is measured by the arc which they intercept on their common secondary. (Art. **14.**) By Art. **77,** the solstitial colure is the common secondary to the ecliptic and equator; therefore, the obliquity of the ecliptic is measured by the arc of the solstitial colure intercepted between the ecliptic and the equator. At the summer solstice the sun stops moving northwards from the equator and begins to move southwards again towards the equator; at the summer solstice, therefore, the sun has attained its greatest distance north of the equator, that is, the sun's northerly declination is then greatest. At the winter solstice the sun's southerly declination is greatest. At the solstices the sun's declination is measured by the arc

of the solstitial colure intercepted between the ecliptic and the equator; wherefore, the sun's declination at the solstices is equal to the obliquity of the ecliptic. In other words, the sun's greatest declination is equal to the obliquity of the ecliptic.

85. Secondaries to the ecliptic are called **circles of celestial longitude.**

The **celestial longitude** of a star is the arc of the ecliptic measured eastwards from the vernal equinox to the circle of longitude passing through the star.

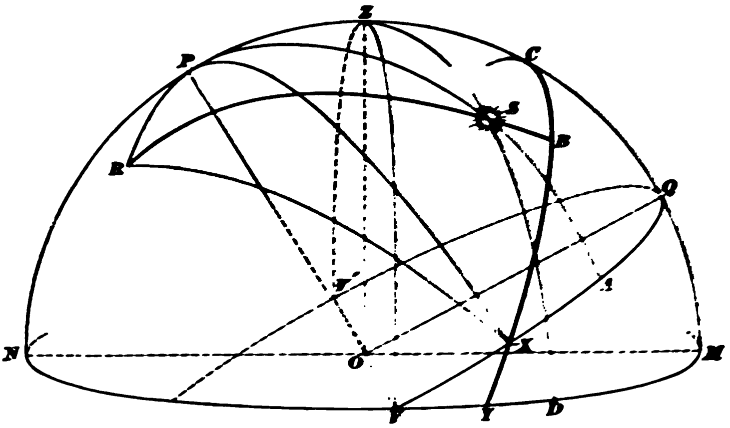


FIG. 13.

The celestial latitude of a star is its angular distance from the ecliptic measured along the circle of longitude which passes through the star.

In Fig. 13, *X B* is the celestial longitude and *B S* is the celestial latitude of the star *S*.

Thus, in this system the position of a celestial body is fixed by its celestial latitude and longitude.

COMPARISON OF THE THREE SYSTEMS.

86. The altitude and azimuth of a star serve to fix its position relative to the earth; but owing to the diurnal motion they are constantly changing.

The polar distance and hour-angle fix the position of a star relative to the earth. The advantage of this system is that for fixed stars the polar distance is constant, and the hour-angle increases at a uniform rate.

Since the vernal equinox and the equator have the same diurnal motion as the stars, the right ascension and declination of a fixed star are constant. For this reason right ascension and declination afford a convenient method of marking the relative positions of the stars on the celestial sphere.

The celestial latitude and longitude of a star are also unaffected by diurnal rotation. The sun is always in the ecliptic, and the moon and planets are always very near to it. Hence, celestial latitude and longitude are very convenient for tracing the paths of the sun, moon, and planets.

Celestial longitude differs from right ascension in that it is measured on the ecliptic instead of the equator, and in that it can not be measured in time, but only in degrees, minutes, and seconds. Before the invention of pendulum clocks, celestial longitude and latitude were the most convenient measurement by which to fix the relative positions of the stars. Since the invention of the pendulum clock, however, the equinoctial system has been found more convenient than the ecliptic system, because right ascension can at once be expressed in time.

ANCIENT AND MODERN ASTRONOMY.

ANCIENT THEORIES: THE PTOLEMAIC SYSTEM.

37. Having now explained the methods by which astronomers record and compare the observed positions of the heavenly bodies, we proceed to explain the astronomical facts and principles which have been derived from these records.

The ancients regarded the earth as the fixed center of the

universe, and conceived that the celestial bodies revolved about the earth in circular orbits. More careful observation, however, disclosed the fact that certain heavenly bodies did not move with this uniform circular motion round the earth. These bodies whose apparent motions are irregular were called *wanderers;* they included the sun, moon, and all the planets known to the ancients.

88. Ptolemy—a famous Greek astronomer and mathematician—was a teacher of mathematics at Alexandria during the first century of the Christian era. In order to account for the irregular motion of the wanderers, he supposed that each of them moved in a circular path, while the center of this circular path itself described a circle about the earth. In this way he was able to explain all the apparent motions of the heavenly bodies, as far as he was acquainted with them; but the system of astronomy which he built up was very complicated and lacking in simplicity. The system of astronomy which regarded the earth as the center of the universe is called the **Ptolemaic system,** and was taught in the schools until the beginning of the sixteenth century.

MODERN DISCOVERIES.

THE COPERNICAN SYSTEM.

89. Nicolaus Copernicus, who was born at Thorn on February 19, 1473, was the founder of the present system of astronomy, which recognizes the sun as the center of the solar system and explains that the apparent diurnal and annual motions of the heavens are caused by the rotation of the earth on its axis and the earth's revolution around the sun. Hence this system is called the **Copernican system.** Copernicus advocated his system [illegible] simpler explanation of the apparent [illegible] bodies than the old Ptolema[illegible] proof of the correctness [illegible] explain by means of it [illegible]

and his early followers held fast to the Copernican theory, and simply ignored those phenomena which they were unable to reconcile with it. By the labors of later mathematicians this theory has now been conclusively established.

90. In the latter half of the sixteenth century, Tycho Brahe, a Danish nobleman, established an astronomical observatory. He made many observations and accumulated a large store of valuable facts which yielded great results in the hands of his successors, though he himself made no advance in theoretical astronomy.

KEPLER'S LAWS.

91. About the year 1610, Kepler, the friend and pupil of Tycho Brahe, discovered three laws which govern the motions of the planets. He worked out these laws from the records of observations made by Tycho Brahe, but he discovered no explanation of them.

These laws, known as **Kepler's Laws,** are:

I. *The orbit of each planet is an ellipse, having the sun in one of its foci.*

II. *The radius vector joining the sun to the planet sweeps over equal areas in equal times.*

III. *The squares of the periodic times of the several planets vary as the cubes of their mean distances from the sun.*

92. In order that the student may fully understand the meaning and importance of these laws, we shall now explain each of them in detail.

93. An ellipse can be very conveniently constructed in the following manner: Tie the ends of a piece of fine inextensible string together so as to form a loop; place the loop over two pins fixed at the points S and S' (Fig. 14), place the point of the pencil in the loop, and move the pencil so as to keep the thread always stretched; the curve described by the pencil will be an ellipse having S and S' for its foci. Produce the line S' S to meet the curve in the

points A' and A, bisect $S'S$ in C, and through C draw a perpendicular to $S'S$, cutting the curve in the points B and B'. Then $A'A$ is called the **major axis**, and BB' the **minor axis** of the ellipse.

If the points S' and S are brought closer together, the curve becomes more nearly circular in shape; and if the

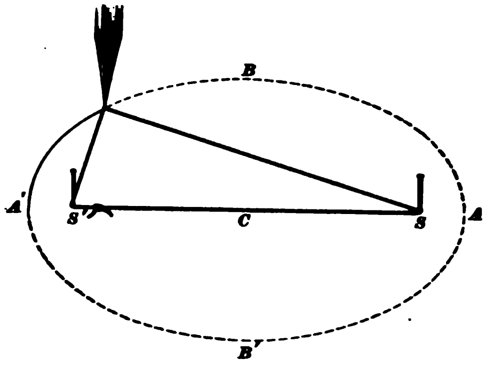


FIG. 14.

points S' and S actually coincide at C, the ellipse becomes a circle having C as its center. On the other hand, if the distance between S' and S is increased, the curve becomes flatter, until finally when the distance $S'S$ is half the length of the string forming the loop, the ellipse reduces to the straight line $S'S$ traced twice over.

Hence it appears that the shape of the ellipse depends upon the relation existing between the length of $S'S$ and the length of the string. Let the length of the string be denoted by l. When the pencil is at A, it is evident that

$$\tfrac{1}{2}l = S'S + SA. \qquad (a)$$

When the pencil is at A',

$$\tfrac{1}{2}l = A'S' + S'S. \qquad (b)$$

By addition of (a) and (b),

$$l = A'S' + S'S + SA + S'S = A'A + S'S.$$

Whence, $A'A = l - S'S.$

We have seen that the shape of the ellipse depends upon the relation between $S' S$ and l; and therefore, from the last equation, we conclude that the shape of the ellipse depends upon the relation between $S' S$ and $A' A$. The shape of the ellipse does not depend upon the actual lengths of $S' S$ and $A' A$, but only on their ratio. If we cut from paper an ellipse having $S' S$ equal to 4 inches, and $A'A$ equal to 6 inches, and also cut out another ellipse in which $S' S$ is 2 inches and $A' A$ is 3 inches; then if we hold the greater ellipse twice as far from our eye as the smaller ellipse, keeping their centers in the same straight line as our eye and their planes parallel, then the smaller ellipse will *exactly* conceal the larger one. That is, the two ellipses differ in size, but have the same shape.

Now, $$S' S = 2\, C S,$$

and $$A' A = 2\, C A.$$

Therefore, $$\frac{S'S}{A'A} = \frac{2\, C S}{2\, C A} = \frac{C S}{C A}.$$

But the shape of the ellipse depends upon the ratio $\frac{S' S}{A' A}$ and is therefore determined by the equal ratio $\frac{C S}{C A}$.

94. The ratio $\frac{C S}{C A}$, which determines the shape of an ellipse, is called the **eccentricity** of the ellipse.

95. We have seen that as $S' S$ diminishes the ellipse becomes more and more nearly circular in form, and when $S' S$ becomes zero the ellipse becomes a circle. Since the eccentricity $\frac{C S}{C A}$ is equal to the ratio $\frac{S' S}{A\, A'}$, it follows that *the smaller the eccentricity the more nearly the ellipse approaches the circular form.*

96. In Art. **93** we saw that as $S' S$ increases the ellipse becomes flatter, and when $S' S$ is equal to $A' A$ the ellipse becomes a straight line described twice over. Evidently in

an ellipse $S' S$ can never be greater than $A' A$. Whence, we conclude

I. *The eccentricity of an ellipse can never be greater than unity.*

II. *The greater the eccentricity, the flatter does the ellipse become.*

97. In Kepler's first law nothing is said about the actual size of a planet's orbit; the law simply deals with the shape of the orbit.

98. If the ellipse in Fig. 14 represents the orbit of a planet, S being the focus occupied by the sun, the point A', where the planet is at its greatest distance from the sun, is called **aphelion,** and the point A, where the planet is nearest to the sun, is called **perihelion.** The line $A' A$ is the major axis of the ellipse, the line obtained by producing $A' A$ indefinitely in both directions is called the **line of apsides;** thus the major axis of the ellipse is a limited portion of the line of apsides.

99. Shape of the Earth's Orbit.—If we are considering the relative motions of the earth and sun alone, it is a matter of indifference whether we regard the earth as describing an ellipse having the sun in one of its foci, or regard the sun as describing an exactly equal ellipse having the earth in one focus. (Arts. **30** and **74.**) Hence, the same curve which represents the sun's apparent path about the earth will also represent the earth's actual path about the sun.

Take a point E (Fig. 15) to represent the position of the earth. From E draw the line $E P_0$ in the direction of the vernal equinox, which is the direction of the sun on the twenty-first of March, and is the point from which celestial longitude is measured. (Art. **85.**) The direction of the vernal equinox is marked thus ♈. Observe the sun's longitude on several days during the year, and draw lines through E, making angles with $E P_0$ equal to the observed longitudes of the sun; that is, make the angle $P_0 E P_1$ equal to the

sun's longitude at the time of the first observation, etc. Then the lines $E\,P_0$, $E\,P_1$, $E\,P_2$, etc., show the directions of the sun on those days.

On each of the days on which observations are made observe also the sun's angular diameter. (Art. **43.**) If the actual diameter of the sun were known, its distance

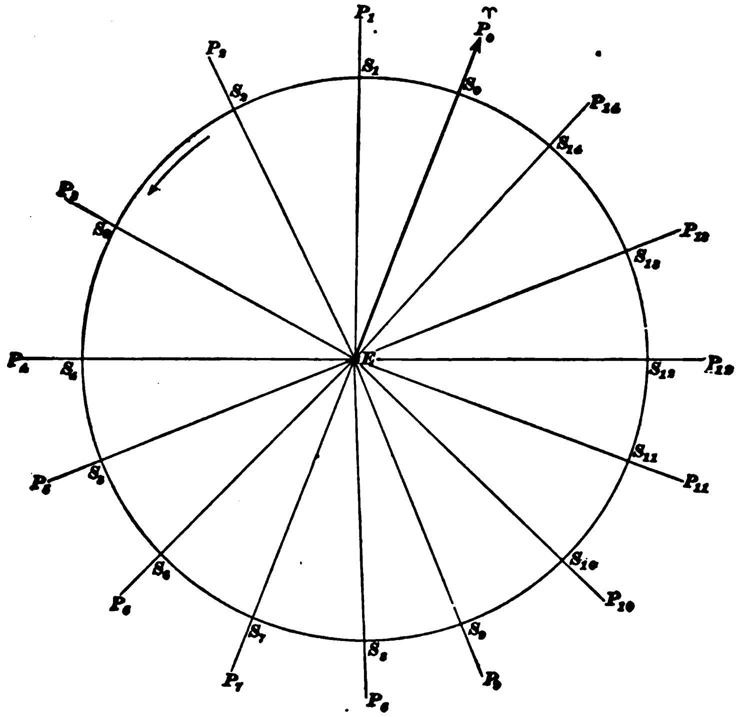


FIG. 15.

from the earth could be calculated from its observed angular diameter. (Art **44.**) When the sun's actual diameter is not known, the observations of its angular diameter do not determine its actual distance, but they do determine its relative distance on the different days. From Art. **44,** we know that the sun's angular diameter varies inversely as its distance; hence, if $E\,S_0$ is taken to represent the distance of the sun at the vernal equinox, the distances $E\,S_1$, $E\,S_2$,

etc., can be laid off to represent its distance on the other days to the same scale. The curve joining these points represents the sun's apparent path; but, since the sun's actual distance has not been determined, we can not mark the map with a scale of miles.

When the path of the sun is drawn in this way, it is found to be an ellipse whose eccentricity is about one-sixtieth; this eccentricity is so small that when the ellipse is accurately drawn it is impossible to distinguish it by the eye from a circle.

The earth's orbit, which is exactly the same size and shape as the apparent path of the sun, is represented in Fig. 16.

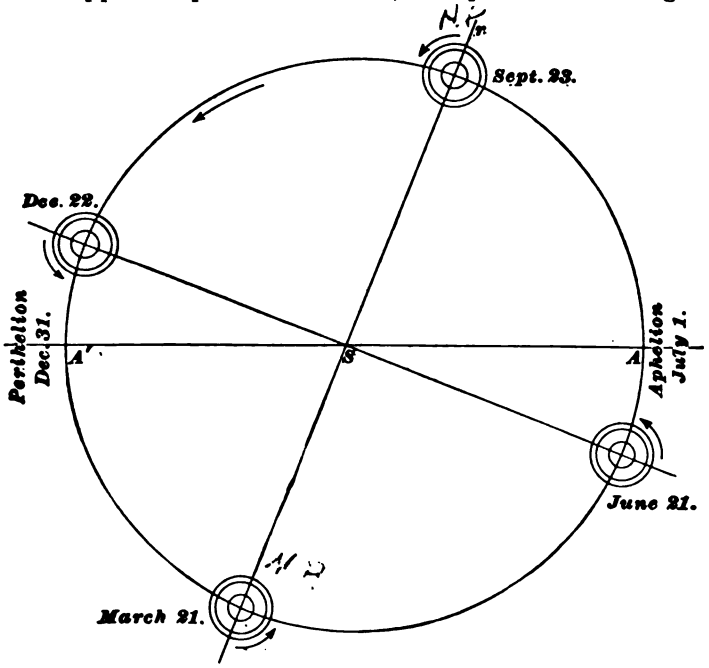


FIG. 16.

On March 21, the line from the earth to the sun points to the vernal equinox; consequently, at this date the line from the sun to the earth points to the part of the heavens diametrically opposite to the vernal equinox. Six months later,

on Sept. 23, the line from the sun to the earth points to the vernal equinox. In Fig. 16, P is the north pole of the earth, and is above the plane of the paper. The earth rotates in the direction indicated by the arrows.

100. Let the ellipse $A\ B\ C\ D\ E$, Fig. 17, represent a planet's orbit, having the sun at the focus S. Suppose the time occupied by the planet in passing from B to C is equal to the time taken in moving from D to E. It is found by observation that the distances $B\ C$ and $D\ E$ are not equal; that is, the planet does not move with the same velocity in all parts of its orbit. From the observations of Tycho Brahe, Kepler found that the area of the sector $S\ B\ C$ is equal to the area of the sector $S\ D\ E$; that is, the area swept over by the radius vector while the planet moves from B to C is equal to the area swept over by the radius vector while the planet moves from D to E, provided the time from B to C is equal to the time from D to E. This is Kepler's second law.

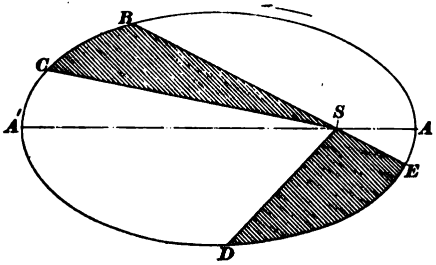


FIG. 17.

101. By examining Fig. 16, we can deduce from Kepler's second law the important fact that a planet moves faster in that part of its orbit where it is nearer to the sun than in that part of its orbit where it is more remote from the sun. Thus, the earth moves faster in winter than in summer.

102. Let P_1 and P_2 be two planets, and let
D_1 = mean distance of P_1 from the sun;
D_2 = mean distance of P_2 from the sun;
T_1 = periodic time of P_1;
T_2 = periodic time of P_2.

Then Kepler's third law is expressed by the equation

$$\frac{T_1^2}{T_2^2} = \frac{D_1^3}{D_2^3}.$$

EXAMPLE.—The mean distance of the earth from the sun is approximately 92 millions of miles. The periodic time of the planet Jupiter is nearly 12 years. Find approximately the mean distance of the planet Jupiter from the sun.

SOLUTION.—Let P_1 be the earth, and P_2 Jupiter. Then, using one million miles as unit of length, and a year as unit of time, we have

$$D_1 = 92,$$
$$T_1 = 1,$$

and

$$T_2 = 12.$$

Substituting these values in the equation,

$$\frac{T_1^2}{T_2^2} = \frac{D_1^3}{D_2^3},$$

we get

$$\frac{1^2}{12^2} = \frac{92^3}{D_2^3}.$$

Solving, $\qquad D_2^3 = 12^2 \times 92^3 = 144 \times 92^3.$

Therefore, $\qquad D_2 = \sqrt[3]{144} \times 92 = 5.24 \times 92$ nearly,

or $\qquad D_2 = 482$ (nearly).

Jupiter's mean distance = 482 millions of miles. *Ans.*

103. The invention of the telescope by the distinguished Italian philosopher Galileo (born 1564, died 1642) opened a new path for the students of astronomy. On Jan. 7, 1610, Galileo, by means of his new telescope, discovered the four moons of Jupiter.

Sir Isaac Newton (born 1642, died 1727) proved that Kepler's laws are a direct consequence of the law of gravitation. (*Elementary Mechanics*, Art. [illegible].) This discovery forms the basis of the whole science of gravitational astronomy.

THE STARS.

104. The solar system is like an island surrounded by a great void which separates it from the stars. The distance of the nearest fixed star is more than [illegible] times the distance of the sun. Modern research has shown that the stars are *suns*, shining by their own light and comparable in size to our sun. If the nearest fixed star has a retinue of planets similar to those which constitute the solar system, [illegible]

no telescope ever constructed would enable us to see a single one of the star's planets.

105. For purposes of reference, it is convenient to divide the stars into groups, just as a territory is divided into states and counties. These star groups are called **constellations.** Many of the constellations have been recognized from prehistoric times, and have received fanciful names. Sometimes the arrangement of the stars bears a resemblance to the object after which the constellation is named; in general, however, no reason can be given for the way in which the stars have been grouped and named.

106. The Zodiac.—A zone 16° wide, 8° on each side of the ecliptic, is called the **zodiac.** The name is derived from a Greek word which means *a living creature*, and was suggested by the fact that the constellations in this zone are, with one exception, figures of living animals. The ancient astronomers made the zodiac of this particular width because the moon and the then known planets are never more than 8° from the ecliptic.

107. The Signs of the Zodiac.—The length of the zodiac is divided into twelve parts, of 30° each. These twelve parts are called the **signs of the zodiac** and are named after the constellations which occupy them. The names of the signs of the zodiac are: Aries, Taurus, Gemini, Cancer, Leo, Virgo, Libra, Scorpio, Sagittarius, Capricornus, Aquarius, and Pisces. (See star map.)

108. On the star map it will be seen that the vernal equinoctial point (♈) is situated in the constellation Pisces, and that the autumnal equinoctial point (♎) is situated in the constellation Virgo; the direction of the sun's annual motion in the ecliptic is indicated by the arrow.

It has been stated that the equinoctial points are fixed, but this is not quite true, for they have a very slow motion along the ecliptic in the direction opposite to the sun's motion. Since the motion of the equinoctial points is opposite to the sun's motion, it is called a **retrograde** motion.

This retrograde motion is so slow that it will take the equinoctial points about 26,000 years to make a complete circuit of the ecliptic.

Since the equinoctial points move to meet the sun, the effect is to make the equinoxes happen at shorter intervals than they would if the equinoctial points were fixed; this is called the **precession of the equinoxes.**

The earliest astronomers whose records have come down to us found the vernal equinoctial point in the constellation Aries, instead of in the constellation Pisces where it now is; consequently they called the vernal equinoctial point the **first point of Aries,** and it still retains this name. The sun enters Aries—an expression often found in almanacs—means that the sun passes through the vernal equinoctial point.

PARALLAX.

109. Let A and B (Fig. 18) be two points of observation on the earth's surface. Suppose an observer at A and an observer at B to determine the position of the moon M relatively to the star S, their observations being made at the same time. To these observers the moon will appear on opposite sides of the star S; hence it is evident that observations made at A can not be compared directly with observations made at B. To avoid this difficulty, the position of a celestial body is always specified by its direction from some determined point of reference. The positions of the moon, planets, and stars are always referred to the earth's center, and the direction of the line joining the earth's center to a celestial body is called the body's **geocentric direction.** The position of a celestial body may be referred to the sun, and the direction of a celestial body from the sun is called its **heliocentric** position.

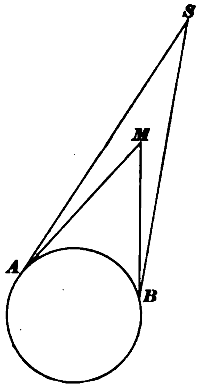


FIG. 18.

110. The angle between the line joining a celestial body to the point of observation and the line joining the celestial body to the determined point of reference is called the **parallax** of the celestial body. Thus, if P (Fig. 19) is the celestial body, A the point of observation, and C the point of reference, then the angle $A P C$ is the parallax of the body P.

111. When the center of the earth is taken as the point of reference, the parallax of a body is called its **geocentric parallax.** Since the position of the moon or of a planet is always referred to the earth's center, the geocentric parallax of one of these bodies is called simply its **parallax.** The student must remember that the parallax of the moon or of a planet always means its geocentric parallax.

112. The geocentric parallax of a body depends upon its distance from the zenith. Let A (Fig. 19) be the position of the observer, C the center of the earth, and Z the zenith. The lines $Z A$ and $Z C$ coincide; hence, the parallax of a body in the zenith is zero. The greater the zenith distance of a body the larger is its parallax; and the parallax is maximum when the body is on the horizon, as at P.

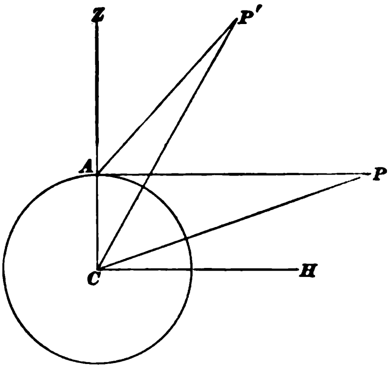


Fig. 19.

The parallax of the body P' is the angle $A P' C$, and we have

$$A P' C = Z A P' - Z C P'.$$

That is, the parallax of a body is equal to the difference between its zenith distance as seen by the observer and its zenith distance as seen from the center of the earth.

From this equation we get

$$Z C P' = Z A P' - A P' C.$$

Hence, *the geocentric zenith distance is found by subtracting the parallax from the observed zenith distance.*

In the plane $Z\ C\ P'$ draw the horizontal line $C\ H$ through the center of the earth. Then $P\ A\ P'$ is the observed altitude of P', and $H\ C\ P'$ is its geocentric altitude. It is easy to show that

$$H\ C\ P' = P\ A\ P' + A\ P'\ C.$$

That is, *the geocentric altitude is found by adding the parallax to the observed altitude.*

Owing to this relation between parallax and altitude, geocentric parallax is frequently called **parallax in altitude.** Geocentric parallax is also frequently called **diurnal parallax,** because it passes through a complete cycle of values every day, being maximum when the body is on the horizon and minimum when the body is on the meridian.

113. The **horizontal parallax** of a body is its geocentric parallax when it is on the horizon. Thus, in Fig. 19, the angle $A\ P\ C$ is the horizontal parallax of the body P. Evidently, then, we may define the horizontal parallax of a body as *the angular semidiameter of the earth as seen from that body.* For instance, when we say that the moon's horizontal parallax is 60′, we mean that, seen from the moon, the earth's diameter appears to be 2 × 60′ or 120′.

114. When the horizontal parallax of a celestial body and the radius of the earth are known, the distance of the celestial body from the earth can be found by the formula of Art. **44.** Let S denote the parallax in seconds, d the distance of the celestial body from the earth, and r the radius of the earth. Then,

$$\sin S'' = \frac{r}{d}. \quad \text{(See equation } (a)\text{, Art. } \mathbf{44.})$$

Whence,
$$d = \frac{r}{\sin S''}.$$

If the parallax is very small, we have approximately

$$S = 206{,}265\,\frac{r}{d}. \quad \text{(See equation } (b)\text{, Art. } \mathbf{44.})$$

Whence,
$$d = r \times \frac{206{,}265}{S}.$$

EXAMPLE.—Being given that the sun's horizontal parallax is 8.8", and the earth's radius 3,960 miles, find the sun's distance.

SOLUTION.—Using the above formula, we have,

$$d = 3,960 \times \frac{206,265}{8.8}.$$

Whence, approximately $d = 92,800,000$ miles. Ans.

NOTE.—It is useless to calculate the sun's distance to a closer degree of approximation, because the values given above for the sun's parallax and the earth's radius are only approximate.

115. The method of determining the distance of a celestial body explained in Art. **114** is not applicable to a fixed star, for the distance of a fixed star is so enormous that its horizontal parallax is utterly inappreciable. No fixed star has a horizontal parallax amounting to as much as $\frac{1}{10000}$ of a second.

Therefore, in case of a fixed star, the center of the sun instead of the center of the earth is taken as point of reference.

116. The greatest angle subtended at a star by the radius of the earth's orbit is called the star's **annual**

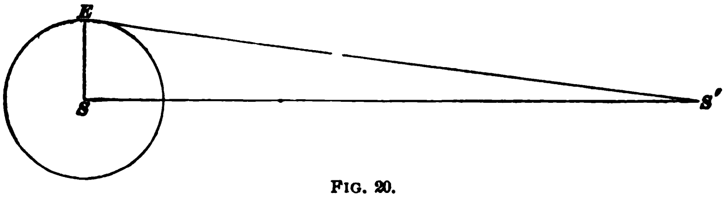


FIG. 20.

parallax. Thus, in Fig. 20, if S represents the sun, S' the star, and E the earth at the time when $E\,S$ subtends the greatest angle at S', then the angle $E\,S'\,S$ is the annual parallax of the star S'.

117. When the annual parallax of a star is known, its distance can be found. Let r denote the radius of the earth's orbit, d the distance of the star, and S its annual parallax in seconds. Then, as in Art. **114**,

$$d = r \times \frac{206,265}{S}.$$

118. The parallax of only a few stars has yet been determined, and in no case does it amount to as much as one second.

REFRACTION.

119. A ray of light travels in a straight line so long as its path is in a medium of uniform density; but when a ray of light passes obliquely from one medium into another medium of different density, or from one stratum of a medium into another stratum of different density, it undergoes a change of direction at the surface of separation. This change of direction or bending of a ray of light is called **refraction.**

When a ray of light from a celestial body enters the earth's atmosphere obliquely, it is always bent *downwards.* For example, if S, Fig. 21, is a celestial body, O the place of the observer, Z the zenith, and A A' the surface of the atmosphere, then the ray of light from S to the observer, instead of traveling in a straight path, travels in the bent line S A O. Now the apparent position of a body depends upon the direction in which its light enters the observer's eye; hence, the celestial body S appears to be at S' instead of in its true position.

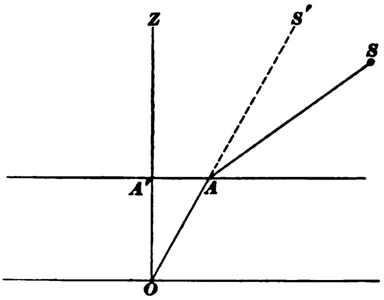


FIG. 21.

If, however, the celestial body is exactly in the zenith, a ray of light from the body to the observer enters the atmosphere perpendicularly and not obliquely; under these circumstances the ray does not suffer refraction, and consequently the position of a body in the zenith is not affected by refraction.

Hence, *the apparent altitude of a star which is not in the zenith is increased by refraction.*

120. The true altitude of a celestial body is obtained from the observed altitude by subtracting a correction for

refraction. The amount of this correction is zero when the body is in the zenith, and increases gradually as the body approaches the horizon, where the correction attains a maximum value of about 35′. As shown by the following table, refraction increases slowly at great altitude, but rapidly near the horizon:

Altitude.	Refraction.	Altitude.	Refraction.
90°	0′	15°	3′ 32″
80°	0′ 10″	10°	5′ 16″
70°	0′ 21″	8°	6′ 30″
60°	0′ 33″	5°	9′ 47″
50°	0′ 48″	3°	14′ 15″
40°	1′ 09″	2°	18′ 09″
30°	1′ 40″	1°	24′ 25″
20°	2′ 37″	0°	34′ 54″

121. Twilight, or the glow of light after sunset and before sunrise, is caused by refraction and the reflection of the sun's rays on the clouds and on the particles in the upper strata of the air. Twilight continues until the sun is about 18° below the horizon. In countries of high northern latitude, twilight lasts all night at midsummer, because the sun does not descend so much as 18° below the horizon during the entire night. The twilight at the equatorial parts of the earth remains almost constant the whole year round, and its duration is about one hour.

TIME.

MEASUREMENT OF TIME.

122. We are all familiar with the ordinary method of measuring time by means of a clock. The importance of the accurate determination of time can hardly be overestimated; upon it depends the safety of trains on land and of ships at sea. How, then, are we to test the correctness

of a clock? Clocks throughout the world are regulated by comparing them, directly or indirectly, with the clocks of the great national observatories; and these observatory clocks are regulated by means of astronomical observations. To the astronomer, therefore, belongs the duty of regulating the determination and measurement of time; and this is one of the most important problems of practical astronomy.

123. The passage of a celestial body across the celestial meridian is called its **transit.** A **day** is defined as the interval of time between two successive upper transits of the same celestial body, and time is measured by the hour-angle of this body.

Three different kinds of day and three different kinds of time are recognized, depending upon the celestial body whose transit is selected to determine the day. The three kinds of day are: the *sidereal day*, the *apparent solar day*, and the *mean solar day;* the corresponding kinds of time are: *sidereal time*, *apparent solar time*, and *mean solar time.*

SIDEREAL TIME.

124. As defined in Art. **48,** a sidereal day is the period occupied by a fixed star in its apparent revolution about the earth; in other words, a sidereal day is the interval between two successive upper transits of the same fixed star. If the vernal equinoctial point were a fixed point, the interval between two successive upper transits of the vernal equinoctial point would be exactly equal to a sidereal day; but owing to the precession of the equinoxes, the interval between two successive upper transits of the vernal equinoctial point is less than a true sidereal day by a little less than the one-hundredth part of a second. In practice this slight difference is neglected, and the following definition of a sidereal day is given:

A **sidereal day** *is the interval between two successive upper transits of the vernal equinoctial point and begins when the vernal equinoctial point is on the meridian.*

The difference between the day as determined by the equinoctial point and the true sidereal day amounts to about one day in twenty-six thousand years.

125. From Art. **123,** it follows that the *sidereal time is the hour-angle of the vernal equinoctial point expressed in time.*

The **sidereal clock** used in observatories shows sidereal time. The hands point to 0 *h* 0 *m* 0 *s* when the vernal equinoctial point is on the meridian, and the hours are reckoned from 0 *h* up to 24 *h* when the vernal equinoctial point is again on the meridian. In Fig. 11, the sidereal time is measured by the arc $X Q$ of the equator, or by the hour-angle $X P Q$.

126. Suppose, in Fig. 11, we had a star in transit, that is, on the meridian between P and M. Then the right ascension of this star would be the arc $X Q$. (Art. **78.**)

Hence, *the right ascension of a star, when expressed in time, is equal to the sidereal time of its transit.*

127. Since both the hour-angle and the right ascension of a star can be expressed either in time or in angular measure, it is necessary to be able to reduce time to angular measure and angular measure to time. From Art. **71,** the following rule is easily deduced:

Rule.—I. *To reduce time to angular measure multiply by 15.*

II. *To reduce angular measure to time divide by 15.*

EXAMPLE.—The hour-angle of a star is 7 h. 40 m. 55 s.; express this angle in angular measure.

SOLUTION.—

7	40	55
		15
115	13	45

In multiplying by 15, we have $55 \times 15 = 825$; dividing 825 by 60, the quotient is 13 and the remainder is 45; set down 45 and carry 13. Then $40 \times 15 + 13 = 613$; dividing by 60, the quotient is 10 and the remainder is 13; set down 13 and carry 10. Then $7 \times 15 + 10 = 115$. Hence the hour-angle is 115° 13′ 45″. Ans.

EXAMPLE.—The right ascension of a certain star is 278° 18′ 42″. Express this in time.

SOLUTION.— $15\,\underline{)\,278°\ 18'\ 42''}$

$\qquad\qquad\qquad\qquad 18\ \ 33\ \ 14.8$

Therefore, the right ascension is 18 h. 33 m. 14.8 s. Ans.

APPARENT SOLAR TIME.

128. **Apparent noon** is the time of the sun's upper transit across the meridian; **apparent midnight** is the time of the sun's lower transit across the meridian.

129. An **apparent solar day** is the interval between two successive apparent noons, or between two successive apparent midnights.

130. From Art. **123** it follows that **apparent solar time** *is measured by the sun's hour-angle.* Apparent solar time is the time shown by a sun-dial.

131. At noon the sun is on the meridian; hence, by Art. **126,** *the sidereal time of apparent noon is equal to the sun's right ascension at noon.*

At any instant we have

$$\text{Sidereal time} = \text{hour-angle of } \Upsilon;$$
$$\text{Apparent solar time} = \text{hour angle of sun.}$$

Hence, (sidereal time) − (apparent solar time) = sun's right ascension. (Art. **78.**)

132. Let a and $a + x$ denote the sun's right ascensions at two successive apparent noons. At the first noon the time shown by the sidereal clock is a, and at the second noon the time shown by the sidereal clock is $a + x$; meanwhile a whole sidereal day or 24 sidereal hours have elapsed; hence, the interval of time is

$$24\text{ h.} + a + x - a, \text{ or } 24\text{ h.} + x.$$

That is, (apparent solar day) = (sidereal day) + x.

Thus, *the apparent solar day is longer than the sidereal day by the amount of the sun's daily increase in right ascension.*

133. As already stated, the length of the sidereal day is constant. But the sun's daily increase in right ascension is not uniform throughout the year; this want of uniformity is due to two causes. First, in accordance with Kepler's second law, the sun's apparent motion in the ecliptic is not uniform; secondly, even if the sun did move uniformly in the ecliptic, the increase of its right ascension would not be uniform, owing to the inclination of the ecliptic to the equator.

Hence it follows that *the length of the apparent solar day is not the same at all times of the year.* For example, December 23d is 51 seconds longer from apparent noon to apparent noon than September 16th.

MEAN SOLAR TIME.

134. Disadvantage of Sidereal Time.—The sidereal time of apparent noon on any day is equal to the sun's right ascension on that day, and, consequently, it gets later by 24 hours during the year. Thus, the sidereal time of apparent noon

On March 21st is 0 h.
On June 21st is 6 h.
On September 23d is 12 h.
On December 22d is 18 h.

We see, then, that sidereal time bears no simple relation to the phenomena of day and night, and is therefore unsuitable for every-day use.

135. Disadvantage of Apparent Solar Time.—Since the length of the apparent solar day is not constant, apparent solar time can not be measured by a clock whose rate is uniform. Therefore, apparent solar time is unsatisfactory for scientific and practical purposes.

136. Sidereal time and apparent solar time having been found unsatisfactory, another kind of time called **mean time** has been devised, which is defined by reference

to what is called the **mean sun.** The mean sun is not a body of any kind, but merely a point which is imagined to move *uniformly* round the celestial *equator* in the same time that the sun moves round the ecliptic.

137. Mean noon is the time of the mean sun's upper transit across the meridian. A **mean solar day** is the interval between two successive mean noons. *Mean solar time, or mean time, is measured by the hour-angle of the mean sun.* (Art. **123.**)

Mean time is the time shown by our clocks, and is now used for all practical and scientific purposes except in certain astronomical work.

When mean time is employed in astronomical work, the day begins at mean noon and is called the **astronomical day;** astronomical mean time is reckoned continuously up to 24 hours.

When mean time is employed in the ordinary affairs of life, it is called **civil time,** and the **civil day** begins at midnight, 12 hours earlier than the astronomical day. Thus, the astronomical date June 8th 19 h. is the same as the civil date June 9th, 7 o'clock A. M.

138. Equation of Time.—The equation of time is the name given to the amount which must be added to the apparent solar time to obtain mean time. Mean time is the time shown by the clock, and apparent time is that shown by a sun-dial. Hence,

(time by clock) = (time by dial) + (equation of time).

The student will observe that the equation of time is not an equation in the correct sense of the word, but simply a correction to be added to the apparent time in order to obtain the mean time.

The equation of time is positive when the sun-dial is slower than the clock, and is negative when the sun-dial is faster than the clock.

In Art. **133** we have mentioned the two principal causes of the variability of apparent solar time which gives rise to

the equation of time. The equation, therefore, is principally due to these same two causes, viz., the variability of the sun's motion in the ecliptic and the inclination of the ecliptic to the equator.

Four times during a year the equation of time is zero, viz., April 15th, June 14th, September 1st, and December 24th.

Its maxima values are: February 11th, + 14 m. 32 s.; May 14th, −3 m. 55 s.; July 26th, +6 m. 12 s.; and November 2d, − 16 m. 18 s.

Both the dates and the amounts are subject to slight variations from year to year.

139. From Art. **48** it follows that a mean solar day is longer than a sidereal day by 4 minutes of mean time.

140. Local Time.—Mean time is measured from the instant the mean sun is on the meridian; hence, at any instant the mean time will not be the same at two places on the earth's surface, unless those two places lie on the same meridian. For this reason mean time as above defined is called **local time.** The difference between the local times of two places depends only on their difference of longitude.

Terrestrial longitude can be expressed in time exactly as right ascension or hour-angle is expressed in time.

The longitude of a place on the earth's surface expressed in time is equal to the difference between the local time of that place and Greenwich local time.

A **chronometer** is a watch that shows *Greenwich mean time*, while the ordinary clock or watch shows local mean time.

THE CALENDAR.

141. A **year** is defined as the period of a complete revolution of the sun in the ecliptic. In order, however, to complete this definition, it is necessary to specify the starting-point from which the revolution is measured. By taking different starting-points, we are led to different kinds of year.

142. A **tropical year** is the interval of time between two successive passages of the sun through the vernal equinoctial point. The length of a tropical year at the present time is very approximately 365 d. 5 h. 48 m. 45.51 s. of mean time.

143. A **sidereal year** is the period of a complete revolution of the sun, starting from and returning to the same fixed point among the constellations. If the vernal equinoctial point were a fixed point, the tropical year and the sidereal year would be the same. But in Art. **108** we saw that the vernal equinoctial point has a retrograde motion, completing the circuit of the ecliptic in about 26,000 years; therefore, its retrograde motion amounts to about $\frac{3,600}{26,000}$, or 50.2′ a year. This causes the vernal equinox to happen earlier than it otherwise would by $\frac{50.2}{360 \times 60 \times 60}$ of a year; that is, by about 20 minutes. Therefore, the tropical year is shorter than the sidereal year by about 20 minutes.

144. The Civil Year.—For ordinary purposes, it is important that the year should contain an exact number of days, and that it should bear a simple relation to the recurrence of the seasons. Neither the sidereal nor the tropical year contains an exact number of days. The sidereal year has the additional disadvantage of not marking the recurrence of the seasons.

For this reason the **civil year** has been introduced. The length of a civil year is sometimes 365 days and sometimes 366 days.

The Roman emperor Julius Cæsar ordered that three successive years should have 365 days each, and the fourth year should have 366 days. The fourth year, which contains 366 days, is called a **leap year**, and the calendar constructed on this principle is called the **Julian calendar.** For convenience the leap years are chosen to be those whose number is exactly divisible by 4; as, 1872, 1684, etc.

A simple arithmetical calculation shows that 3 ordinary years and 1 leap year exceed 4 tropical years by 44 m. 57.96 s. Therefore, 400 years of the Julian calendar exceed 400 tropical years by 3 d. 2 h. 56 m. 36 s.

Therefore, Pope Gregory XIII in 1582 amended the Julian calendar by omitting 3 days in every 4 centuries, and ordered that:

Every year whose number is a multiple of 100 shall be an ordinary year of 365 days; unless the number of the year is divisible by 400, in which case the year is a leap year.

The calendar constructed in accordance with this correction is called the **Gregorian calendar.** The error in the Gregorian calendar is very small, and will not amount to more than 1 d. 5½ h. in 4,000 years.

In the Gregorian calendar the year 1900 will not be a leap year, because the number 1900 is not exactly divisible by 400; but the year 2000 will be a leap year, because 2000 is exactly divisible by 400. The Gregorian calendar is now adopted by all nations except Russia.

145. Old Style and New Style.—At the time of the Council of Nice (325 A. D.) the sun was in the vernal equinoctial point on March 21st, by the Julian calendar; in 1582 the sun was at the same point on March 11th. Pope Gregory, therefore, in correcting the calendar, ordered that the day after Oct. 4th, 1582, should be called Oct. 15th, 1582.

In England the Gregorian calendar was not adopted till the year 1752, when the error of the Julian calendar amounted to eleven days. In 1751 the English Parliament enacted that the day after Sept. 2d, 1752, should be called Sept. 14th, 1752.

During the period immediately following the adoption of the Gregorian calendar, to avoid confusion, writers usually specified whether their dates were given in **old style** (according to the Julian calendar) or in **new style** (according to the Gregorian calendar). Thus, Jan. 4th, 1626, O. S., means Jan. 4th, 1626, of the Julian calendar.

THE SOLAR SYSTEM.

THE SUN.

146. The sun is the most important body in the solar system. Its attraction holds the planets in their orbits and controls their motions; its rays supply the heat and light which make the earth habitable and the energy which maintains every form of activity upon the earth's surface. The sun supplies, directly or indirectly, the power that drives every locomotive and every piece of machinery in the world.

147. The sun is a hot, self-luminous body of enormous magnitude in comparison with the earth or any other member of the solar system. The fixed stars are also hot and self-luminous, and in many ways bear a close resemblance to the sun; hence, the sun is classed among the fixed stars, and the fixed stars are often called *suns*.

We are dependent on the sun for our very existence, as well as for every comfort and luxury we enjoy, and, therefore, the sun is far the most important of the stars to us. Yet there is good reason to believe that there are stars which surpass our sun in magnitude and grandeur.

148. Distance.—The sun's distance is determined by its horizontal parallax; assuming that quantity to be 8.8″, the sun's mean distance from the earth is 92,800,000 miles. (Art. **114.**) When the earth in its orbit is at aphelion, the distance is about 3,000,000 miles more than when the earth is at perihelion.

149. Diameter.—According to the latest calculations, the sun's mean apparent diameter expressed in fractions of degrees is 32′ 4.56″ ± 32.13″; and since 1 second is equivalent to 45,036 miles, its diameter in miles is 866,500, or about 109½ times that of the earth. It is a well-known fact that the sun is diminishing in diameter at the approximate rate of 250 feet per year. In volume the sun is about 1,300,000 times larger than the earth.

150. Mass and Density.—The sun does not exceed the earth in mass nearly so much as it does in volume. The mass is only about 332,000 times that of the earth. The sun's density is about one-fourth, or 0.255 that of the earth. This means that if a body weighing 1 pound on the earth's surface was transferred to the surface of the sun, it would weigh nearly 28 pounds; and whereas a body falls 16 feet the first second at the earth's equator, the same body if transferred to the sun would fall 444 feet in the first second.

151. Rotation.—The sun rotates on its axis once in a little more than 25¼ days. However, some eminent authorities have not agreed by a small fraction on that interval. What are now termed sun-spots were supposed by early astronomers to be intra-Mercurial planets passing over the sun's disk. Recent observations have conclusively dispelled that theory and have proved that these spots are attached to the sun, and by them the sun's period of rotation has been determined. By careful observations of the sun-spots, a foundation has been established on which the theory is based that the body of the sun is enveloped in a luminous gaseous atmosphere, the spots being the dark solid seen through the fissures. (See Fig. 22.) In the same manner it has been proved that the whole surface of the sun does not rotate uniformly like that of an absolutely solid body. The spots near its equator make a complete revolution in a much shorter time than those near the poles. The reason for this is that the outer surface of the sun not being solid, can not rotate equally at all points. These spots have also been successfully resorted to in finding the inclination of the sun's axis to the plane of the ecliptic. Instead of passing over the disk as an interior planet does (in a straight line), the spots describe an oval path. From this the inclination of the axis is computed to be 7° 15′.

152. Nature and Dimension of Sun-Spots.—On examining the sun's surface, or *photosphere*, as it is called, with a telescope, there is seen a greater or less number of dark spots, as shown in Fig. 22. If observed from day to

ıy, they are seen not only to move slowly across the disk, ıt to change their form and general appearance very equently. Sometimes a large spot is divided into two or ore smaller ones, and again a group is united into a single rge spot. Sometimes a spot disappears suddenly, and at her times a spot is seen in the midst of the disk, where ıere was none the day before. On certain occasions, spots e found which show a motion of rotation, and several ientists have advanced a theory that the motion of these

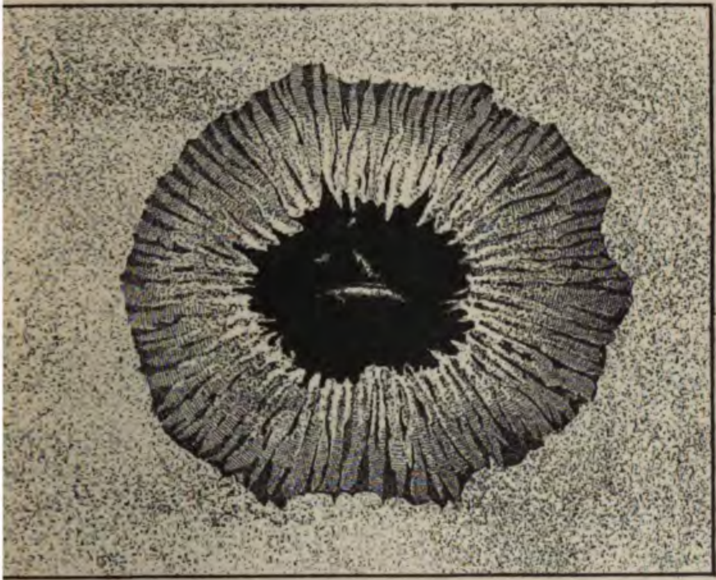


FIG. 22.

ɔots is more or less analogous to that of storms and cyclones pon the earth. However this may be, no substantial ıeory has yet been found which accounts fully for the ɔserved motions of the spots. The dimensions of sun-spots ıry considerably. The diameter of a very small spot is ldom less than 500 miles, while, in the case of a large spot, can be fully 40,000 miles. Quite frequently spots are large ıough to be seen with the naked eye, if observed through

a fog or a colored glass. According to eminent astronomers, the depth of a sun-spot seldom exceeds 2,500 miles, the average being between 500 and 1,500 miles. The number of spots varies exceedingly in different years. At times none are to be seen for days and weeks, then again for months they are to be found all over the disk. Their duration also is extremely variable. In general, the spots have a short existence. Sometimes they last but a few days; sometimes they exist two or three months. One recorded instance tells us of a spot lasting nearly eighteen months. At times violent disturbances are observed in the neighborhood of spots. Luminous masses appear and disappear in quick succession, moving with great velocity, their dazzling brilliancy far exceeding that of the general surface of the sun. Magnetic storms on the earth are supposed to be influenced by these solar disturbances, though physicists are not yet able to explain the nature of the relation.

153. There are immense irregular masses overhanging the sun's edges, the general appearance of which are

(a) (b)

FIG. 23.

illustrated in (*a*) and (*b*), Fig. 23. They are called **solar prominences,** and are not visible with a simple telescope, except at the time of an eclipse. Prominences are generally considered to be fountains of gas spurting out from the interior of the sun. They are distributed all over the disk,

hough visible only at its limb on account of the sun's bright urface. By observations of spots and prominences and hrough the laws of spectrum analysis, discovered by Kirchhoff, the chemical constituents of the sun have been ound to be similar to those composing the earth. As a esult of those investigations, it has been proved beyond loubt that such common substances as iron, carbon, hydrogen, nickel, and copper are the principal ingredients of he sun.

154. Heat and Light.—The sun is the power that egulates all the motions of the bodies in the solar system; ot only their motions in their orbits, but also the physical henomena which take place upon their surfaces. On he earth in particular, the currents of air and those of the vater, the development of vegetation, the production of the orce which results from combustion, are all due to the nfluence of the sun's heat. It is therefore on the sun that ll the phenomena of nature depends, our own existence not xcepted. By letting the sun shine upon an apparatus composed of a box blackened inside and of several pieces of glass aid one upon the other, it is possible to raise water to boiling-point any fine, cloudless day. Experiments in this line ave demonstrated that if the total quantity of heat which he earth receives from the sun in the course of a year were uniformly distributed over all parts of the globe, it would e sufficient to melt a coat of ice enveloping the whole globe o a depth of about one hundred feet. The light received y the earth is only $\frac{1}{2,300,000,000}$ of the total light radiated by he sun; that is, the light emitted by the sun in all directions s 2,300,000,000 times greater than that received by the earth. t is supposed that the heat of the sun is produced by an normous process of condensation; and it has been calculated hat 10,000,000 years from the present time the sun will not e able to supply heat enough to support life upon the earth. This calculation, however, is only an approximation, as we ave not sufficient data to enable us to calculate the future uration of the sun with any degree of exactness.

THE PLANETS.

CLASSIFICATION.

155. Including the earth, there are eight known planets, divided into two classes, inferior and superior planets.

Inferior planets are those whose orbits lie within that of the earth, and **superior planets** are those whose orbits are greater than that of the earth, and, consequently, lie outside of it. When we consider the planets themselves, and not their orbits, they fall into two divisions, *major* and *minor* planets. Commencing from the sun, the planets appear in the following order:

Minor	Mercury	Interior or Inferior.
	Venus	
	The Earth	
	Mars	Exterior or Superior.
Major	Jupiter	
	Saturn	
	Uranus	
	Neptune	

156. Between the orbits of Mars and Jupiter there are a number of small planets, called **asteroids.** At present about 300 asteroids are known; it is supposed that they are fragments of a burst planet.

DEFINITIONS.

157. Before entering upon the description of the several planets, we shall explain the terms usually employed in reference to them.

In Fig. 24, let the outside circle represent the orbit of a superior planet, the solid circle the orbit of the earth, the dotted circle that of an inferior planet, and S the sun. We also assume that the earth is situated at E.

When a planet appears to be close to the sun, it is in **conjunction.** A superior planet is then at C or beyond

the sun, while an inferior is either at A or B. If the planet is at A, we have a **superior conjunction**, and if the planet is at B, an **inferior conjunction**.

When a planet is at O, directly opposite to the sun, it is said to be in opposition.

The **elongation** of a planet is the angle formed by lines drawn from the earth to the sun and to the planet. The greatest elongation of an inferior planet occurs when the planet is at D or at F. The elongation of a superior planet when at L is the angle SEL.

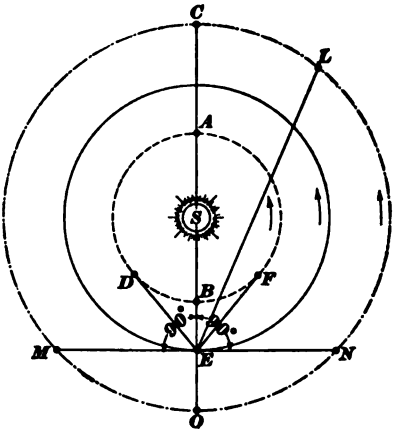


FIG. 24.

When the elongation of a superior planet is 90° (either at M or N) the planet is in **quadrature**.

158. The **sidereal period** of a planet is the time required by the planet to make a complete revolution around the sun from a star to the same star again, *as seen from the sun.*

The **synodic period** of a planet is the time between two successive conjunctions of the planet and sun, *as seen from the earth.* The relation between the sidereal and synodic periods is

$$\frac{1}{s}=\frac{1}{p}-\frac{1}{e},$$

where s denotes the synodic period of a planet, and p and e denote, respectively, the sidereal periods of a planet and the earth. If p is larger than e, as in the case of the superior planets, we may write the relation

$$\frac{1}{s}=\frac{1}{e}-\frac{1}{p}.$$

EXAMPLE.—The sidereal period of Mercury is 88 days. What is his synodic period?

SOLUTION.— $$\frac{1}{s} = \frac{1}{88} - \frac{1}{365\frac{1}{4}} = \frac{277\frac{1}{4}}{32,142};$$

whence $$s = \frac{32,142}{277\frac{1}{4}} = 116.$$

Therefore, Mercury's synodic period is 116 days. Ans.

159. As stated before, the apparent motions of the planets are very irregular; generally they seem to move from west to east, but at times they move in the opposite direction. The cause of the irregularity of the apparent motion of a planet can be easily explained. For this purpose we may assume that the planets move in the plane of the ecliptic.

In Fig. 25, the earth's orbit is represented by the solid circle, and that of the planet Mars by the dotted circle.

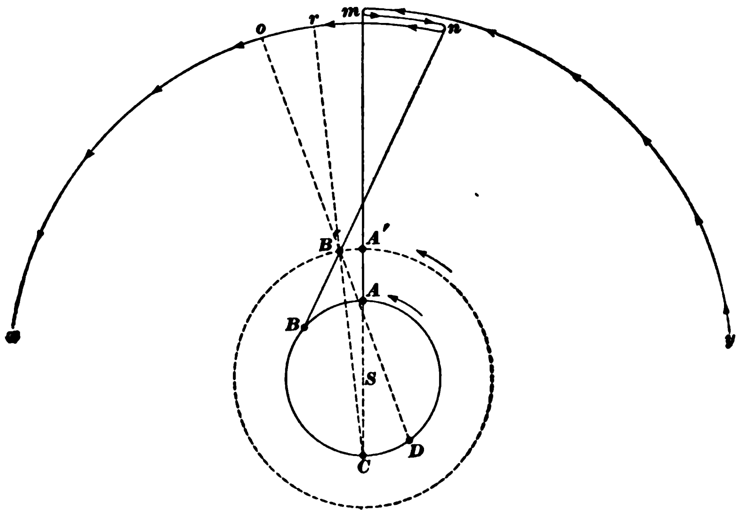


FIG. 25.

The curve $x\,y$ is the ecliptic and the north pole of the celestial sphere is above the plane of the paper. If a watch is laid upon the paper, with its face upwards, the planets revolve about the sun in the direction opposite to the motion

of the hands of the watch. At the time when Mars is in opposition, the earth is at A, and Mars is at A'. To an observer on the earth, Mars then appears on the celestial sphere at m. After a short interval the earth has moved to B, and Mars has moved to B'. An observer then sees Mars in the direction $B\ B'$, and, consequently, Mars appears on the celestial sphere at n. During this interval the real motion of Mars has been forwards from A' to B', yet its apparent motion has been backwards from m to n. This apparent backward motion of the planet is called a **retrograde** motion.

The retrograde motion of the planet is most rapid when it is in opposition, and becomes gradually slower as the interval from the time of opposition increases. At a certain time the retrograde motion ceases, and the planet maintains for a short time the same position relatively to the earth; the planet is then said to be **stationary.** The position of the stationary point depends upon the relative sizes of the orbits of the earth and the planet. After the stationary point is passed, the apparent motion of the planet becomes direct.

When a planet is in superior conjunction, its apparent motion is most rapid and is direct. Suppose, for example, that Mars is at A' when the earth is at C. Mars then appears on the celestial sphere at m. In a short interval the earth moves to D and Mars moves to B'. The observer then sees the planet in the direction $D\ B'$, and, consequently, Mars appears on the celestial sphere at o; hence, the apparent motion of Mars during this interval is $m\ o$. But if the earth had remained motionless at C, the planet would be observed in the direction $C\ B'$, and would appear on the celestial sphere at r; its apparent motion then would be $m\ r$. Thus, at superior conjunction the apparent motion of a planet is direct, and is greater than the planet's real motion. The direct apparent motion of a planet becomes more and more rapid from the stationary point to the point of superior conjunction; then the rapidity of its direct motion diminishes until it again becomes stationary.

After passing the second stationary point, it begins again to retrograde; and the rate of retrogression reaches its maximum again when the earth and the planet are in line with the sun, and on the same side of it. For a superior planet, like Mars, this happens when the planet is in opposition; for an inferior planet, like Mercury, this happens when the planet is in inferior conjunction.

MERCURY.

160. Mercury is an inferior planet; its orbit being far within that of the earth. Its mean distance from the sun is 36,000,000 miles, while its actual distance, on account of the orbit's great eccentricity, varies about 7,500,000 miles on each side of the mean.

The diameter of Mercury is 3,000 miles; and, therefore, its surface is equal to about one-seventh of that of the earth. Its sidereal period is 88 days. Hence, Mercury requires less than $\frac{1}{4}$ of our year to perform a revolution around the sun. As to its rotation upon its axis, astronomers are not agreed. One opinion, rendered a century ago, was that a rotation on its axis was made in 24 hours, but recent observations have not confirmed it. A noted Italian astronomer says he has reason to believe that Mercury, like the moon, rotates on its axis in the same time that it makes a revolution about the sun. The **synodic period** of Mercury, or the time required from one conjunction to another, is 116 days. The greatest *elongation*, as given in the Nautical Almanac, occurs about 22 days before and after the inferior conjunction, and varies from 18° to 28°.

161. Mercury is so near to the sun that it is seldom seen with the naked eye. It is visible as a very bright star at the time of its greatest elongation. It is best seen in the evening at those eastern elongations, which occur in March and April.

It is difficult for an observer in northern Europe to see Mercury with the naked eye; and it is said that Copernicus,

at the close of his life, lamented the fact that he had never been able to see it.

Mercury is the smallest of the planets, excepting the asteroids; it has the smallest mass and the greatest density of any planet. It receives from the sun the greatest amount of light and heat. Its orbit is inclined to the ecliptic at an angle of about 7°. The eccentricity and the inclination of its orbit are greater than those of the orbit of any other planet. Mercury also moves more swiftly than any other planet.

162. The appearance of Mercury in the telescope is similar to that of the moon. At superior conjunction the illuminated surface of the planet is towards the earth, and it appears like a full moon; at inferior conjunction the dark side is towards the earth; at its greatest elongation it looks like a half moon; between superior conjunction and greatest elongation it is gibbous; and between inferior conjunction and greatest elongation it shows the crescent phase. This proves that Mercury shines, not by its own light, but by reflecting the light of the sun.

163. At inferior conjunction, Mercury passes nearly between the earth and sun, and at times the three bodies are in a straight line. The planet is then seen as a black round spot traveling across the sun's disk. This phenomenon is called a **transit** of Mercury. In the nineteenth century occurred 13 transits, and the twentieth century will see 12; the next transit will occur on November 12th, 1907.

VENUS.

164. Venus is the brightest and most conspicuous of the planets, and has been known from ancient times as the morning and evening star. It is, indeed, the most magnificent star of our sky, and its brilliancy sometimes even pierces the azure and shines in full daylight, in spite of the presence of the sun above the horizon. The mean distance of Venus from the sun is 67,200,000 miles. Its actual

distance varies very little from the mean, because the eccentricity of its orbit is small.

The diameter of Venus is 7,700 miles, and its surface is therefore nearly as large as that of the earth. Its sidereal period is 225 days and its synodic period 584 days, or a year and seven months.

The time of rotation is put by some astronomers at 23 hours, 21 minutes; while others claim that Venus, like Mercury, rotates on its axis in the same period that it revolves round the sun. The inclination of the orbit is about 3° 28′; and the orbital velocity is about 22 miles per second.

165. The phases of Venus, as seen in the telescope, are similar to those of the moon and of Mercury. The greatest brightness of Venus is observed when about 40° from the sun; that is, between the point of greatest elongation and inferior conjunction. At that place it is frequently seen during the whole day, and looks like a moon about five days old.

166. Transits of Venus do not occur as frequently as those of Mercury. Since the sun passes the nodes, or points where the planet's orbit cuts the ecliptic in June and December, the transits must occur in those months. The last transit took place Dec. 6th, 1882, while the next will not occur until June 8th, 2004. By observations of the planet when entering upon a transit, it has been proved that Venus is surrounded by an atmosphere the density of which is little less than twice that of the earth's, and this atmosphere is evidence of the existence of water on its surface. The amount of heat and light received by Venus is about twice as great as the amount received by the earth.

MARS.

167. Mars is an exterior or superior planet, its orbit being outside of the earth's. Its mean distance from the sun is somewhat more than one and a half times that of the earth, or 141,500,000 miles. Its orbit is the most eccentric of

all the planetary orbits, with the exception of Mercury, and, therefore, its distance varies a great deal—nearly 13,000,000 miles on each side of the mean.

168. The diameter of Mars, in comparison with the other minor planets and the moon, is shown in Fig. 26; it is 4,200 miles, or a little more than half the earth's. Its sidereal period is 687 days, which is very nearly two of our years, and its synodic period is 780 days. The sidereal day of Mars has been determined with great exactness; it is 24 hours, 37 minutes, 22.67 seconds. The inclination of the planet's orbit to the ecliptic is 1° 31′, and the inclination of its equator to the plane of its orbit is nearly 24° 50′. Mars is remarkable among the planets because of its reddish

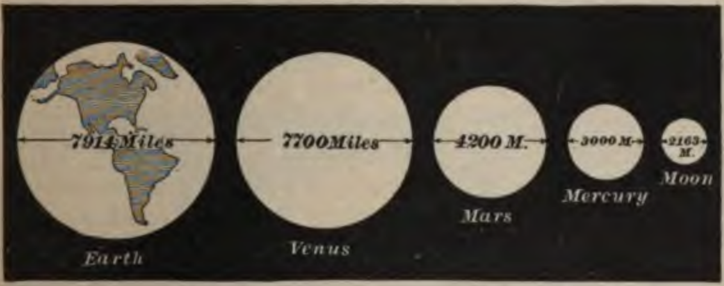


FIG. 26.

appearance. For telescopic observations, it is more favorably situated than any other celestial object, and especially so at the time of opposition, when it is nearest the earth. At that time it shines with a brilliant red light, surpassing even Jupiter in splendor. Some permanent markings on its surface have been revealed by the telescope, and from these its daily rotation has been determined. Several of those markings are commonly called "canals," and by some scientists are believed to contain water; while others suggest that the marks are strips of vegetation. However, no satisfactory explanation of their nature has yet been given. At times the polar regions, when turned away from the sun, exhibit a small white spot, generally believed to be snow or

one which gradually disappears when the pole is turned towards the sun.

169. The existence of an atmosphere on Mars has been [illegible] being about one-fourth of that of the earth [illegible].

170. Mars, in its travels around the sun, is accompanied by two satellites or moons, which were discovered in 1877 at the [illegible] by the eminent astronomer, Professor Hall. Of these the nearest one is 5,897 miles away from the center of Mars; the distance of the outer one, [illegible]. Their diameters, though not exactly known, are very small, the greater one not exceeding [illegible] miles.

THE ASTEROIDS.

171. The large space between Mars and the next planet, Jupiter, was, up to the close of the last century, a puzzle to astronomers. It seemed to break the continuity of the series of planets, and a strong belief was entertained that there must be a missing planet in that very space. In the latter part of the eighteenth century a systematic search for the planet began, which resulted in the discovery of the asteroids. Within the first seven years of this century four of these bodies were found; they were named *Ceres*, *Pallas*, *Juno*, and *Vesta*. To the Italian astronomer Piazzi belongs the honor of discovering the first asteroid. Then a number of years passed without any success to the asteroid hunters; in 1845, however, another was found and named *Astræa*. Since then asteroids have been discovered nearly every year, and at the present time about 300 asteroids are known.

172. The eccentricities of their orbits are much greater than those of the planets, and their mean distances from the sun differ considerably. They are not to be seen with the naked eye, and in the telescope they appear like faint fixed stars, but are easily distinguished from stars by their motion. Their dimensions are quite insignificant. The diameter of

the largest, Ceres, is 500 miles, while that of Vesta, the brightest one, is about 248 miles.

173. The origin of the asteroids is still a problem of the future. The hypothesis which is now regarded as most probable is that they are pieces of a planet about the size of Mars, which was broken into fragments by a series of explosions.

JUPITER.

174. The planets with which we have made the student acquainted in the preceding pages are, in comparison with those that follow, but pygmies. The diameters of the minor planets, including the earth, range from 3,000 to 8,000 miles; while the major planets, Jupiter, Saturn, Uranus, and Neptune, have diameters ranging from 32,000 to 90,000 miles, as illustrated in Fig. 27. Jupiter, the nearest

FIG. 27.

one, has a mean diameter of 88,000 miles, and its mean distance from the sun is 483,000,000 miles. The eccentricity of its orbit is about $\frac{1}{20}$; and, therefore, the actual distance varies about 21,000,000 miles on each side of the mean. The inclination of its orbit to the plane of the ecliptic is 1° 19′. The sidereal period of Jupiter is 11.86 years, while its synodic period is 399 days. Its sidereal day is 9 hours, 55 minutes, approximately. The exact value is not obtainable, because the surface of the planet does not move at a uniform rate. The volume of Jupiter is 1,300 times that of the earth, and its mass about 316 times greater than that of our planet.

175. Jupiter rotates on its axis in less than 10 hours, which means a velocity of 7¾ miles per second at a point situated on its equator. This rapidity of rotation causes it to be quite noticeably flattened at the poles. The polar diameter of Jupiter is only 83,000, while the equatorial is 88,200 miles; hence, its disk is distinctly oval.

176. The appearance of Jupiter when near opposition is brilliant. Its light is white, and presents quite a contrast to that of Mars. No permanent markings are found on its surface. A belt usually, but not always, of uniform breadth appears entirely across its disk. The matter of which Jupiter is composed is supposed to be in a fluid state and of a high temperature; the chief indication of this being the abundance of clouds and their swift transformation.

177. Jupiter has *four moons*, which revolve in almost circular orbits and lie very nearly in the plane of the planet's equator.

Eclipses of Jupiter's moons are very frequent because of the long and large shadow cast by the planet. Two other classes of phenomena are the eclipses of Jupiter itself when one of its moons casts a shadow upon it, and the occultation of the planet when a moon passes between it and the earth. An eclipse of Jupiter always precedes the occultation when the planet is east of opposition; and when the planet is west of opposition, the order is reversed.

178. An important use has been made of the eclipses of Jupiter's moons; by means of them the **velocity of light** has been determined. It was observed in 1675 by the Danish astronomer Ole Roemer that when the earth is in that part of its orbit which is nearest Jupiter, eclipses of Jupiter's moons occurred 8 minutes, 18 seconds **earlier** than the calculated time; and when the planet is in conjunction, the eclipses occur 8 minutes and 18 seconds **later** than the calculated time.

This periodical error of time he attributed to the fact that light travels with a finite velocity, and the correctness of this

view has since been established. Thus the time required for light to cross the earth's orbit is twice 8 m. 18 s., or 16 m. 36 s. (= 996 seconds). The diameter of the earth's

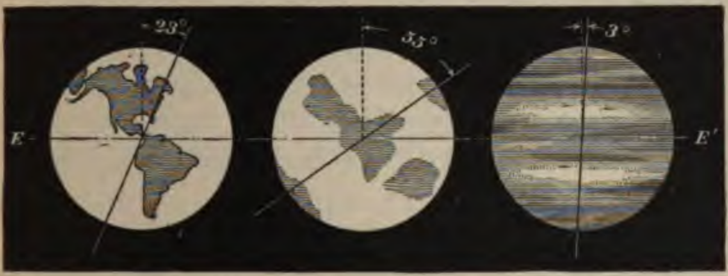


FIG. 28.

orbit is about 185,600,000 miles, and 185,600,000 divided by 996 gives 186,000, approximately. Hence, the velocity of light is about 186,000 miles per second.

In Fig. 28 is shown the inclination of the axis of the earth, Venus, and Jupiter to the plane of their respective orbits. From this can be seen that Jupiter has no seasons so far as the sun is concerned.

SATURN.

179. The mean distance of Saturn from the sun is about 886,000,000 miles, and the actual distance varies nearly 100,000,000 miles on account of the eccentricity of the orbit. The inclination of its orbit to the ecliptic is about 2° 30′.

The sidereal period is 29½ years and the synodic 378 days. Saturn's sidereal day is hard to determine because of the absence of well-defined spots on its surface.

According to Professor Hall, the time of rotation is 10 hours, 14 minutes, 23.8 seconds. The mean diameter of Saturn is 74,000 miles. It is more flattened at the poles than any other planet, and the difference between the equatorial and the polar diameter is about 7,000 miles. The surface of Saturn is about 84 times that of the earth, and its volume about 770 times greater than the earth's.

180. The most noticeable feature of this planet is the system of broad, thin rings which surround it. They were discovered by Galileo in 1610, but owing to the smallness of his telescope he was unable to discover their shape or character. Soon afterwards the edge of the rings was presented to the earth, and he could no longer see them. Forty years later they were again discovered and their nature explained by the Dutch astronomer Huyghens. They present an elliptical appearance to the earth, as shown in Fig. 29,

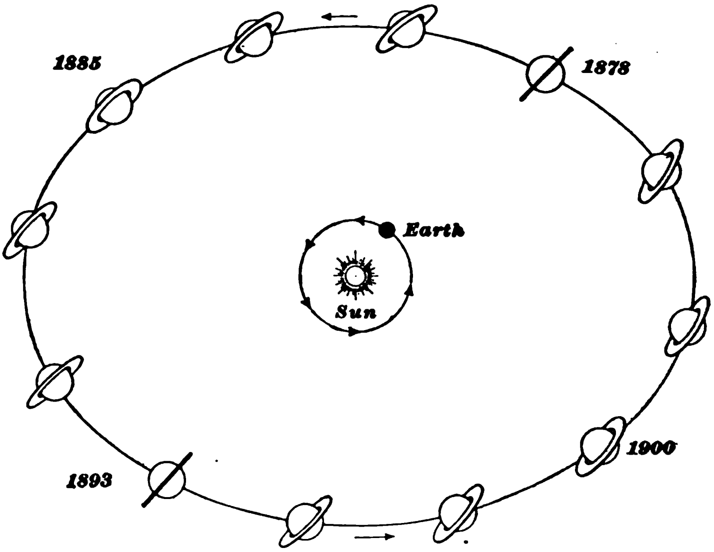


FIG. 29.

because of their inclination to the ecliptic, which amounts to about 28°. According to Professor Young, the exterior ring has a diameter of about 168,000 miles and a width of 10,000; the central, a width of 17,000, and the interior, 9,000 miles. The thickness of the rings does not exceed 100 miles. Researches have demonstrated that the rings are composed of millions of minute bodies, too small to be seen separately —an enormous mass of closely packed moons, each pursuing an independent orbit around the planet.

Sometimes the rings disappear; this phenomenon occurs

when the plane of the rings passes across the orbit of the earth—which happens twice in a revolution of the planet, and generally lasts nearly a year each time. Whenever the plane of the rings passes between the sun and the earth, the dark side of the rings is turned towards us, and, consequently, only the edge can be seen. The disappearances occur at intervals of about 15 years.

181. In the matter of satellites, Saturn is the most favored of planets. It has not less than eight attendants or moons; the largest of them, Titan, is nearly as large as Mercury. They all move in the plane of the rings, with the exception of Iapetus, the outermost, whose orbit has an inclination of 10° to the plane mentioned. The distance of Iapetus from the center of Saturn is very nearly 2,225,000 miles; while Mimas, the one nearest to the planet, is about 118,000 miles distant.

182. The physical condition of the planet is about the same as that of Jupiter, its density being less than that of water. It is surrounded by masses of clouds, which make observations of its surface very difficult.

URANUS.

183. Uranus was discovered about a century ago by William Herschel. It had, however, been seen before by earlier astronomers, who mistook it for a fixed star. By the addition of this new planet, the diameter of the solar system was doubled, and many were the honors bestowed upon the discoverer.

184. The mean distance of Uranus from the sun is about 1,782,000,000 miles. The eccentricity of its orbit is very nearly the same as that of Jupiter, $\frac{1}{21}$, and the actual distance varies about 35,000,000 miles on each side of the mean. The inclination of its orbit to the ecliptic is 0° 46′. Its diameter is 33,300 miles, and its mass is about 14½ times that of the earth. The sidereal period of Uranus is 84 years,

8 days, and its synodic is 369¼ days. The length of its sidereal day is unknown.

185. The appearance of the planet is like that of a small star, hardly visible to the naked eye. With the aid of a powerful telescope, its disk appears oval, but, according to Professor Young, there are no distinctive marks on it, except some faint traces resembling belts.

186. The atmosphere of Uranus has been investigated by means of the spectrum. It differs from ours by its powers of absorption, and resembles that of Saturn and Jupiter rather than that of the earth. It contains a gas that is not to be found on our planet.

Uranus has four moons, the largest of which has a diameter of about 500 miles. The distance of the outermost is 389,000, and of the innermost, 127,000, miles from the center of the planet. The remarkable facts relating to these moons are that their orbits are almost at right angles to the plane of the ecliptic, and that they move backwards, that is, from east to west—quite in contrast to moons of other planets.

NEPTUNE.

187. Neptune was found in 1846. The existence of this planet was revealed by mathematics, and its path marked out long before it was actually discovered. The discovery of Neptune is regarded as the greatest triumph of mathematical astronomy. The circumstances which led to the discovery are as follows: After the orbit of Uranus had been computed and corrected for the disturbing influence of Jupiter and Saturn, it was found that the planet departed from its calculated path and was misguided by some unknown force. This led to the belief that there must exist a planet superior to Uranus whose attraction caused the change of its orbit. Two eminent mathematicians, Le Verrier of France and Adams of England, began working, each without any knowledge of the other's intention, on the enormous task of calculating the mass and orbit

of the unknown planet which was disturbing the motion of Uranus. The results which they reached independently agreed remarkably well. Le Verrier communicated his results to the Berlin Observatory, and the planet was found by Galle within a degree of the predicted position, and within a half hour after the search was begun.

188. The mean distance of Neptune from the sun is 2,792,000,000 miles. The inclination of its orbit is about 1° 47′. Its diameter is 35,000 miles, and the volume is nearly 90 times that of the earth. Its mass, compared with that of the earth, is as 18 to 1. The sidereal period of the planet is 164 years, 281 days. Its time of rotation is unknown, because there are no visible markings upon its surface. Neptune is attended by one moon, the plane of whose orbit is inclined 35° to the ecliptic. This moon, like those of Uranus, moves backwards. From its brightness, the size of this moon is estimated to be about the same as that of our moon.

THE EARTH.

FORM OF THE EARTH.

189. The form of the earth is that of a nearly spherical globe. This fact can be proved in several ways. To an observer who is limited to its surface, the earth appears to be a flat plane, more or less diversified. If, however, we could change our point of view to a position in space, for instance at the moon, the earth would then appear as a round, luminous disk sprinkled with spots—the bright ones marking the continents and the darker ones indicating the oceans. Of the facts which prove the rotundity of the earth, we will mention the curvature of the surface of the sea, which manifests itself in a striking manner. Suppose yourself on board of a vessel at sea, or at the summit of a hill along the coast. Then, when a steamer appears on the horizon, the first indication of its presence is the smoke from its funnel. After awhile you see the upper parts of its spars

and rigging, but the lower part of the masts, the smoke-stack, and the hull are invisible. As the steamer approaches, these lower parts come into view, and later on the entire steamer can be seen, as shown in Fig. 30. In the same

FIG. 30.

manner the successive appearances of the different parts of the coast are manifest to the sailor, who from the steamer observes the land. As the curvature of the ocean is the same in every direction and at any place, it follows that the earth has really the form of a sphere, or at least differs from it very slightly. Another evidence of the earth's spherical form is the outline of the earth's shadow seen upon the moon during a lunar eclipse.

190. It has been found that the form of our planet, though very approximately a sphere, differs therefrom to an appreciable extent, and that its true form is nearly that of the solid produced by the revolutions of an ellipse about its minor axis; the polar diameter is about $\frac{1}{295}$ part shorter than the equatorial. According to recent determinations, the dimensions of the earth are as follows:

Equatorial radius = 3,963.307 miles.
Polar radius = 3,949.871 miles.

191. This flattening of the earth at the poles and bulging at the equator is the inevitable result of its rotation upon its axis. At the present time the earth is solid, and its rotation can hardly affect its shape. But in the ages before the earth became habitable, it passed through various stages of development, and it has been conclusively proved that it was not always solid as it is now. Before the earth became solid, it is evident that its rotation would have a great effect in causing it to bulge at the equator.

The rotation of the earth causes different points on the earth's surface to move with different velocities, the velocity of any point being determined by its latitude. A point on the equator moves round the equator, whose length is about 25,000 miles, in 24 hours; this is equivalent to a velocity of about 17 miles per minute. A point in the latitude of London, England, moves at the rate of 11 miles per minute; and a point at either of the poles has no motion due to the earth's rotation.

MOTION OF THE EARTH.

192. In Art. **48** it was pointed out that the apparent diurnal motion of the heavens is due to the earth's rotation; this rotation causes the phenomena of day and night.

The annual motion of the earth in its orbit about the sun has already been described; this annual motion of the earth determines the length of the year and produces the phenomena of the seasons.

The diurnal and the annual motion of the earth are its most considerable and conspicuous motions; yet it has other motions which, though smaller and slower, must be observed and explained.

193. Precession and Nutation.—It has been stated that the equinoctial points have a slow retrograde motion along the ecliptic; in other words, the equinoctial points are moving to meet the sun, and, consequently, the equinoxes occur at shorter intervals than they otherwise would; this phenomenon is called the precession of the equinoxes.

If the earth were a perfect sphere, its axis would constantly preserve the same direction, and there would be no such thing as precession. But the attraction of the sun and moon on the bulging matter at the equator causes the earth's axis to have a slow conical motion; and, therefore, the pole of the equator describes a small circle about the pole of the ecliptic, completing the circle in a period of 25,868 years.

A result of the precession of the equinoxes is the apparent annual change of position of all the stars in the heavens, returning to the same point only at the close of this great secular cycle. The explanation of this remarkable motion is due to the genius of the immortal Newton.

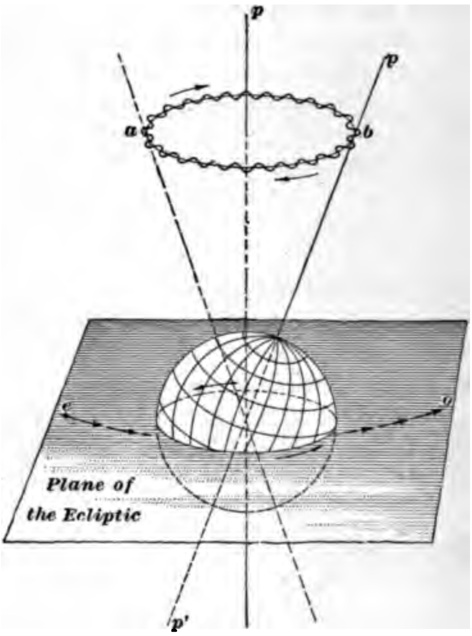


FIG. 31.

By precession alone the axis of the earth would move in the circumference of a circle *a b* (Fig. 31) about the pole of the ecliptic *p*. But this motion is modified by the unequal influence of the moon on the equatorial parts of the earth, which produces a vibration of about 9″ on each side of the circumference. Thus, the line described by the pole, as it advances, is a delicate wave lying along on the arc *a b*. This vibratory motion is called **nutation**, or nodding; the time required by the pole to describe one of these waves is 18 years and 8 months. The waves in the figure are of course greatly exaggerated. Represented in their true form, they would be small enough to cross the arc about 700 times.

194. Besides these motions, there are other disturbances which the earth suffers, the description of which lies beyond the scope of this Paper. They are principally caused by the attractive forces of the other planets.

THE SEASONS.

195. The earth, in its travel around the sun, keeps its axis always nearly parallel to itself. The axis is inclined to the plane of the orbit at an angle of 23° 27′. The position of the earth on the twenty-first of March, which also corresponds to the time of the vernal equinox, is represented in the lower part of Fig. 33. At that time the boundary circle of the illuminated portion of the earth passes through the two poles, and the result is that day and night are equal all over the globe. As the earth advances in its orbit, the north pole is gradually turned more and more towards the sun, while the south pole is turned away. This process continues until the twenty-first of June; this is the time of

FIG. 32.

the summer solstice, when the north pole is turned as much as possible towards the sun, and the south pole is turned away from the sun. The result is that everywhere in the northern hemisphere the days are long, while in the southern hemisphere they are short. As the earth continues its revolution, the north pole gradually turns away from the sun, while the south pole turns towards it; and when the twenty-first of September, the time of the autumnal equinox, Fig. 32, is reached, day and night are equal everywhere.

After passing that point, the north pole continues to turn away from the sun, and at the time of the winter solstice, December 21st, the days everywhere in the northern

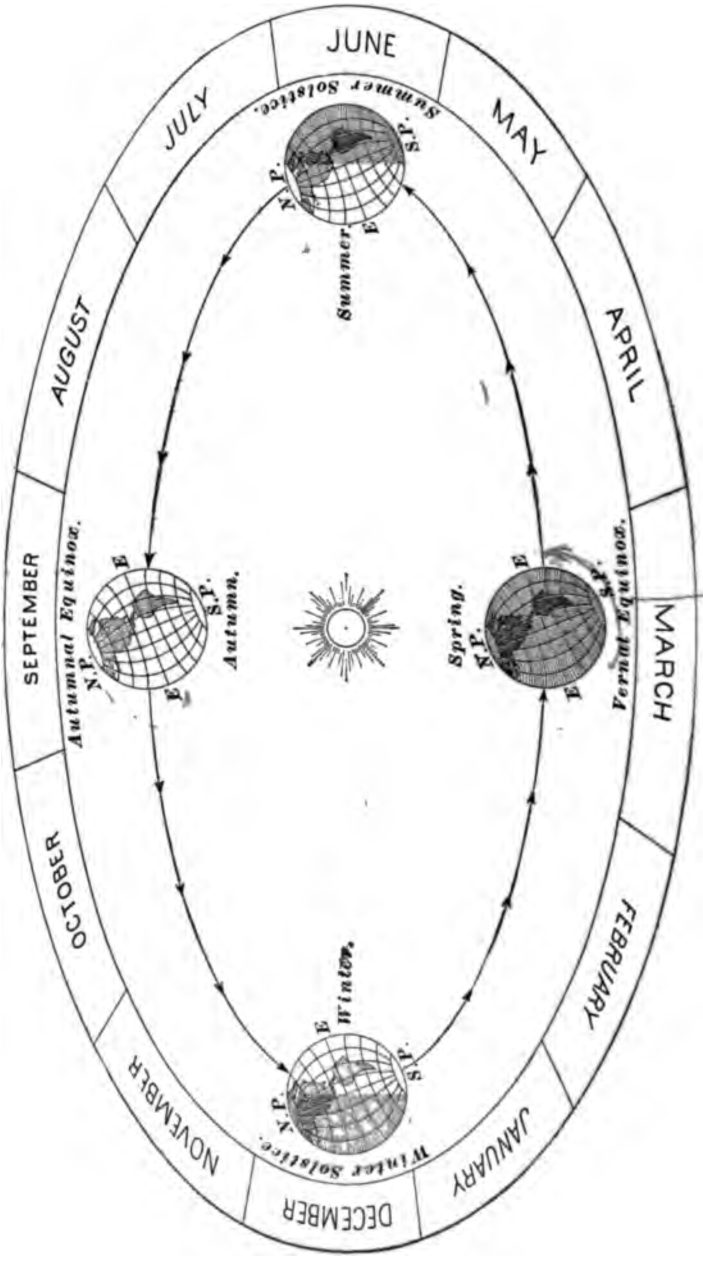

JUNE
MAY
APRIL
MARCH
FEBRUARY
JANUARY
DECEMBER
NOVEMBER
OCTOBER
SEPTEMBER
AUGUST
JULY
Summer Solstice.
N.P.
S.P.
E
Summer.
Autumnal Equinox.
N.P.
E
S.P.
Autumn.
Spring.
N.P.
E
S.P.
Vernal Equinox.
Winter.
E
N.P.
S.P.
Winter Solstice.

FIG. 33.

hemisphere are short, while the southern hemisphere, on the other hand, is enjoying long summer days. At the equator, night and day are of equal length the whole year round, and seasons, in the proper sense of the word, do not exist there. If a small circle parallel to the equator were drawn at a distance of 23° 27′ from each pole, it would form the boundary of the region of perpetual day and night at those places. On account of the eccentricity of the earth's orbit, the lengths of the different seasons are unequal. During spring and summer of the northern hemisphere, the earth is in that portion of its path where it moves less rapidly, and during the autumn and the winter it moves with a greater velocity. Hence, the spring and summer are of longer duration than the autumn and winter. The difference is not considerable, though still sufficient to be appreciable.

SURFACE AND VOLUME OF THE EARTH.

196. Because of the earth's nearly spherical form, its surface and volume can be calculated by using the formula for a perfect sphere. The surface thus found contains about 196,950,000 square miles. Of this immense surface, the seas and oceans embrace more than three-quarters; the remainder is occupied by the continents and islands. By the same method, the volume of the earth is computed to be about 259,900,000,000 cubic miles.

Experiments and calculations have shown that the most probable value of the **mean density** of the earth is $5\frac{1}{2}$ times that of water; or, in other words, that 1 cubic mile of the earth weighs $5\frac{1}{2}$ times as much as 1 cubic mile of water. Hence, we are able to get the approximate weight of the earth, which is about $6{,}096 \times 10^{18}$ tons.

THE ATMOSPHERE.

197. Our planet is entirely enveloped by a gaseous body known as the atmosphere. The height of this atmosphere is far greater than any height to which we can attain, though we can ascertain to some degree its approximate

limit. By measuring the thickness of the penumbra which surrounds the shadow of the earth on the moon at the time of an eclipse of the moon, the height of the atmosphere is estimated to be from 50 to 60 miles. It covers everything upon the earth, and its pressure upon each square inch of the earth's surface is nearly 15 pounds. The density of the atmosphere is a maximum at the surface of the earth, and gradually diminishes until the confines are reached, where the density is zero.

THE MOON.

198. The earth's companion in its trackless path around the sun is called the **moon.** The average distance of the moon from the center of the earth is about 60.3 times the earth's equatorial radius, or 238,840 miles.

The average velocity of the moon's motion, when the size and form of her orbit are known, is easily computed, and it is found to be 2,287 miles an hour. The moon's orbit, like the earth's, is an ellipse, the earth being one of its foci. Its eccentricity is estimated to be 0.0549, and its inclination to the plane of the ecliptic 5° 8.7′. The moon has only $\frac{1}{49}$ of the size and $\frac{1}{81}$ of the weight of the earth. Its influence upon the ocean and the atmosphere is, nevertheless, comparable with that of the sun, and perhaps, to a certain degree, even more important as regards the production of tides.

199. The revolution of the moon around the earth in relation to *the stars* takes place in 27 days, 7 hours, and 43 minutes. This period is called a **sidereal month.** But during this time the earth has not been motionless, and, consequently, the sun appears to have advanced a certain distance. The moon requires about 2 days more to make up this distance and to return to the same point in relation to *the sun;* this period is called a **synodic month;** its average length is 29 days, 12 hours, 44 minutes, and 2.9 seconds.

200. As stated before, the inclination of the moon's orbit to the ecliptic is somewhat more than 5°, and the points

where the orbit crosses the circle of the ecliptic are called the *moon's nodes.* The point where the moon passes the ecliptic from the south to the north side is called the **ascending node,** and the point where the moon passes from north to south of the ecliptic is the **descending node.** These nodes, however, are in constant motion—sliding westwards on the ecliptic, like the vernal equinox, and completing their revolution in 18½ years.

201. The moon rotates on its own axis in the same period in which it makes a revolution about the earth. Since the axis of the moon is very nearly perpendicular to the line joining the center of the moon to the center of the earth, the result of the moon's rotation is that the same face of the moon is presented to the earth. Fig. 34 will explain how

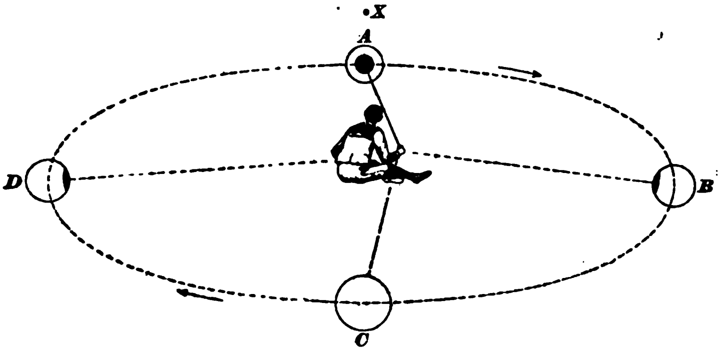


FIG. 34.

the moon accomplishes this feat. The string is attached to the center of the black spot upon the ball, and the boy causes the ball to revolve in the circle *A B C D*, starting from *A* and returning to it again. During this revolution the black spot is always turned towards the boy. When the ball is at *A*, the black spot is turned away from the point *X*; when the ball arrives at *C*, the black spot is turned towards the point *X*. Consequently, in revolving from *A* to *C*, the ball has made a half rotation; and it is evident that when the ball arrives at *A* again, it will have made a complete rotation.

202. Though the moon always presents the same face to the earth, yet parts of the moon's surface, along the edge of the face which is turned towards the earth, which are invisible at one time are visible at other times. This phenomenon is called **libration.**

203. Libration in Latitude.—We have already shown how the north and the south pole of the earth are alternately presented to the sun. In like manner the north and the south pole of the moon are alternately turned towards the earth, owing to the inclination of the moon's axis to the plane of her orbit. The consequence is that at one time we see about $6\frac{1}{2}°$ beyond the moon's north pole, and at another time we see about $6\frac{1}{2}°$ beyond its south pole. This phenomenon is called **libration in latitude.** The small circle of $6\frac{1}{2}°$ radius at the moon's north pole and the corresponding one at the moon's south pole are related to the earth as the arctic and the antarctic circles of the earth are related to the sun.

204. Libration in Longitude.—The moon's rotation is uniform; but since her orbit is eccentric, her motion of revolution is not uniform. The effect of this is that a few degrees of the eastern edge and a few degrees of the western edge of the moon's face become visible alternately. This is called the moon's **libration in longitude.**

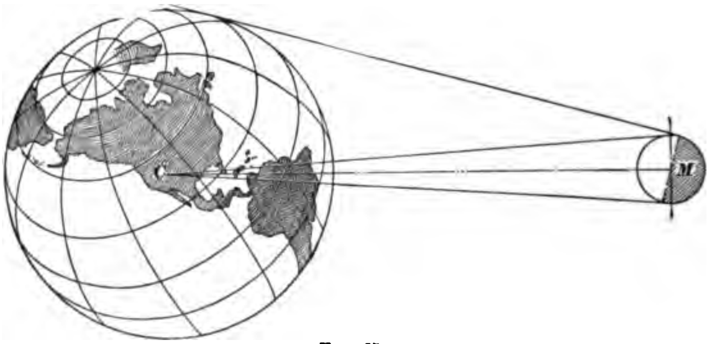


FIG. 35.

205. Diurnal Libration.—When the moon is on the horizon, the effect of horizontal parallax is to enable us to

look over the western edge of the moon when it is rising, and to look over the eastern edge when it is setting. This is called **diurnal libration.**

Diurnal libration is illustrated in Fig. 35, where *O* is the observer, *C* the earth's center, and *M* the moon. When the moon is on the meridian, we view it nearly as from the center *C*; but when it is on the horizon, we see it from a point 4,000 miles higher, and this extends our vision a short distance over its limb when it is rising and setting. Astronomers claim to have seen about 59 per cent. of the moon's surface.

PHASES OF THE MOON.

206. The moon is not a self-luminous body; and the light coming from it—moonshine—is simply reflected

FIG. 36.

sunlight. The various forms of the visible portion of the moon's illuminated surface are called **phases,** and are caused by the moon's continual change of position in relation to the sun and the earth. Fig. 36 shows the various phases of the moon, the direction of the sun's rays being indicated by the arrows.

When at A the moon is in conjunction, and her dark side is turned towards the earth, which renders her wholly invisible; this is called *new moon.* At B the illuminated part commences to be visible, and at C half of her illuminated hemisphere is seen; this phase is called the *first quarter.* When the moon is at F she is in opposition, and the whole of her illuminated surface is turned towards us; we then have *full moon.* From F to A the phases are repeated in reverse order, H being the *last quarter.* When less than half of the illuminated part is visible, we have the *crescent* phase. When more than half of the illuminated part is visible, we have the *gibbous* phase.

ECLIPSES.

207. Eclipses of the Moon and Sun.—The moon is **eclipsed** when it is obscured wholly or in part by the earth's shadow. This can only occur at opposition or full moon. An eclipse of the sun occurs when the moon comes between it and the earth; this can happen only at conjunction or at new moon. There are two kinds of lunar eclipses, *partial* and *total.*

An eclipse is partial when only a portion of the moon enters into the shadow, and it is total when she passes completely into the shadow. Before going further, let us study the shape of the shadow cast by earth and moon, respectively.

In Fig. 37, S represents the sun, E the earth, M' the moon at conjunction, and M the moon at opposition. The darkly shaded conical portion $a\ b\ c$ of the earth's shadow is called the **umbra.** The lightly shaded portions $d\ a\ c$ and $e\ b\ c$, bounded by tangents drawn across the opposite sides of the earth and sun, are called the **penumbra of the shadow.**

When the moon is at M', or in conjunction, the dark space enclosed by $g\ x\ r$ is the umbra, and portions $h\ g\ x$ and $n\ r\ x$ the penumbra of the moon's shadow. Eclipses of the moon are caused, as mentioned before, by the earth passing between her and the sun. If the orbit of the moon were in the same plane as the ecliptic, a lunar eclipse would occur every month. However, her orbit is inclined to the ecliptic, and as a result lunar eclipses are not frequent—seldom more than two in a year. An eclipse of the moon is possible only when opposition happens near the line of nodes, so that some part of the three bodies lies in a straight line. At other times the moon passes north or south of the shadow without even touching it. A lunar eclipse can never occur when

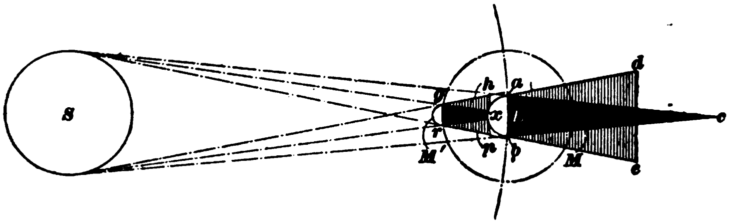


FIG. 37.

the moon's latitude exceeds 63′. That an eclipse of the sun may occur, the moon's latitude must be less than 94′, otherwise the moon's shadow would pass over the earth or under it. Solar eclipses are consequently more frequent than lunar eclipses. But a solar eclipse is visible only from a small portion of the earth, and an eclipse of the moon can be seen over more than half the earth; hence, the number of lunar eclipses visible at any place exceeds the number of solar eclipses visible at that place. In a period of 18 years, 70 eclipses are possible, of which 41 are solar and 29 lunar. The greatest number of eclipses in a year is 7, and the smallest number is 2.

The sun in its annual progress must pass through one of the moon's nodes about every 6 months. Thus the eclipses of any year always occur in *clusters*, with an interval of half a year. In the year 1863, a solar eclipse occurred on May 17th

and a lunar eclipse on June 1st, a solar eclipse on Nov. 10th and a lunar eclipse on Nov. 24th.

When the umbra $g\,x\,r$ (Fig. 37) of the moon's shadow is not long enough to reach the earth, which occurs when the moon's angular semidiameter is less than that of the sun, the eclipse is called **annular.**

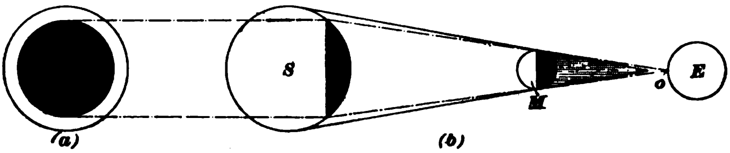


FIG. 38.

The annular eclipse is illustrated in Fig. 38. To an observer at *o* the moon will appear smaller than the sun, and the effect will be as shown in (*a*).

208. The Tides.—The tides arise from the combined attraction which the moon and the sun exercise upon the water with which the earth's surface is partially covered. They consist in a regular rise and fall of the waters of the ocean, and the average interval between two successive high waters at the same place is about 24 hours and 50 minutes. The sea rises for about 6 hours—covering the shores and forcing its way up the mouth of the rivers. Then, after having reached its extreme height and remained unchanged for a quarter of an hour, it begins gradually to retreat. The water retreats for about 6 hours, and after a repose of 15 minutes at its lowest point, it again begins to rise. When the water in this daily oscillation has reached its highest point, it is called **high water**; at its lowest point, it is called **low water.** While the water is rising, it is called **flood,** and while falling, **ebb.** On opposite sides of the globe there are two tide-waves, moving around it from east to west, and arriving at any place at intervals of which the mean value is about 12 hours and 25 minutes.

The effect produced by the attraction of either the moon or the sun is to heap up the waters of the ocean in the direction of the disturbing body, and also in the opposite direction,

so as to cause the earth to assume the form of an ellipsoid, whose longer axis is in the direction of the disturbing body. Although the mass of the moon is very small in proportion to that of the sun, it produces a greater effect in the production of the tides, because it is so near to the earth. When the moon is nearest to the earth, the tides are nearly 20 per cent. higher than when it is farthest away.

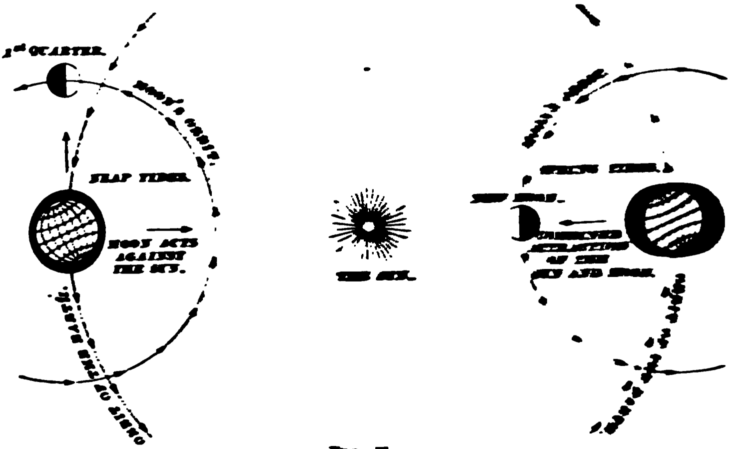


FIG. 39.

When the attraction of the moon and the sun acts in the same straight line, which happens at the times of new and full moon, the effect produced is the greatest possible, and the tides at such junctures are called **spring tides** (Fig. 39). When the moon and sun act in directions at right angles to each other, which occur when the moon is in first or last quarter, the effect of the disturbing bodies is the least, and tides at such junctures are termed **neap tides**. High water at any place does not occur when the moon is actually on the meridian, but about two hours afterwards; and the reason for this is that the waters of the ocean being very inert do not yield immediately to the attraction which the moon exercises upon them. The wind also has a certain influence upon the tides. If a strong gale of wind is blowing in the direction of the tide, the water reaches a greater height than during calm weather. The height which the

tides reach is different at different points of the earth. For instance, near St. Malo, in the English Channel, the tides are sometimes 50 or 60 feet, while in the Bay of Biscay they rarely exceed 28 feet. At the Island of Roumon and other places in the great Southern Ocean, the height of the tides is but 18 inches. In enclosed seas, as the Baltic and the Black Sea, there are hardly any tides; in the Mediterranean and Red Sea, the range of elevation is quite small, at no time exceeding 4 inches. The height of the tide at any place depends largely upon local conditions. Thus, the ports of the English Channel are subject to strong tides, because the waters rise higher when they meet with an obstacle in the narrowing of the coasts; and the farther the gulf is penetrated, the greater is the height of the tides. A knowledge of the tides is of great importance to navigators, because at high tide the waters of the ocean are forced up the rivers and bays, thus enabling ships of heavy tonnage to pass over the bars and banks which are generally situated at their entrances. The tides also prevent the sea from becoming stagnant by distributing in all directions the salt which the ocean contains.

DETERMINING TERRESTRIAL LATITUDE AND LONGITUDE.

209. The position of a place on the earth is determined by its latitude and longitude; hence, the determination of these quantities is one of the important problems of practical astronomy. We shall now explain how terrestrial latitude and longitude are found by astronomical observations, and in the first place describe the method of determining the quantities upon which the **latitude** depends.

210. To Find the Altitude and the Zenith Distance of a Celestial Body.—Observe the altitude of the body by means of a sextant or transit instrument. If the sun is the observed body, begin to measure a few minutes before noon. The altitude will then constantly increase till apparent noon, when it will stop and begin to decrease. The

highest altitude attained is the meridian altitude. To this observed altitude must be applied several corrections in order to get the true altitude.

1°. A correction for "index-error" in case a sextant is used.

2°. If the lower limb of the sun has been brought in contact with the horizon (which is the most usual), the sun's angular semidiameter must be **added** to the observed altitude; if the upper limb is observed, the semidiameter must be **subtracted** from it to get the altitude of the sun's center. The semidiameter is found in the Nautical Almanac for each day of the year.

3°. If the sea horizon is used, a correction for the "dip" of the visible horizon (Art. **38**) must be applied. This correction is always subtractive from the observed altitude and depends entirely upon the elevation of the observer. Table II at the end of this Paper gives the amount of dip corresponding to several elevations.

4°. There is also a correction to be applied for refraction (Art. **119**). Since refraction tends to increase the observed altitude, this correction is subtractive; its amount is given in Table I, corresponding to altitudes from 5′ to 90°.

5°. If the body is not a star, its observed altitude must also be corrected for parallax. Table III gives the sun's parallax; it is always additive to the observed altitude.

Having obtained the true altitude, it is subtracted from 90°. The result is the zenith distance of the observed body.

211. The zenith distance of a body may be either north or south, and is **named** accordingly. *If the observer's face is turned northwards when measuring the altitude, the zenith distance is south; if the observer's face is southwards, the zenith distance is north.* Thus, in Fig. 40, if the zenith is at Z, the zenith distance $S_1 Z$ of the star S_1 is south, and the zenith distance $S_2 Z$ of the star S_2 is north. If the zenith is at Z_1, the zenith distance $S_3 Z_1$ of the star S_3 is south, and the zenith distance $S_4 Z_1$ of the star S_4 is north.

212. To Find the Sun's Declination.—The sun's declination at Greenwich apparent noon is given in the

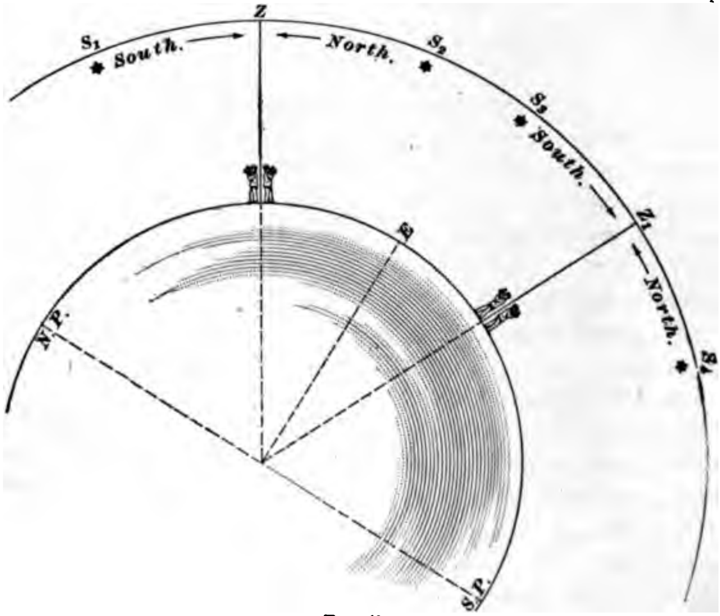


FIG. 40.

Nautical Almanac, and also the hourly variation of the declination. To find the sun's declination when it is apparent noon with the observer, he must know the interval of time between Greenwich apparent noon and his apparent noon.

If the longitude is known approximately, the interval may be found by converting the longitude into time (Art. **71**); or it may be taken from a chronometer registering Greenwich time. It is convenient to express this interval as hours and decimals of an hour.

We may now lay down the steps in finding the declination in the following rule:

Rule.—I. *From the Nautical Almanac take the declination at Greenwich apparent noon, and its hourly variation.*

II. *From the chronometer, or from the longitude, find the*

difference 'ime between Greenwich apparent noon and local apparent n.

III. *Multiply the hourly variation of declination by the difference of time, and apply this product to the declination in the following manner:*

When	*the*	*long.*	*is*	*west*	*and*	*declination*	*increasing*,	*add correction.*
"	"	"	"	"	"	"	*decreasing*,	*subtract* "
"	"	"	"	*east*	"	"	*increasing*,	*subtract* "
"	"	"	"	"	"	"	*decreasing*,	*add* "

Example.—Suppose an observer is stationed in longitude west 100° 30′ on April 3d, 1898. Find the sun's declination at noon for that place.

Solution.—Sun's Decl., April 3d = N 5° 25′ 49.9″
Correction for 6.7 h. = + 6′ 23.9″
Corrected Decl. = N 5° 32′ 13.8″. Ans.

Long. turned into time, 100° 30′ = 6 h. 42 m. = 6.7 h.

Diff. for 1 hour 57.31″
6.7
40117
34386
60)383.977
Cor. 6′ 23.9″

In this example the longitude is west and the declination increasing; hence, the correction is to be added. Had the longitude not been known, and the chronometer at noon showed 6 h. 42 m., the result would have been the same, provided the observer had known whether he was east or west of Greenwich and had applied the correction according to above rule.

Example.—Suppose the observer to have been in longitude east 154° 30′; what is the sun's declination for apparent noon at that place on August 30th, 1898?

Solution.—Sun's Decl., August 30th = N 8° 54′ 36.8″
Correction for 10.3 h. = + 9′ 14.4″
Corrected Decl. = N 9° 3′ 51.2″. Ans.

Long. turned into time, 154° 30′ = 10 h. 18 m. = 10.3 h.

Diff. for 1 hour 53.83″
10.3
16149
5383
60)554.449
Cor. 9′ 14.4″

In this case the longitude is east and the declination decreasing; hence, the correction is to be added.

213. Declination may be either north or south, and is **named** north or south accordingly. The declination of the sun can never exceed 23° 27′ 30″ in either direction. On the twenty-first or twenty-second of March the sun is on the equator, and its declination is zero. From this date to the twenty-first of June, the sun's declination is *north* and *increasing;* from June twenty-first to the twenty-second or twenty-third of September it is *north* and *decreasing.* On the twenty-second or twenty-third of September the sun is again on the equator, and its declination is zero. From this date to the twenty-first of December the sun's declination is *south* and *increasing;* and from the twenty-first of December to the twenty-second of March it is *south* and *decreasing.*

214. To Find the Latitude by a Meridian Altitude of a Celestial Body.—The general formula for finding the latitude from an observed meridian altitude is

$$\text{Latitude} = \text{zenith distance} \pm \text{declination}.$$

This formula can be proved from Fig. 41. In this figure, N is the north point, S the south point, Z the zenith, P the pole, and Q is the point where the observer's meridian cuts the equator.

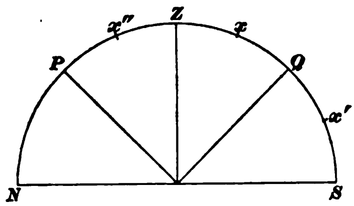


FIG. 41.

If a celestial body is situated at x, the declination of x is the arc $Q\,x$, and is northerly; the zenith distance of x is the arc $x\,Z$, and is also northerly. In this case, therefore, the latitude $Q\,Z$ is equal to the sum of the declination and zenith distance.

If a celestial body is situated at x', its declination is the arc $Q\,x'$, and is southerly; its zenith distance is the arc $x'\,Z$, and is northerly. Hence, in this case, the latitude $Q\,Z$ is equal to the difference between the zenith distance and the declination.

For a celestial body at x'', the declination $Q\,x''$ is northerly, and the zenith distance $x''\,Z$ is southerly. In this case

the latitude [illegible] Z is [illegible] declination and [illegible]

Thus we get [illegible]

Rule.—*[illegible] they are of the same [illegible] from the larger [illegible] result is the [illegible] quantity.*

215. Should the [illegible] distance is the [illegible] the declination is the [illegible] 90° unless by some great [illegible]

216. In the [illegible] mined by the meridian [illegible]

EXAMPLE.—On April [illegible] meridian altitude of the sun's center was [illegible] south. Find the latitude.

SOLUTION.—Decl. = N [illegible]
Cor. for [illegible] = — [illegible]
Cor. Decl. = N [illegible]

Long. [illegible] = [illegible]

True merid. alt. = [illegible]

Zenith Dist. = [illegible]
Declination = N [illegible]
Latitude = N [illegible]

EXAMPLE.—On May 3d, [illegible] ian altitude of the sun's center was [illegible] south. Find the latitude.

SOLUTION.—Decl. = N 15° 45′ 44.7″
Cor. for 2.3 h. = — 1′ [illegible]
Cor. Decl. = N 15° 47′ 35.5″

1 4 [illegible]

Long. turned into time = 2.3 h.

True merid. alt. =	58° 20′ 6″
	90° 0′ 0″
Zenith Dist. =	N 31° 39′ 54″
Declination =	N 15° 47′ 25.6″
Latitude =	N 47° 27′ 19.6″ Ans.

EXAMPLE.—On May 10th, 1898, in longitude west 114° 3′, the observed altitude of the sun's lower limb was 67° 14′ 20″, the observer facing south. The index-error was + 1′ 20″, and the height of the eye 20 feet. Find the latitude.

SOLUTION.—The observed altitude must be corrected as explained in Art. **210.**

Decl. =	N 17° 42′ 0.0″	Diff. for 1 hour 3 9.0 4″
Cor. for 7.6 h. =	+ 4′ 56.7″	7.6
Cor. Decl. =	N 17° 46′ 56.7″	2 3 4 2 4
		2 7 3 2 8
		60)2 9 6.7 0 4″
		4′ 5 6.7″

Long. turned into time = 7.6 h.

Obs. alt. sun's lower limb =	67° 14′ 20″
Index-error =	+ 1′ 20″
	67° 15′ 40″
Dip of the horizon =	− 4′ 23″
App. alt. lower limb =	67° 11′ 17″
Sun's semidiameter =	+ 15′ 52″
App. alt. sun's center =	67° 27′ 9″
Refraction =	− 24
	67° 26′ 45″
Parallax in alt. =	+ 3″
True alt. sun's center =	67° 26′ 48″
	90 0 0
Zenith dist. =	N 22° 33′ 12″
Cor. decl. =	N 17° 46′ 56.7″
Latitude =	N 40° 20′ 8.7″ Ans.

It will be noticed in this example that the parallax does not amount to much. In altitudes of 70° or more it can simply be omitted.

217. When the latitude is found from the meridian altitude of a fixed star, the observed altitude has not to be corrected for parallax or for the angular semidiameter of

219. Example.—On January 29th, 1898, in the evening, the observed meridian altitude of the star *Aldebaran* was 52° 36′, the observer facing south, and the height of his eye being 30 feet. Find the latitude.

Solution.—

Obs. alt. of *Aldebaran* =	52° 36′ 0″
Dip of horizon =	− 5′ 22″
App. alt. of the ∗ =	52° 30′ 38″
Refraction =	− 0′ 43″
True alt. of the ∗ =	52° 29′ 55″
	90 0 0
Zenith dist. of the ∗ =	N 37° 30′ 5″
The ∗ declination Jan. 29 =	N 16° 18′ 21.6″
Latitude =	N 53° 48′ 26.6″. Ans.

Example.—On the 18th of July, 1898, in the morning, the observed meridian altitude of the star *Fomalhaut* was 73° 36′, the observer facing south, the height of his eye 35 feet, and the index-error of the instrument being + 1′ 30″. Find the latitude.

Solution.—

Obs. alt. of *Fomalhaut* =	73° 36′ 0″
Index-error =	+ 1′ 30″
	73° 37′ 30″
Dip of the horizon =	− 5′ 48″
App. alt. of the ∗ =	73° 31′ 42″
Refraction =	− 0′ 17″
True alt. of the ∗ =	73° 31′ 25″
	90 0 0
Zenith dist. of the ∗ =	N 16° 28′ 35″
The ∗ declination =	S 30° 9′ 19.3″
Latitude =	S 13° 40′ 44.3″. Ans.

220. Thus far the meridian altitudes have been taken at the upper culmination of the celestial bodies, but it is sometimes possible to get an altitude of a star when it is crossing the meridian below the pole; and in high latitudes this can also be done with the sun. When taking an altitude below the pole, it must be remembered that until the body reaches the meridian, the altitude decreases continually, and after it passes the meridian the altitude increases continually; in this case, therefore, the lowest altitude is the meridian altitude. In northern latitudes the polar distance

is reckoned from the north [illegible] pole. The latitude in such case [illegible] true altitude to the polar distance, which is the [illegible] to declination.

EXAMPLE.—On September [illegible] star *A Ursa Majoris* was [illegible] height of the observer's eye was [illegible]. Find the [illegible].

SOLUTION.— Obs. alt. of *A* [illegible] = [illegible]

The $*$ decl. = N [illegible] 17 [illegible] [illegible]

Polar dist. = [illegible]

[illegible] = [illegible]

NOTE.—On the last page of the [illegible] given a table for computing the [illegible] *Polaris* at any time whether the star is [illegible] hour-angle being approximately known; [illegible] accompany the table, these [illegible] well to remember that the table given in the [illegible] only for that year and [illegible] priate corrections.

221. Longitude by the [illegible]. [illegible] tionably the best method [illegible] opportunity, is to make a direct [illegible] between his local time and that [illegible] tude of which is known. The [illegible] times, corrected for [illegible] true difference of longitude [illegible] common way to determine the [illegible] local mean time with the [illegible] by a chronometer. The [illegible] accurate watch, but [illegible] The observer must know the [illegible] be able to apply the necessary [illegible] true Greenwich time and the [illegible] clocks are in some [illegible] eters, but even they are [illegible]

The difference between [illegible] mean time is the longitude [illegible]

greater, the longitude is *west;* if Greenwich time is the smaller, the longitude is *east.*

The observer finds his mean local time by observations of celestial bodies. If it were possible to determine exactly the moment when the sun is on the meridian, this would be 12 o'clock apparent local time, and by applying the equation of time as found in the Nautical Almanac, the observer could find his mean local time. The exact moment when the sun's altitude has attained its maximum value can not, however, be determined within two or three minutes, at least not with a sextant, and at the equator one minute of time is equivalent to fifteen miles of longitude. Hence, the longitude can not be found in this manner, though many persons believe this is the usual method. True local time is usually found by measuring the altitude of the sun, not at or near noon, but when the sun is nearly east or west, and about fifteen or twenty degrees above the horizon, which usually happens about 8 in the morning or 4 in the afternoon. At the instant of taking the sun's altitude, the time registered by the chronometer is *carefully* noted. From the sun's observed altitude, the sun's hour-angle, or the apparent time,

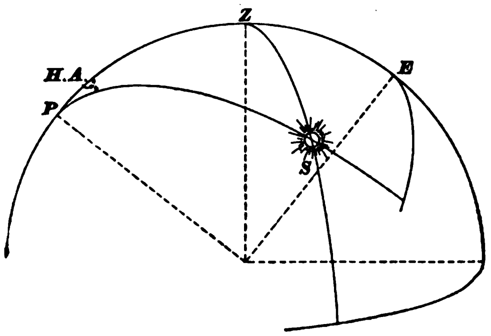


FIG. 42.

is found by solving a spherical triangle. In Fig. 42, P is the pole of the celestial sphere, Z the zenith, PZE the meridian, and S the sun or other celestial body. In the spherical triangle PZS, PZ is the complement of the latitude, PS is the polar distance of the celestial body, and ZS

is its zenith distance. Hence, PZ is known when the observer's latitude is known; PS is the complement of the declination of the celestial body which is found in the Nautical Almanac, and ZS is the complement of the observed altitude. Thus, the three sides of the triangle PSZ are known, and the hour-angle ZPS is found by the following rule:

Rule.—*From the sine of the altitude subtract the product of the sine of the latitude and the cosine of the polar distance, and divide the remainder by the product of the cosine of the latitude and the sine of the polar distance. The quotient is the cosine of the hour-angle.*

This rule is expressed in the formula

$$\operatorname{Cos} ZPS = \frac{\sin a - \sin l \cos p}{\cos l \sin p},$$

where a = true altitude of celestial body;
l = latitude of place;
p = polar distance of celestial body.

EXAMPLE.—On May 17th, 1861, about 5 P. M., an observer in latitude N 48° 51′ found the true altitude of the sun's center to be 25° 32′ 48″ when the true chronometer time was $6^h\ 51^m\ 37.7^s$. Find his longitude.

SOLUTION.—We have a = true altitude = 25° 32′ 48″,
l = observer's latitude = 48° 51′.

From the Nautical Almanac, sun's declination =	N 19° 23′ 26″
Cor. for 6.9 h. =	+ 3′ 38″
Cor. decl. =	N 19° 27′ 4″
	90°
Hence, p = sun's polar distance =	70° 32′ 56″

From the table of Natural Functions, we find

$\sin a = .43125$,	$\sin l = .75299$,	$\sin p = .94292$,
	$\cos l = .65803$,	$\cos p = .33300$.

Substituting these values in the formula, we get

$$ZPS = 73° 5′ 20″,$$

which, when converted into time, gives

local apparent time = $4^h\ 52^m\ 21.3^s$.

comets the nucleus is very brilliant and forms a conspicuous object in the sky even by day.

225. Almost 400 comets were recorded before the invention of the telescope; since the invention of the telescope the number has increased enormously, and there seems no limit to the number existing in space. Sometimes as many as eight telescopic comets are discovered in a year, and almost every day there is at least one in sight.

Brilliant comets, however, are comparatively rare; according to Prof. Newcomb, between 1500 and 1800 there were 79 visible to the naked eye.

226. In ancient times, comets were regarded with terror, and their appearance was considered a token of coming disasters, of war, pestilence, or death. Modern science has completely dispelled this superstition, and proved that comets produce not the slightest effect upon terrestrial conditions.

227. The time during which they are visible varies greatly. The comet of 1811 was visible for seventeen months, that of 1861 was visible for twelve months, that of 1882 for five months. Some do not remain visible for more than a few weeks.

228. Their Orbits.—Comets enter the solar system from enormous distances, and after passing round the sun, retreat to enormous distances again. Some comets visit the sun but once, and then disappear forever into the depths of space from which they came; others return again and again to the sun at regular intervals, and are therefore called **periodic** comets.

Periodic comets move in elliptic orbits, as the planets do, and their orbits can be computed and the time of their reappearance predicted. If the comet returns at the predicted time, we know that its orbit and period have been correctly calculated.

The orbits of comets that are not periodic can also be

computed, but as they never return, we have no means of testing the correctness of the computed orbit.

The inclinations of comets' orbits to the ecliptic vary from 0° to 90°. The motions of some comets are in the same direction as that of the planets, while the motions of others are retrograde.

229. The head of a comet is seldom less than 10,000 miles in diameter; their diameters usually range from 40,000 to 100,000 miles. The diameter of the head of the comet of 1811 measured 1,200,000 miles. The comet of 1882 had a tail 100,000,000 miles long.

The diameter of the nucleus of different comets ranges from 100 miles to 8,000 miles.

From these figures, it is evident that the volume of a comet is enormous. The mass of a comet, however, is exceedingly small.

230. The tail of a comet resembles the cloud of smoke puffed out from a smokestack of a steamer seen when disappearing at the horizon. Like the smoke from a steamer, the particles of the tail do not return, and, therefore, the comet must ultimately be reduced in mass as well as brightness. Comets, like the planets and moons, shine by reflected sunlight, which is proved by their diminishing brightness when they recede from the sun. In some cases, however, they are self-luminous, especially when near the sun, a fact proved by spectrum analysis.

231. The following table gives the elements of the orbits of thirteen periodic comets which have been observed at more than one visit to the sun. The first column gives the name of the comet, the second column its periodic time in years, the third shows the inclination of its orbit to the ecliptic, the fourth its greatest distance from the sun expressed in terms of the radius of the earth's orbit as a unit, and the fifth column gives its least distance from the sun in terms of the same unit.

Names of Comets.	Period. Years.	Incli-nation.	Aphelion Distance.	Perihe-lion Distance.
Encke's	3.3	13°	4.097	0.343
Temple's, 1873	5.2	13°	4.665	1.346
Temple's, 1867	6.5	11°	4.897	2.073
Temple-Swift's	5.5	5°	5.163	1.073
Winnecke's	5.8	14°	5.582	0.883
Brorsen's	5.5	29°	5.613	0.590
D'Arrest's	6.7	16°	5.772	1.326
Faye's	7.5	11°	5.970	1.738
Biela's	6.6	13°	6.182	0.860
Tuttle's	13.7	55°	10.460	1.025
Olber's	72.6	45°	33.616	1.200
Pons-Brooks'	71.4	74°	33.616	0.775
Halley's	76.4	162°	35.411	0.589

232. Halley's comet is the only one of long period the elements of whose orbit (see Fig. 44) are all known. It was observed by Halley in 1682, who came to the conclusion that its path was nearly identical with those of the comets of 1607 and 1531, and that the three were one and the same, and he predicted that it would return in 1759, which it did. It also returned again in 1835. On its last return, the period of this comet had been increased nearly two years by the attractions of Jupiter and Saturn.

Encke's comet has the shortest period of all and is insignificant in appearance. By laborious computations, Encke showed that its periodic time was shortened nearly three hours at each revolution, and this may be due to collisions with small bodies coming across its path. One of the most remarkable occurrences in the history of comets was the splitting of Biela's comet into two distinct comets in 1845. At first the two comets were unequal in size, but after a while the smaller one became brighter and brighter until it surpassed the other for a time. After the splitting, the two

comets moved in separate and independent orbits. The great comet of 1882 was discovered by observers in the southern hemisphere. It was of unusual brilliancy, and moved swiftly right up to the edge of the sun, and then became invisible as it swept across the disk, giving no evi-

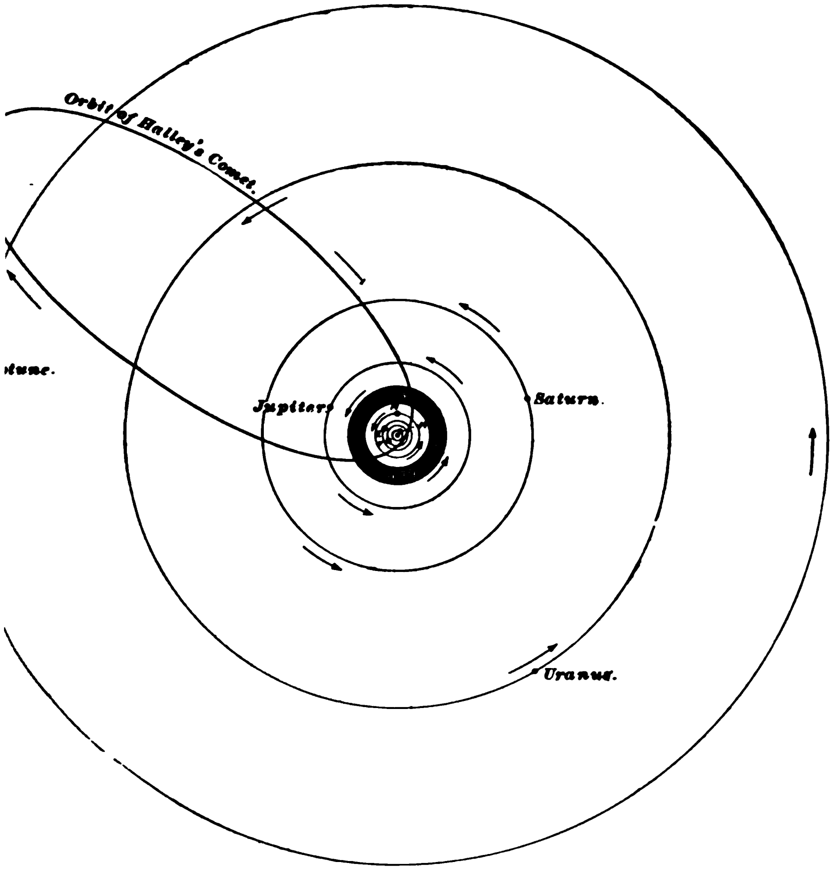


FIG. 44.

dence of any kind of its existence on the solar surface. The following day it was seen close to the sun, and for a whole week it was visible to the naked eye in full sunshine. It took nearly six months before it became too faint to be observed.

233. As to the possibility of a collision between the earth and some of the comets, it can be stated that such a catastrophe is highly improbable; and it is by no means certain that a collision with a comet would inflict any very great injury upon the earth.

METEORS AND SHOOTING STARS.

234. **Meteorites** are bright bodies which dash through the atmosphere with great velocity, and after being heated to incandescence by the violent friction, usually explode, scattering their fragments in all directions. They are also known under the name of **aerolites.** The less conspicuous of those bright bodies which move across some part of the sky for a second or two and then disappear are known as **shooting stars.** They can be seen any clear night, and no special attention is paid to them. **Meteor** is the common name for a meteorite and a shooting star.

235. Meteorites, if touched immediately after their fall, are very hot, but they cool very rapidly; their shape is very irregular, and, generally, they are enveloped in a crust of a dark hue, and sometimes they appear as if covered with thin varnish.

No consensus of opinion exists among scientists as to the origin of meteors. They consist of matter of the same kind as that of which the earth and the other bodies of the solar system are composed.

Formerly they were thought to be stones hurled from the volcanoes of the moon, but serious objections of more than one kind have since appeared which render this hypothesis untenable. By means of observations taken from a number of different stations, the path of a meteorite can be computed. Usually they appear at a height of about 90 or 100 miles, and disappear when about 5 or 10 miles from the earth. The average velocity ranges from 10 to 50 miles per second when first seen, but it gradually decreases.

236. Shooting stars do not appear in like numbers on all nights of the year, for, according to observations, there are

yearly, monthly, and daily periods of recurrence of certain sets of them. They are seen in all parts of the sky; but if the directions from which they seem to come are examined, it is found that the different parts of the sky furnish different numbers. More shooting stars come from the east than

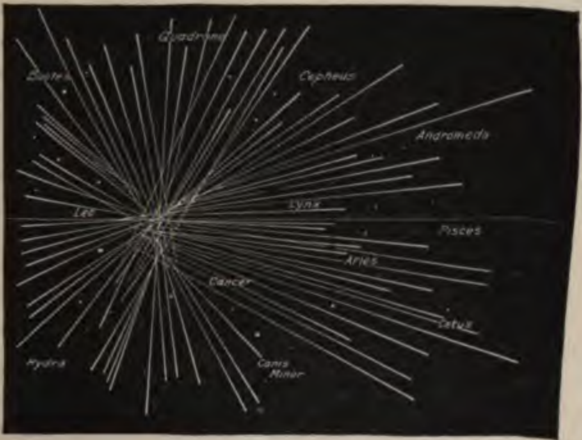


FIG. 45.

from the west, but the numbers from the north and the south are nearly equal. At periods when shooting stars are unusually abundant, about the 9th and 10th of August and 12th and 13th of November, they nearly all come from given directions instead of appearing in all the regions of the sky indifferently.

A peculiarly interesting feature of a meteoric shower is that all the meteors seem to diverge from a single point in the heavens, as indicated in Fig. 45, and this point—known by astronomers as the "radiant," or point of emanation—apparently keeps its position unchanged during the whole continuance of the shower. For the shower of November, the radiant is situated in the constellation of Leo; for the August displays it is in that of Perseus. For this reason the shooting stars of November are frequently termed the *Leonids*, and those of August the *Perseids*. There are

several other showers besides these, but they are of less brilliancy and duration.

Shooting stars appear more frequently at 6 A. M. than at 6 P. M.; and the reason for this is that in the morning the observer is in the front of the earth, as it moves in its orbit, while in the evening he is in the rear, as shown in Fig. 46. From observations, it has been estimated that the speed of meteors in general is equal to that of comets descending towards the earth. A similarity in orbits has also been observed; for instance, the shower of shooting stars seen on the 10th of December describes in space the same ellipse as Biela's comet. The orbit of the meteoric systems known as Leonids and Andromeds is a long ellipse, the perihelion of which is near the orbit of Uranus, its periodic time being 33 years and a fraction. A current of these shooting stars which crosses the earth's orbit at a certain point, and the different parts of which take several years to pass this point of meeting, must be crossed by the earth each year at the same time. Therefore, the periodical showers of meteors which are seen from year to year are simply the various parts of this current which the earth successively reaches.

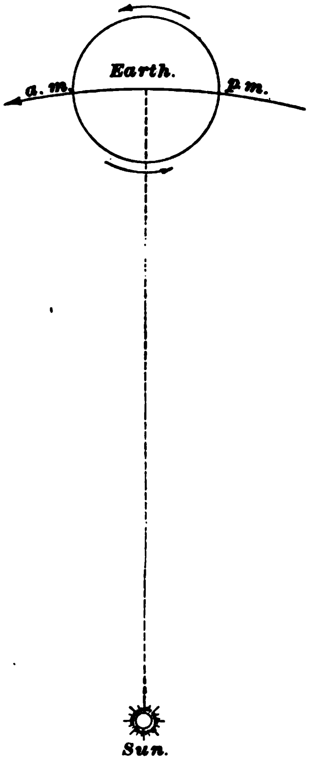


FIG. 46.

237. The Zodiacal Light.—After sunset in early spring and after sunrise in the fall, the celestial sphere sometimes displays a faint, hazy band of light inclined towards the horizon and in the plane of the zodiac. This is called **the zodiacal light,** and is one of the most beautiful of the

celestial phenomena, its color being pure white. It is not visible in Europe or the Northern States of America during the summer, owing to its inclined position upon the southern horizon. In February its appearance is most complete. In the tropics the shortness of twilight and the elevation of the ecliptic cause the zodiacal light to be visible all the year round. Scientists have advanced several hypotheses as to its cause. The most accepted one is that the phenomenon is due to the milliards of meteoric bodies revolving about the sun and reflecting the light of that luminary to us.

THE UNIVERSE.

CLASSIFICATION OF THE STARS

238. We have already spoken of the division of the stars into star groups or constellations; modern astronomers recognize 67 constellations, of which 48 were transmitted to us by Ptolemy and 19 have been added by later astronomers.

239. The stars have also been classified according to their brightness. Twenty of the brightest stars in the sky are called stars of the **first magnitude.** A number of stars which are a little less bright than those of the first magnitude are said to be of the **second magnitude,** and so on. The faintest stars visible to the naked eye are of the **sixth magnitude.** The same system of classification has been extended to telescopic stars. The forty-inch Yerkes telescope of the University of Chicago reveals stars of the seventeenth magnitude.

It must be carefully borne in mind that the *magnitude* of a star has nothing whatever to do with its real or apparent size. Even in the most powerful telescope a star shows no sensible disk; the telescope does not make a star appear larger, but only makes it brighter by collecting more of its light.

The stars of the first magnitude are fewest in number,

and the smaller the magnitude the larger the number of stars included in it, as shown in the following table:

Number of stars of the 1st	magnitude,	20
" " " " " 2d	"	59
" " " " " 3d	"	182
" " " " " 4th	"	530
" " " " " 5th	"	1,600
" " " " " 6th	"	4,800

According to this estimation, the number of stars visible to the ordinary eye is about 7,000. With the aid of a common opera-glass, however, the number increases to at least 100,000, and a 2½-inch telescope brings out about 300,000. A telescope 36 inches in diameter increases the number enormously, probably revealing about 100,000,000.

240. When measuring the distance of a star, the earth's diameter, and even the diameter of the earth's orbit, is too small to be a convenient unit. The distances are altogether too enormous, and therefore the light-year is generally used as a unit when computing the distance of a star. By **light-year** is meant the distance over which light travels in one year. Thus, when a star's distance is said to be 9 light-years, it means that the star's light, traveling at the rate of

Name of Star.	Distance in Diameters of the Earth's Orbit.	Distance in Billions of Miles.	Distance in Light-Years.
Alpha Centauri...	137,500	25	4.35
Sirius............	262,500	58	8.36
Procyon.........	380,500	71	12.00
Aldebaran.......	473,000	81	13.80
Altair............	543,000	101	17.10
Vega............	687,500	128	21.70
Capella..........	937,500	174	29.60
Arcturus.........	1,097,000	204	34.70
Pole Star........ (2d Magnitude)	1,159,000	215	36.60

early 186,000 miles per second, would require 9 years o reach the earth. Of the stars whose distances have been letermined, the nearest to the earth is Alpha Centauri; its ight requires more than 4 years to reach the earth. The ight of Sirius, a star of the first magnitude, requires about 8 years to traverse the distance to the earth. The preceling table gives the distances of some of the stars of the first nagnitude, which will give the student an idea of the grandeur of space.

241. There are a few other stars of different magnitudes the distances of which are about equal to those named above, but as to the rest we only know that they are still more distant. In all probability the light of the remotest telescopic stars occupies hundreds or thousands of years in coming to us. And when we at any time detect a change in position or appearance of a star, that change does not occur at the time of our observation, but occurred ten, a hundred, or a thousand years before. As stated previously, the distance of any star can be determined when its annual or stellar parallax is known. To determine the parallax of a star is, however, one of the most difficult problems of sidereal astronomy, because, among all the stars, there is not one that shows a parallax of one second. The distances corresponding to an angle of 1″ and fractions thereof are given below:

	Units.
An angle of 1″ corresponds to a distance of	206,265
" " " 0.9″ " " " " "	229,183
" " " 0.8″ " " " " "	257,830
" " " 0.7″ " " " " "	294,664
" " " 0.6″ " " " " "	343,750
" " " 0.5″ " " " " "	412,530
" " " 0.4″ " " " " "	515,660
" " " 0.3″ " " " " "	687,500
" " " 0.2″ " " " " "	1,031,320
" " " 0.1″ " " " " "	2,062,650

(Radius of the earth's orbit as unit.)

When a star's parallax is given, this table can be used in finding its approximate distance.

T. VIII.—8

242. The parallaxes of some of the principal stars are as follows:

Name of Star.	Parallax	Name of Star.	Parallax.
Alpha Centauri...	0.75″	Vega............	0.15″
Sirius............	0.39″	Capella..........	0.11″
Procyon.........	0.27″	Arcturus.........	0.094″
Aldebaran........	0.24″	Pole Star........	0.089″
Altair............	0.19″	61 Cygni.........	0.45″

Expressed in light-years, the distance of a star with a parallax of 1″ is 3.262; so the formula of Art. **117** in this case is

$$\text{Distance} = \frac{3.262}{\text{parallax}}.$$

MOTIONS OF THE STARS.

243. The general host of the stars have been termed fixed stars, in contradistinction to the planets the apparent position of which are obviously in a state of continual change. By the researches of modern astronomers, however, it has been established that many of the stars are perpetually but very slowly undergoing a change of apparent position in the celestial sphere. Every year fresh indications of the existence of such a proper motion are emerging from the observations of astronomers, and the probable conclusion is that in the lapse of ages all the stars of the firmament will be found to have a proper motion. By **proper motion** is meant not the real motion of the star in space, but the shifting of its position in relation to other stars. This shifting has all sorts of directions, but naturally only the angular shifting can be observed by us, because, if a star is moving directly towards us or going directly away from us, it would appear as a fixed star on the celestial sphere, and, consequently, would have no proper motion at all. Furthermore, it has been established that many stars are displaced with unequal velocities, or, in other words,

that their proper motion is not uniform. The proper motion of a star can not be detected by a single observation; it requires, indeed, years of patient observations to establish it. To illustrate this fact, we will mention that the bright star Arcturus requires a whole century to traverse a distance equal to only the eighth part of the diameter of the moon, and Alpha Centauri requires the same interval of time to traverse a fifth of the moon's diameter. Other stars move more slowly still. Thus the stars have a real motion of their own besides their apparent motions, owing to the revolution of the earth in its orbit and the combination of this motion with the velocity of light.

244. When we consider the proper motion of the stars, another question of profound interest suggests itself to our mind. According to the Copernican theory of the universe, the sun is a star. Then there is good reason to suppose that our sun also has a proper motion in space.

Setting out from this point of view, astronomers of the nineteenth century have established the fact of the existence of a real motion of the solar system in space, and have ascertained the point in the celestial sphere towards which it is transported from year to year. According to the more recent calculations, this point is situated in the constellation Hercules. The velocity of its motion can not be determined with the same accuracy as the direction, but it is estimated to be about 10 miles per second.

DOUBLE AND VARIABLE STARS.

245. If two stars are separated by a smaller interval than 2′, they appear to the naked eye as a single star. When such a pair of stars are viewed in a telescope of sufficient power, they are seen as separate stars; some of these stars are so close that they can be separated only by a telescope of very high magnifying power. Such a pair is called a **double star.**

246. The components of a double star may be at an enormous distance from each other, and entirely unconnected

with each other. In this case they form a double star, simply because they are in line with each other. Such stars are said to be **optically** double. Formerly all double stars were supposed to be of this type.

247. About one hundred years ago, Sir W. Herschel began to examine the double stars for the purpose of determining stellar parallax. He found, very much to his surprise, that the components of the double star he was examining were evidently physically connected, and were moving under the influence of their mutual attraction. Such a double star is called a **binary star.**

248. Each of the components of a binary star describes an orbit about the other, in exact accordance with the law of gravitation. Usually one of the components, called the *companion*, is less bright than the other. The periods of revolution of the binary stars vary from 12 years to several centuries.

249. The number of double stars catalogued is about 12,000; about 700 of these are known to be binary stars.

Among the binary stars are *Gamma Virginia* and *Castor*. (See star map.) The period of the former is 180 and of the latter 1,000 years.

250. In several cases we find three or four, or even more stars grouped together in one system. Zeta Cancri is a triple star consisting of a close pair revolving in an orbit nearly circular, while a third star farther away revolves in the same direction around them. Another case is Theta Orionis, located in the constellation Orion, which is composed of six stars. Lately another star has been added to this remarkable system, making it a septuple star.

251. There are some hundreds of stars the brightness of which has been observed to change; these are called **variable stars.** Among them the star Algol in the

constellation Perseus is remarkable. After shining as a star of the second magnitude for two and a half days, it begins to lose its brilliancy, and in the course of about four hours it falls to a star of the fourth magnitude. In this condition it remains for fifteen or twenty minutes, and then begins to brighten. In four hours it has become again a star of the second magnitude, and so remains for another period of two and a half days. Another variable star is called Mira, which means "the wonderful"; it is situated in the constellation Ceti. The periodical change of this star occupies about 331½ days. For about five months of this time the star is quite invisible; it then increases in brightness, and after remaining at its greatest brightness for some time, it gradually sinks down to invisibility. There are a good many other cases of the same kind, too numerous to be described here.

In some cases the loss of light may be due to the fact that the star is accompanied by a dark or nearly dark satellite, which circles around it and periodically eclipses it by passing between it and the earth.

It is well known that there are in the universe dark as well as bright bodies, and the probability is that the luminous world which we see as stars, numbered by millions of millions, forms but a minority of the whole, the greatest number being invisible, because, like the planets which circle around the sun, they have ceased to be luminous.

252. In connection with the variation of brightness of the stars, another phenomenon of interest is the different colors displayed by them. Most of the stars are white, but there are many that shine with yellowish, reddish, or bluish light. For instance, Sirius is white, Vega is bluish, Arcturus is reddish, and Aldebaran is orange. Their colors are probably due to their temperature and composition. Stars displaying white light are supposed to be the hottest, while yellow indicates a lower temperature. Red stars are considered to be aged suns verging towards extinction as luminaries. Several times unknown stars are recorded to

have suddenly appeared, and after a short period of unusual brilliancy sunk into invisibility again. A celebrated case of this class took place in 1572. A new star appeared, nearly surpassing Jupiter in brightness. It then faded slowly, and after about a year and a half entirely disappeared. Other cases less marked than this are on record.

THE NEBULÆ.

253. **Nebulæ** are patches of light, formless and cloud-like. The great majority of these objects are invisible to the naked eye, but with the aid of a powerful telescope thousands of them can be seen. As the powers of the telescope are increased, many nebulæ are resolved into clusters of stars, while others retain their cloudy appearance, no matter how powerful the telescope which is turned on them. There are various forms of nebulæ, the round, globular, or spherical form being most common. Nebulæ vary greatly in size. Some are so small that when viewed in an ordinary telescope they look like a star, while others are among the most gigantic objects seen on the celestial sphere.

254. Some of the celestial objects which have a nebulous appearance are resolved by powerful telescopes into clusters of stars, and it was formerly supposed that all the nebulæ were simply star clusters. It is now known that there are many nebulæ which are not composed of stars, but are gaseous; these are called true nebulæ, to distinguish them from those objects which present a nebulous appearance, though composed of stars.

255. There has been much speculation as to the composition of the true nebulæ. Some of the nebulæ are of a distinctly green color, and in these it is certain that hydrogen and some other gases are present. But no definite and satisfactory explanation of their nature and composition has yet been given.

Sir William Herschel began a catalogue of the nebulæ; this catalogue was completed by his son, Sir John Herschel, and contains about 5,000. Nearly 2,000 have since been added by other observers, and those which are still uncatalogued must be very faint indeed.

It is remarkable that the nebulæ are most numerous where the stars are fewest.

256. Attempts to measure the distance of the nebulæ have hitherto been unsuccessful. Fifty years ago astronomers believed that there was no difference between the nebulæ and the star clusters, except that the nebulæ were infinitely farther away. At present it is held to be probable that the distances of the nebulæ do not greatly exceed those of the stars.

257. The **Galaxy,** or **Milky Way,** is a belt or zone of nebulous appearance which encircles the celestial sphere.

The Milky Way intersects the ecliptic near the solstitial points, and is inclined to it at an angle of 60°.

The edges of the Milky Way are very irregular, and its brightness is far from uniform. The telescope shows that the Milky Way is composed of innumerable small stars, of which very few are greater than the eighth magnitude.

THE PRINCIPAL STARS.

258. It is very desirable that the student should be able to recognize the principal constellations and some of the principal stars. To enable him to do so we shall explain how a few of the brighter and more conspicuous stars can be located.

In order to indicate a particular star, it is usual to mention the constellation in which the star is situated, and to add a letter or number by which that star can be distinguished from other stars in the same constellation.

About the year 1603 a catalogue of stars was published in which the stars of each constellation were distinguished by

the letters of the Greek alphabet. The Greek letters, with their names, are:

α alpha,	ι iota,	ρ rho,
β beta,	κ kappa,	σ sigma,
Γ gamma,	λ lambda,	τ tau,
δ delta,	μ mu,	υ upsilon,
ε epsilon,	ν nu,	φ phi,
ζ zeta,	ξ xi,	χ chi,
η eta,	o omicron,	ψ psi,
θ theta,	π pi,	ω omega.

The brightest star in a constellation is denoted by α, the next brightest by β, and so on.

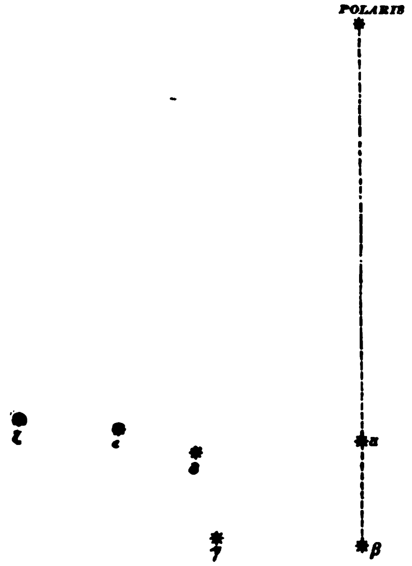


FIG 47.

259. There is no difficulty in recognizing the constellation of Ursa Major, or the Great Bear, of which the seven principal stars are shown in Fig. 47. These seven stars form the **Dipper.** If we imagine a line drawn through the stars β and α of the Dipper and produced about five times the distances from β to α, the end of this line will be near to a bright star. This bright star is Polaris, or the pole star.

For this reason the stars β and α are called the Pointers. The pole star is also the star α of the constellation Ursa Minor.

260. If the handle of the Dipper is produced with a uniform curvature, it will point out the bright star Arcturus in the constellation Bootes. The line joining Polaris to η, the last star in the handle of the Dipper, if produced, passes very near to Arcturus. By means of these two lines, Arcturus can be readily recognized.

261. A line drawn from Polaris perpendicular to the line of the Pointers and on the opposite side to the Dipper passes, at 48° distance, through Capella, another bright star of the first magnitude.

In the opposite direction and about the same distance from Polaris is a large white star called Vega; and at one-third distance from Arcturus to Vega is the bright star Gemma in the constellation Corona. A line drawn from η in the Dipper through Vega and produced to an equal distance beyond them passes through Altair. The line of the Pointers carried through the Polaris passes through Markab, the principal star in the constellation Pegasus.

A line from Polaris through Capella passes through Rigel in the constellation Orion; and a line from Rigel in the direction of the Dipper goes through α Orionis and very near Castor in Gemini. A line drawn through η and ζ in the Dipper, in the direction $\eta\,\zeta$, will pass close to Capella and go through the star Aldebaran in Taurus. A line from Aldebaran through the belt in Orion passes, at about 20° on the other side, through Sirius, the brightest of stars. Sirius and Procyon (to the northwards of Sirius), together with α Orionis, form an equilateral triangle. A line from Rigel through Procyon passes, at an equal distance beyond, very near to Regulus in the constellation Leo.

262. In the southern hemisphere **the Southern Cross** is about as far from the south pole as the Dipper is from the north pole. To the left of the cross when on the meridian and pointing towards it are α and β Centauri, both

of the first magnitude. A line from α Orionis through Rigel passes not very far from Fomalhaut, a very bright star. Achernar, Fomalhaut, and Canopus are in line and nearly equidistant, being about 40° apart. A line from Regulus through Spica passes at 45° distance through Antares, a very bright and reddish star in the constellation Scorpio. When a few stars are known, the rest are easily found by the times of their meridian passage and their declination, as described in Art. **218.** A star may also be found occasionally by means of its altitude or azimuth, computed roughly.

On the star map the hours of right ascension are indicated at the circumferences, being numbered as the figures upon a watch face, but with 24 hours instead of 12. The map of the northern hemisphere is also equipped with the names of the different months of the year. By means of these indications the student may approximately find what stars will pass the meridian at a certain time each month. For instance, in September all stars within the space of the two lines connecting that month with the center of the circle will pass the meridian about 9 o'clock in the evening; in February all stars situated within the boundng lines of that month will be on the meridian about 7 o'clock in the morning.

EXAMPLE.—Find the approximate time when the star *Aldebaran* will pass the meridian during the month of January

Ans. About 4.30 o clock in the morning.

EXAMPLE.—In February you wish to observe the meridian altitude of the star *Sirius*. Find by star map the approximate time of her meridian passage. Ans. About 6.40 o'clock in the morning.

ASTRONOMICAL INSTRUMENTS.

THE SEXTANT.

263. The instruments employed in astronomical observations are divided into three classes. To the first class belong instruments designed to extend our powers of vision; these instruments are called **telescopes.** To the second class belong instruments designed for measuring time; these

are watches, clocks, or chronometers. To the third class belong instruments for measuring angles.

264. The rapid progress of modern astronomy is largely due to the perfection and completeness of astronomical instruments. It would be impossible to give a description in this Paper of the instruments used in a fixed observatory.

In the Instruction Papers on Surveying in this Course, the student will find full descriptions of the instruments used by surveyors for measuring angles. In this Paper it will be sufficient to give a brief description of the sextant.

The sextant is an instrument for measuring the angular distance between two objects by observing them both at the same time.

The sextant (Fig. 48) consists of a metal frame *C D E*; the angle *D C E* is usually 60°, and the arc *E D* is divided into *half degrees*, which are marked as *whole degrees*. Thus the graduations on the arc *E D* read from 0° to 120°. The arm *B I*, called the **index-arm,** is fitted with a vernier and rotates about the center *C* of the sextant; to this index-arm is also affixed the **index-mirror** *B C*. To the arm *C D* is fixed the **horizon-glass** *A*; half of the back of this glass is silvered, and the other half is transparent. To the arm *C E* is attached a telescope *T*, directed towards the horizon-glass *A*. When the index *I* is at *E*, that is, at the zero of the graduated arc *D E*, the mirrors *A* and *B C* are parallel.

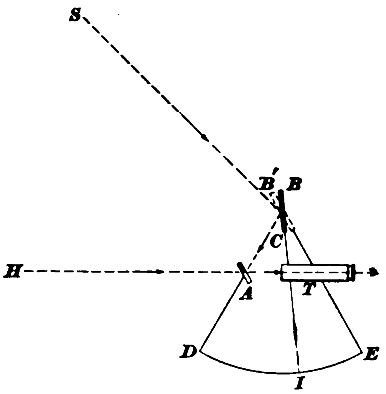


FIG. 48.

Error remaining after the instrument is adjusted is called the **index-error.** If the index stands near but not on zero, and the observer looks through the telescope at a very distant object, he will see two images of the object. One of

these images is formed by the rays of light which pass through the unsilvered half of the horizon-glass *A*; the other image is formed by the rays which are reflected from the index-mirror to the horizon-glass, and there reflected to the telescope *T*. If, now, the observer moves the index a little, the image formed by the direct rays remains fixed, while the image formed by the reflected rays moves. By moving the index, he can make the two images coincide; then if the object is *very distant*, being, for instance, a star, the index should stand at the zero-mark on the graduated scale. If the index, under these circumstances, does not stand at zero, the difference as read on the graduated arc is the index-error of the instrument. It is either + or — and has to be

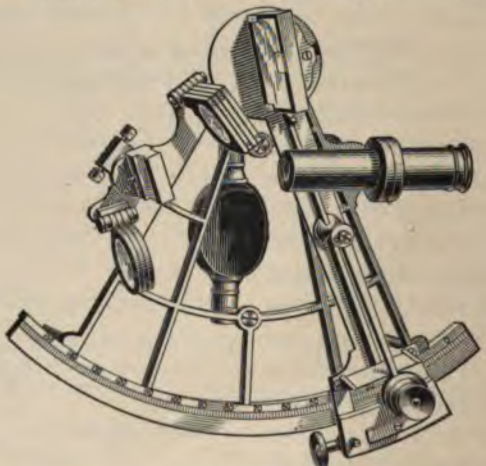


FIG. 49.

applied to the observed altitude accordingly. If zero on the vernier is to the left of zero on the arc, the index-error is to be subtracted; if it is to the right, the error must be added.

To find the angular distance between two objects *H* and *S*, the index *I* is moved until the image of *S* formed after two reflections coincides with the image of *H* formed by the direct rays. Then the angular distance between *S* and *H* is equal to twice the angle *E C I*. On the scale *E D*, every

half degree is marked 1°; hence, the reading of the scale gives double the angle *E C I*, which is the required angular distance between *S* and *H*.

The fineness of the reading as made by means of the vernier, on a good instrument, is 10″. Fig. 49 represents a modern sextant.

265. Artificial Horizon.—An artificial horizon is used on land when the natural sea horizon can not be seen.

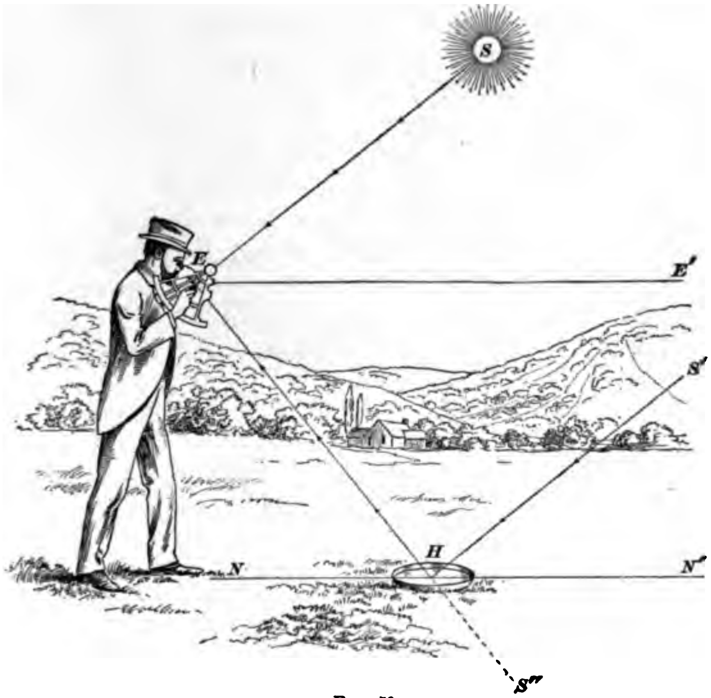


FIG. 50.

It consists of a reflecting surface of some fluid, preferably mercury, in which the image of the object can be seen. Should mercury not be available, an artificial horizon can be obtained simply by pouring a quantity of oil, tar, or syrup into a shallow vessel and then prevent the wind giving a tremulous motion to its surface. When measuring an altitude, the observer stands at a little distance from the vessel,

TABLE I.

MEAN REFRACTION.

(Subtractive from apparent altitude.)

App. Altitude.	Refraction.	App. Altitude.	Refraction.	App. Altitude.	Refraction.	App. Altitude.	Refraction.	App. Altitude.	Refraction.
0° 0′	33′ 0″	3° 20′	13′ 34″	6° 40′	7′ 40″	10° 0′	5′ 15″	16° 40′	3′ 8″
0° 5′	32′ 10″	3° 25′	13′ 20″	6° 45′	7′ 35″	10° 10′	5′ 10″	16° 50′	3′ 6″
0° 10′	31′ 22″	3° 30′	13′ 6″	6° 50′	7′ 30″	10° 20′	5′ 5″	17° 0′	3′ 4″
0° 15′	30′ 35″	3° 35′	12′ 53″	6° 55′	7′ 25″	10° 30′	5′ 0″	17° 10′	3′ 3″
0° 20′	29′ 50″	3° 40′	12′ 40″	7° 0′	7′ 20″	10° 40′	4′ 56″	17° 20′	3′ 1″
0° 25′	29′ 6″	3° 45′	12′ 27″	7° 5′	7′ 15″	10° 50′	4′ 51″	17° 30′	2′ 59″
0° 30′	28′ 23″	3° 50′	12′ 15″	7° 10′	7′ 11″	11° 0′	4′ 47″	17° 40′	2′ 57″
0° 35′	27′ 41″	3° 55′	12′ 3″	7° 15′	7′ 6″	11° 10′	4′ 43″	17° 50′	2′ 55″
0° 40′	27′ 0″	4° 0′	11′ 51″	7° 20′	7′ 2″	11° 20′	4′ 39″	18° 0′	2′ 54″
0° 45′	26′ 20″	4° 5′	11′ 40″	7° 25′	6′ 57″	11° 30′	4′ 34″	18° 10′	2′ 52″
0° 50′	25′ 42″	4° 10′	11′ 29″	7° 30′	6′ 53″	11° 40′	4′ 31″	18° 20′	2′ 51″
0° 55′	25′ 5″	4° 15′	11′ 18″	7° 35′	6′ 49″	11° 50′	4′ 27″	18° 30′	2′ 49″
1° 0′	24′ 29″	4° 20′	11′ 8″	7° 40′	6′ 45″	12° 0′	4′ 23″	18° 40′	2′ 47″
1° 5′	23′ 54″	4° 25′	10′ 58″	7° 45′	6′ 41″	12° 10′	4′ 20″	18° 50′	2′ 46″
1° 10′	23′ 20″	4° 30′	10′ 48″	7° 50′	6′ 37″	12° 20′	4′ 16″	19° 0′	2′ 44″
1° 15′	22′ 47″	4° 35′	10′ 39″	7° 55′	6′ 33″	12° 30′	4′ 13″	19° 10′	2′ 43″
1° 20′	22′ 15″	4° 40′	10′ 29″	8° 0′	6′ 29″	12° 40′	4′ 9″	19° 20′	2′ 41″
1° 25′	21′ 44″	4° 45′	10′ 20″	8° 5′	6′ 25″	12° 50′	4′ 6″	19° 30′	2′ 40″
1° 30′	21′ 15″	4° 50′	10′ 11″	8° 10′	6′ 22″	13° 0′	4′ 3″	19° 40′	2′ 38″
1° 35′	20′ 46″	4° 55′	10′ 2″	8° 15′	6′ 18″	13° 10′	4′ 0″	19° 50′	2′ 37″
1° 40′	20′ 18″	5° 0′	9′ 54″	8° 20′	6′ 15″	13° 20′	3′ 57″	20° 0′	2′ 35″
1° 45′	19′ 51″	5° 5′	9′ 46″	8° 25′	6′ 11″	13° 30′	3′ 54″	20° 10′	2′ 34″
1° 50′	19′ 25″	5° 10′	9′ 38″	8° 30′	6′ 8″	13° 40′	3′ 51″	20° 20′	2′ 32″
1° 55′	19′ 0″	5° 15′	9′ 30″	8° 35′	6′ 5″	13° 50′	3′ 48″	20° 30′	2′ 31″
2° 0′	18′ 35″	5° 20′	9′ 23″	8° 40′	6′ 1″	14° 0′	3′ 45″	20° 40′	2′ 29″
2° 5′	18′ 11″	5° 25′	9′ 15″	8° 45′	5′ 58″	14° 10′	3′ 43″	20° 50′	2′ 28″
2° 10′	17′ 48″	5° 30′	9′ 8″	8° 50′	5′ 55″	14° 20′	3′ 40″	21° 0′	2′ 27″
2° 15′	17′ 26″	5° 35′	9′ 1″	8° 55′	5′ 52″	14° 30′	3′ 38″	21° 10′	2′ 26″
2° 20′	17′ 4″	5° 40′	8′ 54″	9° 0′	5′ 48″	14° 40′	3′ 35″	21° 20′	2′ 25″
2° 25′	16′ 44″	5° 45′	8′ 47″	9° 5′	5′ 45″	14° 50′	3′ 33″	21° 30′	2′ 24″
2° 30′	16′ 24″	5° 50′	8′ 41″	9° 10′	5′ 42″	15° 0′	3′ 30″	21° 40′	2′ 23″
2° 35′	16′ 4″	5° 55′	8′ 34″	9° 15′	5′ 39″	15° 10′	3′ 28″	21° 50′	2′ 21″
2° 40′	15′ 45″	6° 0′	8′ 28″	9° 20′	5′ 36″	15° 20′	3′ 26″	22° 0′	2′ 20″
2° 45′	15′ 27″	6° 5′	8′ 21″	9° 25′	5′ 34″	15° 30′	3′ 24″	22° 10′	2′ 19″
2° 50′	15′ 9″	6° 10′	8′ 15″	9° 30′	5′ 31″	15° 40′	3′ 21″	22° 20′	2′ 18″
2° 55′	14′ 52″	6° 15′	8′ 9″	9° 35′	5′ 28″	15° 50′	3′ 19″	22° 30′	2′ 17″
3° 0′	14′ 36″	6° 20′	8′ 3″	9° 40′	5′ 25″	16° 0′	3′ 17″	22° 40′	2′ 16″
3° 5′	14′ 20″	6° 25′	7′ 57″	9° 45′	5′ 23″	16° 10′	3′ 15″	22° 50′	2′ 15″
3° 10′	14′ 4″	6° 30′	7′ 51″	9° 50′	5′ 20″	16° 20′	3′ 12″	23° 0′	2′ 14″
3° 15′	13′ 49″	6° 35′	7′ 45″	9° 55′	5′ 18″	16° 30′	3′ 10″	23° 10′	2′ 13″

TABLE I—*Continued.*

MEAN REFRACTION.

(Subtractive from apparent altitude.)

App. Altitude.	Refraction.	App. Altitude.	Refraction.	App. Altitude.	Refraction.	App. Altitude.	Refraction.	App. Altitude.	Refraction.
23° 20′	2′ 12″	26° 40′	1′ 53″	34° 0′	1′ 24″	48° 0′	0′ 51″	68° 0′	0′ 23″
23° 30′	2′ 11″	26° 50′	1′ 52″	34° 30′	1′ 23″	49° 0′	0′ 49″	69° 0′	0′ 22″
23° 40′	2′ 10″	27° 0′	1′ 51″	35° 0′	1′ 21″	50° 0′	0′ 48″	70° 0′	0′ 21″
23° 50′	2′ 9″	27° 15′	1′ 50″	35° 30′	1′ 20″	51° 0′	0′ 46″	71° 0′	0′ 19″
24° 0′	2′ 8″	27° 30′	1′ 49″	36° 0′	1′ 18″	52° 0′	0′ 44″	72° 0′	0′ 18″
24° 10′	2′ 7″	27° 45′	1′ 48″	36° 30′	1′ 17″	53° 0′	0′ 43″	73° 0′	0′ 17″
24° 20′	2′ 6″	28° 0′	1′ 47″	37° 0′	1′ 16″	54° 0′	0′ 41″	74° 0′	0′ 16″
24° 30′	2′ 5″	28° 15′	1′ 46″	37° 30′	1′ 14″	55° 0′	0′ 40″	75° 0′	0′ 15″
24° 40′	2′ 4″	28° 30′	1′ 45″	38° 0′	1′ 13″	56° 0′	0′ 38″	76° 0′	0′ 14″
24° 50′	2′ 3″	28° 45′	1′ 44″	38° 30′	1′ 11″	57° 0′	0′ 37″	77° 0′	0′ 13″
25° 0′	2′ 2″	29° 0′	1′ 42″	39° 0′	1′ 10″	58° 0′	0′ 35″	78° 0′	0′ 12″
25° 10′	2′ 1″	29° 30′	1′ 40″	39° 30′	1′ 9″	59° 0′	0′ 34″	79° 0′	0′ 11″
25° 20′	2′ 0″	30° 0′	1′ 38″	40° 0′	1′ 8″	60° 0′	0′ 33″	80° 0′	0′ 10″
25° 30′	1′ 59″	30° 30′	1′ 37″	41° 0′	1′ 5″	61° 0′	0′ 32″	81° 0′	0′ 9″
25° 40′	1′ 58″	31° 0′	1′ 35″	42° 0′	1′ 3″	62° 0′	0′ 30″	82° 0′	0′ 8″
25° 50′	1′ 57″	31° 30′	1′ 33″	43° 0′	1′ 1″	63° 0′	0′ 29″	83° 0′	0′ 7″
26° 0′	1′ 56″	32° 0′	1′ 31″	44° 0′	0′ 59″	64° 0′	0′ 28″	84° 0′	0′ 6″
26° 10′	1′ 55″	32° 30′	1′ 30″	45° 0′	0′ 57″	65° 0′	0′ 26″	86° 0′	0′ 4″
26° 20′	1′ 55″	33° 0′	1′ 29″	46° 0′	0′ 55″	66° 0′	0′ 25″	88° 0′	0′ 2″
26° 30′	1′ 54″	33° 30′	1′ 26″	47° 0′	0′ 53″	67° 0′	0′ 24″	90° 0′	0′ 0″

TABLE II.

DIP OF THE HORIZON.

(Subtractive from observed altitude.)

Height in Feet.	Dip.	Height in Feet.	Dip.	Height in Feet.	Dip.
1	0′ 59″	13	3′ 32″	26	5′ 0″
2	1′ 23″	14	3′ 40″	28	5′ 11″
3	1′ 42″	15	3′ 48″	30	5′ 22″
4	1′ 58″	16	3′ 55″	35	5′ 48″
5	2′ 11″	17	4′ 2″	40	6′ 12″
6	2′ 24″	18	4′ 9″	45	6′ 34″
7	2′ 36″	19	4′ 16″	50	6′ 56″
8	2′ 46″	20	4′ 23″	60	7′ 35″
9	2′ 56″	21	4′ 29″	70	8′ 12″
10	3′ 5″	22	4′ 36″	80	8′ 46″
11	3′ 15″	23	4′ 42″	90	9′ 18″
12	3′ 24″	24	4′ 48″	100	9′ 48″

TABLE III.

SUN'S PARALLAX IN ALTITUDE.

(Additive to observed altitude.)

Altitude.	Parallax.	Altitude.	Parallax.
0°	9″	54°	5″
6°	9″	57°	5″
12°	9″	60°	4″
16°	8″	63°	4″
20°	8″	66°	3″
25°	8″	69°	3″
30°	8″	72°	3″
34°	7″	75°	2″
36°	7″	78°	2″
40°	7″	81°	1″
45°	6″	84°	1″
48°	6″	87°	0″
51°	5″	90°	0″

DRAINAGE.

GENERAL CONSIDERATIONS.

SYSTEMS AND REQUIREMENTS.

1391. Drainage and Sewerage.—In Municipal Engineering, the subjects of Drainage and Sewerage are so closely connected as to be almost inseparable. In general, however, it may be stated that, while the subject of sewerage refers principally to the removal of excrementitious or human refuse and other waste matter common to human habitations, the subject of drainage properly relates to the removal of storm water from the surface and subsoil. All water given by rain storms may, without impropriety, be called **storm water.** This name, however, is commonly applied to the water from rain storms which does not soak immediately into the ground nor evaporate, but flows away over the surface, through natural channels or through artificial conduits.

1392. Municipal Drainage.—In rural districts the drainage may have for its object to reclaim and improve low-lying lands, in order that they may be utilized for agricultural purposes; but the drainage systems constructed within the limits of municipalities are not commonly for this purpose, but to remove the storm water from the surface, where it would cause more or less injury to habitations and inconvenience to the inhabitants, and from the subsoil, where it would become a menace to health.

1393. Systems of Drainage.—In municipalities the storm water is generally removed by means of drains or

For notice of copyright, see page immediately following the title page.

sewers; it is sometimes conveyed wholly by the street gutters and in open conduits without entering the sewers, but such practice is not common; in the majority of cases the water is led into the sewers by the most direct routes. The latter method being the more common and popular, and by far the better, will be the one considered here. Sometimes the storm water is conveyed in a system of conduits separate from that in which the sewage is conveyed; that is, the drainage system and the sewerage system are entirely separate. This method is known as the **separate system** of sewerage. In other cases, the drainage and sewage are removed by the same system of conduits; that is, the drainage and the sewerage systems are combined in one system. This method is known as the **combined system** of sewerage. The latter system is the more common.

1394. Requirements for Drainage.—A system of drainage in a municipality should be adequate for the prompt removal of the rainfall from the surface during violent storms, including also such animal and vegetable refuse as will necessarily be removed with the storm water. If this be accomplished, and the drains be located at sufficient depth, efficient drainage will be provided for the subsoil. Under drainage, therefore, we will treat of the capacities of drain sewers necessary to convey the storm water, but will not treat of sewerage proper. Hence, the word sewer, as herein used, will be understood to mean a storm-water sewer or drain sewer.

1395. Conditions to be Considered.—In designing a system of sewers for the purpose of drainage, the principal conditions which must be considered will usually be as follows:

First. The *area*, *physical outlines*, and *general topographical features* of the district to be drained must be considered with reference to the natural flow of water upon its surface, in order that the sewers may be located as nearly as possible in the natural drainage channels.

Second. The *rainfall* upon the district must be considered with reference to the maximum intensity of precipitation

during a period of time sufficient to completely charge the sewer.

Third. The *general character*, *condition*, and *slope of the surface* must be considered with reference to the proportion of the rainfall that will probably reach the sewer during the time of its maximum flow.

Fourth. The *geological character of the district* must be considered with reference not only to the depth to which it may be desirable to provide efficient subsoil drainage, but also with reference to the difficulties to be encountered in the practical construction of the sewer.

Fifth. The *location of the outlet* of the sewer must be considered with reference to the final disposition of the drainage.

These various conditions, however, are not wholly independent of each other, as the consideration of any one of them often involves some consideration of certain features of the others.

THE PHYSICAL OUTLINE OF THE DRAINAGE DISTRICT.

1396. Elevated Location.—The physical character of the district should be carefully studied. If the location of the district be near a summit or quite elevated, with surface sloping in such a manner as to afford ample channels of natural drainage, the problem of artificial drainage will be very simple. In such a case, the lines of drainage sewers are located along such streets as most nearly follow the natural drainage channels, leading to such natural outfall as may be available. It will often be found that the positions, directions, and grades of the streets will be influenced to some extent by the natural channels of the drainage.

1397. District Without Natural Outlet.—If, however, the drainage district be situated in a valley not having a good natural drainage outlet, the problem of drainage may be much more serious, and the expense of the construction of the system greatly enhanced. If the drainage is to be

discharged into a running stream passing through the town, the drainage should be delivered to the stream at a point *below* the town, whatever may be the direction of the natural drainage. This may require inconvenience and circuitous courses for the sewers, as well as deep excavations. A case of this nature is represented in Fig. 355. As there shown, the town is situated on the bank of a stream. A

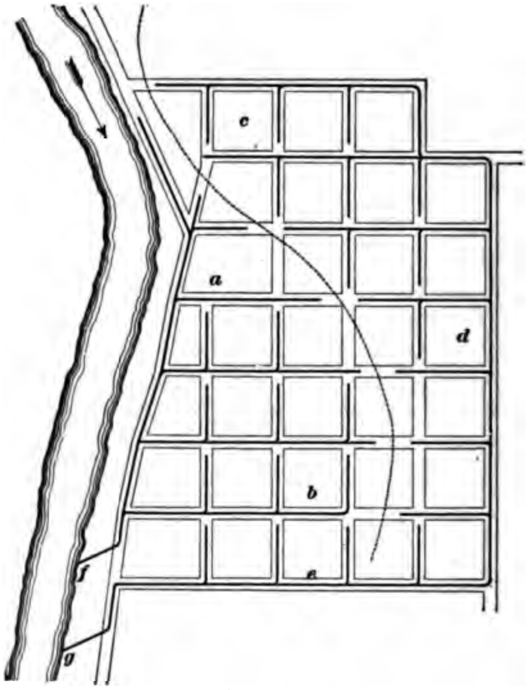


FIG. 355.

crest of elevated ground, in the position indicated by the dotted line, separates the town into practically two drainage districts. The outfall for the district *a b* is at *f*, while the drainage from *c* passes around by the circuitous course *d e* and is discharged at *g*. The drainage from *c* could not pass by a direct course to the outlet without an extremely deep excavation through the high ground.

1398. Low-Lying District.—If a drainage district is very level and at a very slight elevation above tidewater or above the river into which the drainage is to be discharged, the discharge sewer must be constructed to the lowest available point of outfall. This will then be the chief controlling condition, and it may even be necessary to resort to pumping in order to provide sufficient fall for the drainage to cause it to flow to the outlet. In such cases, the drainage is conveyed to tanks, located at the lowest points, and then pumped to a sufficient height to flow to the outlet.

Such conditions will often control the depth at which drains can be located. For the purpose of removing the storm water, it is not generally necessary to locate the drains at any great depth below the surface; but for the purpose of subsoil drainage, it is necessary to locate them as deep as it may be desired to drain the subsoil.

1399. Subsoil Drainage.—The subject of subsoil drainage is very important. If the entire district to be drained is built up and paved, the amount of rainfall reaching the subsoil will be comparatively so small that it is seldom necessary to take it into consideration. But if a portion of the district consists of grassy lawns, wooded tracts, or unpaved streets, the amount of rainfall reaching the subsoil will be very considerable, and provision must be made for its removal. This will be the case especially when the subsoil is quite permeable and is underlain by an impervious stratum; the necessity for drainage will be further increased if the formation is such as to produce springs in the subsoil. The water having no opportunity to escape, the subsoil will be continually soaked with stagnant water, which, as is well known, is very injurious to health. If the formation is such that the water is continually, but gradually flowing away, the injurious effect will not be nearly so marked; but it is in all cases desirable to provide efficient subsoil drainage.

STORM-WATER EFFLUENT.

RATE OF RAINFALL.

1400. Important Condition.—In determining the required capacity for a storm-water sewer, one of the most important conditions to be considered is the maximum rate of rainfall, which is measured by the depth of the water falling during a definite length of time. Thus, a rainfall of 6 inches per hour means that, if all the water that falls in one hour remained on the surface, its depth would be 6 inches. A knowledge of this condition will enable us to determine the amount of storm water reaching the sewer during a storm continuing for a period of time sufficient to completely charge the sewer.

1401. Records of Rainfall.—For this purpose, the records of rainfall are quite incomplete. Records of daily, monthly, and yearly rainfall are numerous; but however valuable such records may be for some purposes, they have little value for the purposes of designing sewers. The records of storms, as generally reported, give the total precipitation for the entire storm, and, possibly, the duration of the storm, but they do not usually give the *maximum rate* of the precipitation. The average rate of precipitation throughout the storm can be obtained by dividing the total precipitation by the duration of the storm. But this will seldom, if ever, be the maximum rate of precipitation; for, as is well known, the greatest intensity of the rainfall is attained only during certain short periods of the duration of the storm. It is often the case that a rain storm will continue through several hours with a very uneven intensity, being sometimes a mere drizzle and again a veritable downpour. Evidently, the total precipitation of such a storm will bear no relation to its maximum rate. A storm which will precipitate 6 inches of rain in 12 hours may precipitate 3 inches in 2 hours, or, possibly, 2 inches in 30 minutes. The average precipitation during the entire

storm would then be at the rate of $\frac{6}{12} = \frac{1}{2}$ inch per hour, while the precipitation during the 30 minutes, $= \frac{1}{2}$ hour, of maximum rainfall would be at the rate of $\frac{2}{\frac{1}{2}} = 4$ inches per hour.

1402. Chief Condition to be Considered.—A locality subject to long-continued drizzling rains may have a large annual rainfall, while short, heavy rains may occur in localities having a much smaller annual rainfall. It is generally this maximum rate, or rapid downpour, during a reasonably short period, that taxes most severely the capacity of a storm-water sewer. The chief condition to be considered in designing a storm-water sewer is the maximum intensity of the precipitation; that is, the maximum rate per second or per hour, during a period of time sufficient for the water from the most remote portions of the district to reach the sewer and flow to the point under consideration.

1403. Self-Registering Rain Gauges.—It is, therefore, evident that in order to intelligently design a storm-water sewer, the designer should have a fairly accurate record of the rainfall in the locality, giving both the rate and duration of the varying degrees of precipitation for each storm. Such records are obtained by means of self registering rain gauges, in which the continuous amount of rainfall and the time are automatically recorded upon a sheet moved by clockwork. In 1889, the United States Weather Bureau placed self-registering rain gauges in the principal cities of this country; they had also previously been in use in a few cities. From the records given by these gauges, valuable data regarding the rainfall are being obtained.

1404. Valuable Data.—A great deal of information relating to the rainfall is given in the *Weather Review*, the official publication of the United States Weather Bureau. Only the records of self-registering gauges, however, can be considered as really accurate. On this subject, much useful information, relating to conditions in the United States,

was also collected and compiled by the Board of Sanitary Engineers appointed by the President in 1889 to report upon the sewerage of the District of Columbia. Diagrams of the rainfall were constructed by this Board, by platting on cross-section paper the rates per hour of excessive rainfall for storms of different lengths, as obtained from records of the precipitation in five of the principal cities of the United States. From the inspection of such diagrams could be obtained a comprehensive and intelligent idea of the relative rates of rainfall given by storms of different lengths.

1405. One important condition indicated by these diagrams is that the *maximum rate of precipitation for a given short period of time is almost uniform throughout the United States*, although this is quite contrary to what has been the commonly accepted opinion. While the *total amount* of annual rainfall varies greatly in different parts of the United States, the greatest amount of rain that falls in a given *short* period of time does not appear to vary greatly in different parts of the country, although the maximum rates of rainfall are generally somewhat greater for the Southern coast States than for the interior. The *frequency* with which the maximum rate of precipitation may be attained, however, varies greatly in different parts of the country, as does also the *total amount* of rain that may fall during a single storm or during a season. But the *maximum rate* of rainfall bears no relation to the *total amount* of rainfall. The *frequency* with which the maximum rate of rainfall may occur, however, should be considered in connection with the design of storm-water sewers, as will again be noticed.

1406. Formula for the Maximum Rate of Rainfall.—The maximum rates of rainfall given by storms of varying lengths may be very closely expressed by the formula

$$y = \frac{a}{x + b}, \qquad (99.)$$

in which a and b are constants and y is the rate of rainfall

in inches per hour during a period of time x. That is, x is the duration of the storm giving a rate of precipitation y. In this formula, the duration x of the storm is considered to be expressed in hours.

The duration x of the storm may be expressed in minutes by multiplying both numerator and denominator of formula **99** by 60, giving it the form

$$y = \frac{60\,a}{60\,x + 60\,b} = \frac{a'}{x' + b'}, \qquad \textbf{(100.)}$$

in which $a' = 60\,a$ and $b' = 60\,b$. The duration of the storm in *minutes* is $x' = 60\,x$, and y remains the rate of rainfall in inches per hour.

Likewise, the duration x of the storm may be expressed in seconds by multiplying both numerator and denominator of formula **99** by 3,600, giving it the form

$$y = \frac{3{,}600\,a}{3{,}600\,x + 3{,}600\,b} = \frac{a''}{x'' + b''}, \qquad \textbf{(101.)}$$

in which $a'' = 3{,}600\ a$ and $b'' = 3{,}600\ b$. The duration of the storm in *seconds* is $x'' = 3{,}600\ x$, and y remains the rate of rainfall in inches per hour, as in formula **99**.

1407. Talbot's Formulas for Rainfall.—If, in formula **99**, the values 1.75 and .25 be given to a and b, respectively, the result will be the equation for the maximum rate of *ordinary* rainfall proposed by Professor Talbot, of the University of Illinois, which is

$$y = \frac{1.75}{x + .25}, \qquad \textbf{(102.)}$$

in which x and y represent the same values as in formula **99**.

If, in formula **99**, values of 6 and .5 be given to a and b, respectively, the result will be Talbot's equation for the maximum rate of *rare* rainfall, which is

$$y = \frac{6}{x + .5}, \qquad \textbf{(103.)}$$

in which x and y represent the same values as in formula **99**.

Formula **103**, however, is of no practical value in the design of storm-water sewers.

Professor Talbot states that it is probable that storms reaching values given by his equation for the maximum rate of ordinary rainfall (formula **102**), will occur at a given point two or three times in ten years, and that the values given by his equation for rare rainfall (formula **103**) will not be exceeded oftener than once in fifty years or more. It must be noticed, however, that the *frequency* with which the values given by formula **102** will be likely to be reached or exceeded by actual storms will vary considerably in different parts of the United States. Storms attaining the rate of precipitation given by this equation may be expected to occur much more frequently in the South Atlantic States than in the Middle or Western States. The values given by Talbot's equation for ordinary rainfall (formula **102**) apply reasonably well throughout most of the Northern and Western States.

1408. Other Formulas for Rainfall.—As a basis for the design of sewers in some localities, it will be advisable to use an equation giving somewhat higher rates of precipitation. For localities in which the rainfall is frequent and severe, values of 2.25 and .3, substituted in formula **99**, for a and b, respectively, appear to be satisfactory. This gives the following formula for the maximum rate of occasional rainfall in such localities:

$$y = \frac{2.25}{x + .3}, \qquad (104.)$$

in which y represents the rate of rainfall in inches per hour, and x represents the duration of the storm in hours, as in formula **99**.

Rates of rainfall given by formula **104** will probably not be exceeded at any given point in the United States oftener than once in about five years. It, therefore, appears to be a satisfactory formula for the maximum rate of *occasional* rainfall in localities where rain storms are of frequent occurrence. In the examples for practice given in this section of the Course, formula **104** will be used to express the maximum rate of rainfall attained by occasional storms, although this

formula is really no more general in its application than formula **102.**

1409. It is quite probable that, for some parts of the United States, neither formula **102** nor formula **104** will satisfactorily express the maximum rate of rainfall to be used in designing storm-water sewers. In such cases, an equation suitable to the locality must be derived from the records of rainfall in that region. The records of rainfall should be obtained from self-registering rain gauges. The equation for rainfall may be obtained for any locality by substituting suitable values for a and b in formula **99.**

A formula for the maximum rate of occasional rainfall which is better adapted to some localities than formula **104** may be expressed by substituting the values 2.00 and .25 for a and b, respectively, in formula **99,** giving the equation

$$y = \frac{2.00}{x + .25}, \qquad \textbf{(105.)}$$

in which x and y have the same values as in formula **99.** Formula **105** will probably apply to more localities than formula **104.**

It should be noticed that all of the above formulas express the *rate* per hour, and *not* the total precipitation.

1410. Violent Storms.—In cases of violent and excessive downpours, a small portion of the storm water can be conveyed in the surface gutters, thus to some extent relieving the sewers, by temporarily affording additional provision for the storm water; hence, no serious damage will usually result if, at rare intervals, the rate of precipitation be such as to give a flow somewhat in excess of the actual capacities of the sewers. It is not customary, and is not generally considered necessary or even desirable, to design drainage systems with capacities sufficient for the prompt removal of the entire rainfall from *excessive* storms by means of the sewers alone. Storms giving an excessive precipitation occur only at long intervals and are of short duration. The excess of storm water above that which

shown as follows: There are 43,560 square feet in an acre. A rainfall of one inch ($= \frac{1}{12}$ ft.) per hour would give $\frac{1}{12} \times 43,560 = 3,630$ cubic feet per hour per acre, or $\frac{3,630}{60 \times 60} = \frac{121}{120}$, or 1.00833 cubic feet per second per acre. An assumed rate of 12 inches, or one foot, per hour would give 43,560 cubic feet per hour per acre, or $\frac{43,560}{60 \times 60} = 12.1$ cubic feet per second per acre. Therefore, in all computations relating to the required capacities of sewers, the number of cubic feet per acre falling in one second may, without material error, be taken the same as the number of inches in depth falling per hour.

The number of cubic feet of rain per second falling upon one acre will hereafter be designated by y_1. For any length of storm, the value of y_1, according to formula **104**, will be taken from the column y or y_1 in Table 29, or proportionally between the values there given.

PROPORTION OF RAINFALL REACHING SEWER.

1414. General Statement.—In the preceding articles, formulas were given for estimating the rate of rainfall. It will now be necessary to notice the proportion of the rainfall that will reach the sewer during the period of its maximum flow. For only a portion of the total rainfall will reach the sewer, and a still smaller portion will reach it during the period of greatest flow, which is the most essential condition to be considered in determining the capacities required for storm-water sewers.

1415. A Common Practice.—In America it is not an uncommon practice, in designing sewers, to provide capacity for one inch of rainfall per hour, or 1 cubic foot per second per acre of the district tributary to the sewer, assuming one-half the rainfall to reach the sewer. This is probably a good average practice, but it can not be adopted as a universal standard, for the conditions will vary in dif-

ferent localities, and a capacity satisfactory for one district may be insufficient for another.

1416. Various Ways in Which the Rainfall Disappears.—As stated above, not all the rainfall will reach the sewer. The water which falls in the form of rain disappears in various ways. Of that which does not reach the sewer, a portion evaporates, a portion is absorbed by growing vegetation, no inconsiderable portion soaks into the ground, and a portion flows away through natural channels, reaching the sewer, or watercourse, farther down.

1417. Evaporation.—Several experiments indicate that under certain conditions the amount of water evaporated from the surface of the ground may be as great as 70 per cent. of the rainfall. But evaporation does not take place to any extent while rain is falling, and for practical purposes it may be assumed not to occur during the maximum flow of storm water. Hence, for present purposes, this condition may be wholly neglected.

1418. Percolation.—The proportion of the rainfall which soaks, or percolates, into the ground will vary according to the nature of the soil; it will be much greater in sandy or peaty soil than in ordinary soil, and will be very much smaller in clay. Under favorable conditions, the percolation in ordinary soil may be 30 per cent. of the rainfall; in chalk, it may be nearly 40 per cent., while in sand or gravel, it may be over 80 per cent. But during a short and rapid downpour of rain, the percentage of percolation will be much smaller. Also, if the conditions are such as to afford a prompt and rapid flow of the water over the surface, the percentage of percolation will be small. A large portion of the water which soaks into the ground may reach the sewer, or watercourse, farther down; but this will occur after the lapse of considerable time, and not during the maximum flow of the storm water.

In the design of sewers, however, we do not deal with the conditions of evaporation and percolation except

indirectly. They need be considered only so far as they affect the amount of storm water reaching the sewer, with which we have to deal directly.

1419. Conditions Affecting the Flow of Storm Water.—The proportion of the rainfall that reaches the sewer varies according to the area, slope, and condition of the surface, and the nature of the subsoil. Wooded tracts, cultivated lands, or those covered by luxuriant vegetation retain a greater portion of the rainfall, and are longer in yielding up what they do not retain, than smoothly cut lawns or areas devoid of vegetable growth. The latter will, therefore, give the greater flow. The amount of the flow is also affected by the nature of the soil. Loose, porous soils, as sand or loam, readily drink in a large proportion of the rainfall, while hard-packed and impervious soils, as clay and cemented material, take in much less of the rainfall and give much greater surface flows. Steep slopes throw off a much greater proportion of the rainfall than flat areas, and carry it more quickly to the channels of flow. Hence, a hilly country will not only yield a greater proportion of the storm water than a level country, but will also deliver it to the sewers much more quickly. Frozen ground may give a surface flow practically equal to the rainfall; in connection with melting snow, it may considerably exceed the rainfall.

1420. Flow of Storm Water from Built-Up Districts.—The conditions noticed above refer principally to suburban districts. In closely built-up districts with paved streets, the proportion of storm water carried to the sewers and the rapidity with which it will be conveyed to them are both greatly augmented. The greater portion (often the entire portion) of the surface upon which the rain falls consists of paved streets and courts, walks, and the roofs of buildings, all of which offer nearly impermeable surfaces to the rainfall, while the systems of surface drains, troughs, and gutters quickly convey it to the sewers. As a result, a large portion of the rainfall is promptly delivered to the

sewers, which may thus be charged to their full capacity by short storms having a high rate of precipitation.

1421. Ratio of Storm Water to Rainfall Found to be Constant.—Gaugings of sewers during the flow of storm water show the important fact that, after the ground has become saturated, the ratio of storm water to rainfall is practically constant for a given district; that is, after saturation, the percentage of rainfall reaching the sewer is practically the same for all rates of precipitation. In the case of roofs and well paved areas, the effect of saturation is slight. As noticed above, the ratio of the storm water to the total rainfall depends largely upon the character and condition of both the surface and subsoil. Gaugings sufficient to definitely establish this ratio for different conditions of surface have never been made. Such reliable information as is available concerning the subject will be briefly noticed.

1422. Professor Baumeister, a German authority, states that "for drainage purposes, the ratio of the storm sewage to the total precipitation can be assumed to be the same as that existing between the impervious and the total area." In this statement, the word "impervious" has an absolute significance, that is, *perfectly* impervious. Area not absolutely impervious should be reduced to an amount of impervious area proportionate to its impermeability.

1423. Mr. E. Kuichling, of Rochester, N. Y., a well-known hydraulic engineer and an eminent authority on storm-water sewerage, gives the percentages in the following table as representing the relations of the impervious surface to the total drainage area, assuming the relations to vary according to the density of the population. These values were obtained from an extended analysis of the conditions found in such cities as Buffalo, Rochester, and Syracuse. The percentages in the last column should be used as the ratios of storm water to total precipitation.

Table Showing Relation of Fully Impervious Surface to Total Area According to Density of Population.

Average Number of Persons per Acre.	Percentage of Fully Impervious Surface.			
	Roofs.	Improved Streets.	Unimproved Streets, Yards, etc.	Total.
15	8.4	3.3	3.0	14.7
(*c*) 25	14.0	7.0	4.3	25.3
(*d*) 32	18.0	10.2	5.0	33.2
(*e*) 40	22.5	14.7	5.4	42.6
(*f*) 50	28.0	19.0	5.6	52.6

1424. According to Knauff, the maximum quantity of storm effluent, in percentages of the rainfall, is as follows:

On flat roofs..............40 to 50 per cent.
(*f*) In courts and squares......50 to 70 per cent.
(*h*) On steep roofs............60 to 80 per cent.

These percentages are probably rather low for small areas.

1425. In London, it has been found from gaugings of the sewers that from 53 to 94 per cent. of the rainfall flows away as storm water.

CONTEMPORARY FLOW.

1426. Ratio of Storm Water to Contemporary Rainfall.—The percentages given above relate to the *total* amount of storm water flowing from a given storm. What is most important to obtain in designing storm-water sewers, however, is the ratio of the *contemporary* storm water to the rainfall; that is, the proportion of the rainfall that will enter the sewer as storm water *during a period of time equal*

to the duration of the storm. This condition will generally produce the maximum flow in the sewer. Of the total amount of storm water flowing to the sewer from a given storm, not more than from one-quarter to three-quarters of it will generally reach the sewer during the continuance of the storm. This ratio can not be stated with any great degree of accuracy.

1427. For cities, Professor Talbot gives the following as the ratio of storm water to contemporary rainfall:

(*b*) Suburban districts, sewered but not paved, As low as .20
(*d*) Suburban districts, paved and sewered, .30 to .40
(*f*) Closely built-up districts, paved and sewered, .40 to .50
(*h*) Roofs of buildings .... Nearly 1.00

1428. Assuming the condition of the surface to vary according to the density of the population, he also gives the following percentages (stated here as ratios) as indicating the proportion of the rainfall flowing off at once:

	Population per Acre.	Ratios.
(*a*)	10	.10
(*d*)	20	.20
(*e*)	40	.30
(*f*)	50	.40
(*g*)	Denser	.50 and upwards.

For convenience of comparison, corresponding items of the ratios and percentages given in this and other articles are designated by the same letter.

The ratios given by Talbot are probably high; for the percentages given by Kuichling, as well as those derived from the gaugings of London sewers, represent approximately the *total* amount of storm water flowing from a given storm, while the quantity of storm water flowing off during a period of time equal to the duration of the storm will be very considerably less.

THE MAXIMUM RATE OF FLOW.

1429. London Gaugings; Rate of Flow.—The period of maximum flow may be assumed to be the same as the time of the duration of the storm, although it will generally require a considerably longer time to carry off the total amount of storm water given by the storm. Of the available data relating to this phase of the subject, probably the most reliable are those obtained from gaugings of sewers in the city of London. From these gaugings, the length of time required for the sewers to carry off the rainfall was found to be from three to four times the duration of the storm; or, in other words, the duration of the storm was found to be from $\frac{1}{4}$ to $\frac{1}{3}$ of the time required for the storm water to flow away. But it was also found that the quantity of storm water reaching the sewer during different portions of this time varied greatly, being for certain short periods as high as 2.4 times the average storm flow; that is, the *maximum* flow per second was found to rise as high as 2.4 times the *average* flow per second from the entire storm. Therefore, in order to provide for this condition, the sewer capacity must be sufficient to carry, *during a length of time equal to the duration of the storm*, from $\frac{1}{4} \times 2.4 = 0.6$ to $\frac{1}{3} \times 2.4 = 0.8$ of the total amount of storm water *given to the sewer* by the storm.

1430. Ratio of Storm Flow to Rainfall.—In different districts of London, the ratio of the total amount of storm water to the total amount of rainfall was found to vary from .53 to .94, as stated in Art. **1425.** As the city of London is one of the most densely populated and completely paved cities in the world, it is probable that the maximum ratio of .94 was for districts consisting of solid stretches of roofs, walks, and pavements in perfect condition, and may be properly taken to represent the absolute maximum of the total storm-water flow. For this condition, the flow of storm water during a length of time equal to the duration of the storm will be from $.6 \times .94 = .56$ to $.8 \times .94 = .75$ of the rainfall. It is to be remembered that these

ratios are for the most extreme conditions of a very densely populated city, and that they will not apply to ordinary cities.

For the minimum ratio of .53, the corresponding flow of storm water will be from $.6 \times .53 = .32$ to $.8 \times .53 = .42$ of the rainfall. This ratio will be assumed to have been for an ordinary district, rather closely built up and ordinarily paved, such as might exist in the outlying portions of London; the population will be assumed at 50 per acre. By applying the same ratio of the *contemporary* to the *total* flow of storm water to Kuichling's percentage of impervious ground, we obtain, for a population of 50 per acre, a ratio of from $.6 \times .526 = .32$ to $.8 \times .526 = .42$, which agrees well with the above.

1431. Coefficient of Storm Flow.—The ratios thus obtained represent, for the given conditions, the proportion of the rainfall to be assumed as flowing off during a period of time equal to the duration of the storm. This proportion of the rainfall will probably not flow away during this period of time, but the *rate of flow* given by this assumption will be such as may be attained by the storm water during short periods of its flow, and for which sewer capacity must be provided. The rate of flow thus obtained will, however, represent the extreme conditions. This ratio will hereafter be called the **coefficient of storm flow,** and will be designated by f. The ratio f is so affected by many and various local conditions that it is impossible to accurately state its value for districts of different character. Taking the above figures, however, in connection with Kuichling's percentage of impervious ground, as a basis, the following approximate values of f are given as a general guide. These values are not to be greatly relied upon, however, and, in practice, the proper coefficients of storm flow for special cases should generally be determined by experiment. The coefficients given here, however, will be valuable as checks.

TABLE 30.

Ratio f, or Coefficient of Storm Flow, for Drainage Districts.

Class.	Population per Acre.	Character of District.	Values of f. Min.	Max.	Mean.	f_1.
(*a*)	10	Unimproved suburbs	.06	.12	.09	
(*b*)	20	Improved suburbs, unpaved	.10	.18	.14	.310
(*c*)	25	Macadamized residence suburbs	.15	.21	.18	.375
(*d*)	32	Ordinary suburban districts, roughly paved	.20	.28	.24	.440
(*e*)	40	Built-up, paved districts	.26	.34	.30	.520
(*f*)	50	Closely built-up and well-paved districts	.32	.42	.36	.625
(*g*)		Densely built-up and exceedingly well-paved districts	.42	.56	.50	.750
(*h*)		Solid stretches of roofs, walks, and pavements, in perfect condition	.55	.75	.65	

In the above table, the various districts of different character are, for convenience of reference, designated by letters. The letters designating the various classes correspond as nearly as possible to those of the ratios and percentages previously given. There is no well-defined distinction between the districts of different character; they merge into one another. Judgment and care must be exercised in selecting the class to which any given district belongs. The values of f given in the table relate to the maximum rates of flow to which the storm water may rise. As this maximum flow will be maintained only during comparatively short periods of time, it follows that, when used with reference to a period of time equal to the duration of the storm, it will generally be sufficient to use the minimum values of f for large districts. The mean values of f will be used here, however. The values f_1, given in the last column of the table, will be explained further on.

1432. Conditions Affecting Ratio of Storm Flow.—The quantity of storm water represented by the ratios given in Table 30 may be assumed to reach the sewer during a period of time equivalent to the duration of the storm. The ratio will, of course, vary somewhat, according to the size of the district and the slope of the surface. If the district be large and its surface comparatively level, considerable time will be required for the storm water to flow over it to the sewer, and opportunity will be afforded for a large amount of evaporation and absorption; hence, the percentage of the rainfall reaching the sewer will be small. Moreover, in large districts the maximum rate of precipitation given by the storm will not always extend throughout the entire district. During storms of great length, the flow of the storm water from such districts will gradually increase, until the water from the most remote portions of the district reaches the sewer. If the storm is of short duration, it may cease before the water from remote parts of the district begins to enter the sewer. It is often the case in large districts that the greatest flow given by a storm of short duration occurs after the storm has abated. If, on the other hand, the district be small and the surface very sloping, the storm water will quickly reach the sewer and the proportion evaporated and absorbed will be small. In such a district, the heavy flow of storm water will begin during the early stages of the storm and will rapidly decrease soon after the storm abates. These conditions affect, to some extent, the *amount* of storm water given to the sewer, and must be considered in using the ratios given in Table 30. In general, it may be stated that the maximum values of f should be used for small districts having very sloping surfaces, and the minimum values for large and comparatively level districts.

1433. Time of Flow; Duration of Storm.—The above conditions will also very materially affect the *time* required for the water from the remote parts of the district to reach the sewer, and, consequently, the length of the

storm that will give the maximum flow in the sewer; for the sewer will not receive its greatest flow until the water from all parts of the district begins to enter it. This condition is also affected by the shape of the district, as well as by the slope and character of its surface. In a long district, it requires a greater length of time for the water to reach the lower portions of the sewer; and, the duration of the storm being greater, the rate of rainfall giving a maximum charge to the sewer will be correspondingly less than in a short district. The flow of the storm water over the surface of a flat district may require double the time required for it to flow over a district of the same size having a sloping surface. The flow of the storm water will be much more rapid over the surface of a paved district than over the surface of an unpaved district, and more rapid over a smooth lawn than over a wooded tract. If in reaching the sewer the water flows through rough and crooked channels, it will require much longer than if led by direct courses through smooth conduits.

The shorter the storm the higher will be the rate of rainfall. But in order to give the highest rate of flow entering the sewer, the storm must continue until the water from the most remote portions of the district begins to flow into the sewer.

1434. Condition Producing Maximum Flow.—From what has been said, it is evident that *the rate of rainfall that will produce the greatest flow in the sewer at any given point will be the maximum rate that will continue (after the ground has become saturated) for a length of time sufficient for the water from the most remote parts of the district to flow to the sewer and down through the sewer to the point under consideration; that is, to any point at which the size of the sewer is to be determined.* Hence, having ascertained the length of time required for this condition, we can readily determine the maximum rate of rainfall for which to provide. Thus, if it requires 10 minutes for the storm water from the remote parts of the district to reach

that point where the size of the sewer is to be determined, the rate of rainfall y that will cause the maximum flow in the sewer, as given by Table 29, Art. **1413**, will be 4.82 inches per hour, or 4.82 cubic feet per second per acre. The high rate of precipitation in this case is due to the fact that the storm water reaches the given point so promptly that a storm of short duration will give the maximum flow.

RATIONAL FORMULA FOR RATE OF FLOW.

1435. General Expression for Contemporary Flow.—From the conditions noticed above, taken in connection with the equation for the rate of maximum rainfall, may be derived a rational formula for determining the rate of flow per second that must be provided for in designing a storm-water sewer.

A **rational formula** is one which can be reasoned out by the application of known laws. An **empirical formula,** on the other hand, is one which is based upon experiments, the laws upon which it is based being assumed. All the preceding formulas are empirical. All formulas relating to the flow of water or any other fluid are more or less empirical, since *all* the laws governing the flow are not known. When, however, a formula is principally dependent upon known laws for its derivation, it is customary to call it a rational formula, even though the constants in it have to be determined by experiment.

As y_1 represents the number of cubic feet of rainfall per second per acre (taken equal to the rate of rainfall in inches per hour), if the duration x of the storm be expressed in seconds, then the value of y_1 will be given in the column y_1 of Table 29, opposite the value of x' given in the column x'.

If now f represents the ratio of the storm water to contemporary rainfall, as given by Table 30, then $f y_1$ will represent the contemporary flow per acre, or the number of cubic feet of storm water per acre, *reaching the sewer each second* during a period of time equal to the duration of the storm. Substituting y_1 for its equivalent value y in formula

101, and multiplying both terms of the equation by f, there will result

$$f y_1 = f \times \frac{a'}{x' + b'}. \qquad \textbf{(106.)}$$

1436. Derived Flow per Acre.—If now F be taken to represent the flow per acre of storm water entering the sewer in cubic feet per second, then

$$F = f y_1. \qquad \textbf{(107.)}$$

From formulas **107, 106, 104,** and **101,** we may write

$$F = f \times \frac{a'}{x' + b'} = f \times \frac{2.25 \times 3,600}{x' + .3 \times 3,600} =$$

$$f \times \frac{8,100}{x' + 1,080}. \qquad \textbf{(108.)}$$

Formula **108** may be readily solved by substituting the values of f and x'. But, for any value of x', the value of the expression $\frac{8,100}{x' + 1,080}$ may be taken directly from Table 29, Art. **1413.** Hence, for a storm of any duration x', the value of F in cubic feet per second, will be given by formula **107,** by taking the value of f from Table 30, Art. **1431,** and the value of y_1 from Table 29.

1437. Flow at Inlet.—If t be taken to represent the length of time in seconds required for the storm water from the most remote parts of the district to reach the inlet of the sewer, then, for the maximum flow of the sewer *at the inlet*, t will equal x' of formula **101,** and may be substituted for x' in formula **108,** by doing which we obtain

$$F = \frac{8,100 f}{t + 1,080}. \qquad \textbf{(109.)}$$

This equation determines the required capacity of the sewer in cubic feet per second at its upper inlet.

1438. Flow at Points Below Inlet.—For determining the required capacity of the sewer at any given point below the inlet, the rate of flow *along the sewer* must also be taken into consideration. This will vary with the grade

and with the character and size of the conduit; it may be determined by applying the ordinary hydraulic formulas, which will be noticed further on.

Let l = length of sewer in feet from inlet to point under consideration;

v = average velocity of flow in sewer in feet per second.

Then, $\frac{l}{v}$ = time in seconds required for the water to flow from the inlet to the point under consideration;

$t + \frac{l}{v}$ = total time in seconds for the water to flow from the most remote parts of the district to the point under consideration.

But $t + \frac{l}{v}$ is the duration of that storm which will give the maximum flow of storm water at the point under consideration. (See Art. **1434.**) By substituting this value for x' in formula **108,** we shall then have for this condition

$$F = \frac{8,100\,f}{t + \frac{l}{v} + 1,080}. \qquad \textbf{(110.)}$$

This is the most rational form of an equation for the required capacities of storm-water sewers that has yet been proposed, though the numerical constants may not apply to all cases.

1439. Talbot's Equation for the Rate of Flow.—Professor Talbot derived an equation for the rate of flow, having the same form as formula **110,** from his equation for the maximum rate of ordinary rainfall; it is as follows, using the same notation as above:

$$F = \frac{6,300\,f}{t + \frac{l}{v} + 900}. \qquad \textbf{(111.)}$$

1440. Objection to Element of Time in Formula.—The objection made to this form of equation for

the flow of storm water is that it includes the element of time. But as the rate of rainfall varies greatly with the duration of the storm, it would seem impossible to propose a rational formula for this purpose that does not include the element of time. As the length of the sewer l, from the inlet to the point under consideration, is always known, and, by assuming dimensions, the velocity v may be calculated by the ordinary formulas of hydraulics, the value $\frac{l}{v}$ may be readily determined. The most difficult feature in applying the formula is to satisfactorily determine the length of time required for the storm water from the remote parts of the district to reach the sewer. This period of time t may be determined by experiment.

Having by experiment determined the value of t for one part of the district, then, for any other part, it may be determined by proportion, all other conditions being the same. If k be taken to represent the length in feet of the path traversed by the water in reaching the sewer, the time t may be taken directly proportional to k.

1441. Surface Velocities for One Per Cent. Slope.—The period of time t in seconds required for the storm water to reach the sewer may also be approximately estimated as follows:

If v_s = the average velocity of the surface flow in feet per second, then the time t will equal $\frac{k}{v_s}$. The value of k is known, and the value of v_s may be approximately determined. For an average surface slope of 1 per cent., that is, for a fall of 1 foot per hundred feet, the approximate effective surface velocities v_1 in feet per second, as estimated for different characters of surface, are given in Table 31. The values given in this table do not represent actual velocities; they represent the *effective* velocities, that is, the velocities which the water would need to have in order to reach the sewer in the same length of time that it actually does, *if it flowed in a direct line.* Under the usual con-

ditions, the water flows to the sewer by quite devious courses and at considerably higher velocities. It will be evident that any such values will be merely rough approximations.

TABLE 31.

Effective Velocities v_1 in Feet per Second for an Average Surface Slope of 1 Per Cent.

Class.	Character of Surface.	Velocities v_1. Min.	Max.	Mean.
(a')	Woodlands and heavy vegetable growth	.11	.22	.16
(a)	Pasture lands and meadows	.14	.28	.21
(b)	Smooth lawns	.19	.37	.28
(c)	Firm gravel and macadam	.28	.52	.40
(d)	Rough stone pavements	.45	.75	.60
(e)	Ordinary pavements	.60	1.10	.85
(f)	Good pavements	.75	1.45	1.10
(g)	Perfect pavements and gutters	.95	1.85	1.40
(h)	Asphalt pavements, perfectly paved gutters, roofs, troughs, and similar surface conduits	1.60	2.40	2.00

NOTE.—It will be noticed that the various classes in Table 31 will generally correspond with the classes designated by the same letters in Table 30, Art. **1431**, so that the classes designated by the same letters can generally be used together. This will not always be the case, however.

For convenience, these values will be used throughout this subject; but they are not to be relied upon in practice. The surface velocities or the time t should, so far as possible, be determined by experiment.

1442. Surface Velocities for Other Slopes.—By the slope S, expressed as a per cent., is meant the fall of the surface in feet per hundred feet in length of slope, or the sine of the angle of the average slope with the horizon, multiplied by one hundred; thus, if h is the total fall in feet along the path k, then $S = \frac{100\,h}{k}$. Having obtained the surface velocity for a slope of one per cent., the surface velocity v_s for any other slope may be obtained by proportion; for

the velocities will be to each other as the square roots of the slopes, all other conditions being equal. Hence, if S be expressed as a per cent., we shall have the proportion $v_1 : \sqrt{1} :: v_s : \sqrt{S}$, from which $v_s = v_1\sqrt{S}$. Consequently, having the velocity v_1 for a slope of one per cent., to find the velocity v_s for any other slope, *multiply v_1 by the square root of the slope*. It is evident that the surface velocity will be influenced by the form as well as by the character of the district. Judgment and extreme caution must be exercised in using the velocities given in Table 31.

1443. Practical Formula for the Flow per Acre.—It will be noticed that the period of time t required for the water to flow to the sewer is equal to $\frac{k}{v_s} = \frac{k}{v_1\sqrt{S}}$. By substituting the latter expression for t in formula **110**, we shall have

$$F = \frac{8{,}100\,f}{\frac{k}{v_1\sqrt{S}} + \frac{l}{v} + 1{,}080}. \qquad \textbf{(112.)}$$

If, in order to meet special conditions, it is desired to use different values for the constants a'' and b'' in the equation for the rate of rainfall, such values may also be substituted for a'' and b'' in the general form of this formula, which is as follows:

$$F = \frac{f a''}{\frac{k}{v_s} + \frac{l}{v} + b''}. \qquad \textbf{(113.)}$$

It will be noticed that formula **113** is simply formula **106** with the expression $\frac{k}{v_s} + \frac{l}{v}$ substituted for x'', and that formula **112** is formula **108** with the expression $\frac{k}{v_1\sqrt{S}} + \frac{l}{v}$ substituted for x''. Consequently, formula **112** may readily be solved in the form of formula **107**, by finding the value of the expression $\frac{k}{v_1\sqrt{S}} + \frac{l}{v}$, which is equal

to x'; then, taking from Table 29, Art. **1413**, the value of y_1 corresponding to this value for x', and the value [illegible] from Table 30, Art. **1431**.

Formula **112** will herein be used for determining the required capacities for storm-water sewers. As thus rationally derived, it is a practical and flexible formula for this purpose. By embracing all essential conditions, it employs all acquired data for determining the flow, and becomes a safe working formula, permitting the intelligent exercise of judgment and discretion in deciding the values of the various quantities. The values of all quantities should be determined as accurately as possible. The values of f and r_1 should be determined by experiment when possible; the correct values of k and [illegible] may be readily obtained.

1444. The **value of v** may be calculated by assuming the character, dimensions, and gradient of the sewer, as will be explained hereafter. For preliminary calculations of small sewers, however, [illegible] be as given in Table 32.

TABLE 32.

Approximate Velocities of Flow in Sewers [illegible] per Second to be Assumed for Different Grades.

Class.	Grade.	[illegible]
(1)	Very flat.	[illegible]
(2)	Flat.	[illegible]
(3)	Moderately flat	[illegible]
(4)	Ordinary . . .	[illegible]
(5)	Steep . . .	[illegible]
(6)	Very steep	[illegible]

In very large sewers, the [illegible] For convenience, the [illegible] used here in calculations [illegible] will not greatly affect the [illegible] of F given by formula **112** [illegible] cubic feet per second, and [illegible] of acres drained in order to give the total flow per second.

to be provided for. The total flow in a sewer is called the **discharge** of the sewer.

1445. Form of Drainage District. — Although drainage districts are usually somewhat irregular in form, yet most districts are approximately rectangular, and, for the purposes of estimating the storm-water flow, may generally be assumed to be rectangular with the main trunk sewer extending longitudinally through the middle of the district. In Fig. 356 *A* may represent the actual form of a drainage district tributary to the main drainage sewer *s*. In computing the required capacity of the sewer, however, the form of the district may generally be assumed to be as shown at *B*. In such a case, the length *l* of the sewer, below the upper inlet that receives water from a remote part of the district, may be taken equal to the length of the district minus *d*, the distance from the inlet to the upper end of the district; while *k* will usually be equal to half the width of the district at its upper end, as shown.

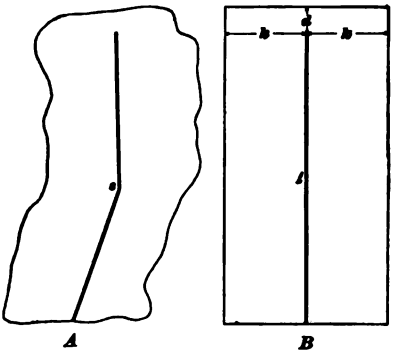


FIG 356.

1446. Area of Drainage District.—There are 43,560 square feet in an acre. Hence, the number of acres *A* in the rectangular drainage district shown at *B*, Fig. 356, will be given by the formula

$$A = \frac{2k(l+d)}{43,560} = \frac{k(l+d)}{21,780}, \qquad \textbf{(114.)}$$

in which *k* is half the width, and $l + d$ is the total length of the drainage district, all in feet.

It is evident that formula **114** will give, approximately, the number of acres in the drainage district shown at *A*, Fig. 356.

The number of acres in a given district may usually be roughly approximated by the formula

$$A = \frac{k\,l}{21,000}. \qquad \textbf{(115.)}$$

If the flow in the sewer is to be determined at some point above the outlet, then, in formulas **110** to **115**, inclusive, l will be the distance from the inlet to the point under consideration.

1447. The Total Effluent.—As F is the flow *per acre* in cubic feet per second, the effluent E, or total flow from the district in cubic feet per second, will be equal to F multiplied by the number of acres, or $A\,F$. By multiplying together the corresponding terms of formulas **112** and **114**, and writing E for $A\,F$, the following formula is obtained:

$$E = \frac{k\,(l+d)}{21,780} \times \frac{8,100\,f}{\frac{k}{v_1\sqrt{S}} + \frac{l}{v} + 1,080} = \frac{.37\,f\,k\,(l+d)}{\frac{k}{v_1\sqrt{S}} + \frac{l}{v} + 1,080}. \qquad \textbf{(116.)}$$

This is a general working formula for the required capacity of a storm-water sewer. It will, however, be well to notice that if the district is very irregular, the number of acres contained in it will not be correctly given by formula **114**, which will also affect the accuracy of formula **116**. In such a case, the flow per acre F should be calculated by formula **112**, and the result multiplied by the number of acres in the district, as calculated from measurement or other available information. The latter method is to be preferred.

It should also be noticed that the distance k, or path of the water flowing over the surface to the sewer inlet, which in B, Fig. 356, is shown as equal to one-half the width of the district, is not usually of nearly so great a length. The two lines marked k in the figure would, in most instances, represent the positions of branch sewers. In such cases, the length k of the path of the surface flow would usually be

the distance from the upper end of the longer branch to the upper margin of the district, which would generally be equal to the distance d. For simplicity in the problems, the path k of the surface flow is assumed as shown in B, Fig. 356.

1448. EXAMPLE.—In a rectangular suburban drainage district, roughly paved with stone pavement (class d) and having an average slope towards the sewer of 2 feet per 100 feet, and in which the distances k, l, and d are equal to 660, 5,000, and 280 feet, respectively, what will be the required capacity in cubic feet per second at the lower end of a storm-water sewer draining the district, assuming a velocity of 4 feet per second for v, and using for f and v_1 the mean values given for class (d) in Tables 30, Art. **1431**, and 31, Art. **1442**?

SOLUTION.—The mean value of f, as given for class (d) in Table 30, is .24, while from Table 31, the mean value of v_1 for the same class is .60. Hence, by formula **116**,

$$E = \frac{.37 \times .24 \times 660 \times (5{,}000 + 280)}{\frac{660}{.60 \times \sqrt{2}} + \frac{5{,}000}{4} + 1{,}080} = \frac{30{,}945}{311} = 99.5 \text{ cubic feet per second. Ans.}$$

EXAMPLES FOR PRACTICE.

NOTE.—In the following examples, in order to avoid confusion, the values of f and v_1 used will always be the mean values for the class designated, as given in Tables 30 and 31. The value of v will be assumed as classified in Table 32, Art. **1444.** Districts will be assumed to be rectangular; and, in each case, the outlet of the sewer will be considered to be at the lower edge of the district. The total discharge will generally be expressed here to the nearest tenth of a cubic foot.

1. If, as in the above example, the distances k, l, and d remain equal to 660, 5,000, and 280 feet, respectively, but the character of the district and surface be assumed to correspond to class f, Tables 30 and 31, while v be taken as in class (*4*), Table 32, and S be taken at 10 feet per hundred, what will be the required capacity or discharge of the sewer at its outlet? Ans. 244.93 cu. ft. per sec.

2. For the same, what will be the flow per acre? Ans. 1.539 cu. ft. per sec.

3. What will be the discharge of the same at a point 2,640 feet above its outlet? Ans. 149.1 cu. ft. per sec.

4. At the same point, what will be the flow per acre? Ans. 1.864 cu. ft. per sec.

5. If, for the same district, the character of the surface be taken to correspond to class c, v be taken as in class (*1*), and S be taken at 0.25

of a foot per hundred, what will be the maximum discharge at the outlet of the sewer? Ans. 33.70 cu. ft. per sec.

6. For the same, what will be the flow per acre? Ans. .212 cu. ft. per sec.

7. In a drainage district 4,000 feet long and 4,356 feet wide, the upper inlet to the sewer is 200 feet below the upper edge of the district; if f and v_1 be taken as in class e, v be taken as in class (*2*), and S be taken at 5 feet per hundred, what will be the maximum discharge at the outlet? Ans. 304.4 cu. ft. per sec.

8. For the same, what will be the flow per acre? Ans. .765 cu. ft. per sec.

9. For a district of the same dimensions, if f and v_1 be taken as in class g, v be taken as in class (*3*), and S be taken at 1 per cent., what will be the maximum discharge at the outlet? Ans. 493.02 cu. ft. per sec.

10. For the same, what will be the flow per acre? Ans. 1.239 cu. ft. per sec.

OTHER FORMULAS FOR EFFLUENT.

PRACTICE IN VARIOUS CITIES.

1449. Buerkli's Formula.—Various other formulas have been proposed for the capacities of storm-water sewers, attempting to state the effluent, or storm-water flow, in mathematical language. Of these, the formula proposed by Buerkli, a German authority, is probably the most reliable; it may be written as follows:

$$F = f_1 r_1 \sqrt[4]{\frac{S_1}{A}}, \qquad \textbf{(117.)}$$

in which F is the flow of storm water per acre in cubic feet per second, S_1 is the average surface slope (presumably towards and along the drain) in feet per *thousand* feet through the drainage district, A is the area of the drainage district in acres, f_1 is the coefficient relating to "the proportion of rainfall that will reach the sewer," and r_1 is a coefficient representing the rate of rainfall in inches per hour "during the period of the greatest intensity of rain." As S_1 is the average slope, or fall, of the surface in feet per thousand feet, it will always be equal to ten times the average slope

expressed in feet per hundred feet; that is, $S_1 = 10\ S$. (See Art. **1442.**)

1450. Values of Coefficients in the Buerkli Formula.—The coefficient f_1 in the Buerkli formula has values ranging from .31 in rural districts and suburbs to .75 in well built-up cities, with a mean value of .62; for purposes of comparison, as applied to various classes of districts, assumed values of f_1 are given in the last column of Table 30, Art. **1431.** A value of .75 has been used for f_1 in St. Louis. When not otherwise specified, a mean value of .625 will be used here. By *mean value*, as used here, is meant that value which best represents the most usual conditions.

The quantity r_1, though commonly stated as the rate of rainfall during the greatest downpour, has been shown to be scarcely more than an arbitrary coefficient. Since in this climate, the intensity of rainfall varies greatly with the length of the storm, it follows that, in using r_1 as the rate of rainfall, it is necessary to fix upon a definite length of time as representing the duration of a typical storm, which is equivalent to arbitrarily fixing the value of r_1. When the length of a typical storm has been decided upon, the value of r_1 will be the same as the value of y given by formula **99,** Art. **1406,** or, generally, the same as given in Table 29, Art. **1413.** In using the Buerkli formula, the European practice is to give r_1 values ranging from 1.75 to 2.5 inches per hour, but recent American practice gives r_1 values of from 2.0 to 3.5, and even higher, for sewers designed to carry all the storm water. As used in St. Louis, the McMath formula (to be noticed further on) is equivalent to the Buerkli formula with a value of .75 for f_1 and values for r_1 varying from 3.02 for a district containing 100 acres to 3.51 for a district containing 2,000 acres. Observations taken in Rochester of rain storms lasting less than one hour indicate that, for the conditions in that city, storms of 51 minutes' duration give the greatest flow. For this length of storm, the value of r_1, taken equal to y, as given by formula **104,** Art. **1408,** will be 1.96, or say 2.0. A value of

2.75, which is about the mean of American practice, will be taken here for r_1.

1451. The Total Effluent.—If both terms of formula **117** be multiplied by A, the number of acres drained, and if in the first term E be written for $A\ F$, the value of the total effluent E, as thus derived from formula **117**, will be as follows:

$$E = f_1 r_1 A \sqrt[4]{\frac{S_1}{A}} = f_1 r_1 \sqrt[4]{S_1 A^3}. \qquad \textbf{(118.)}$$

If both terms of formula **118** are divided by $f_1\ r_1$ it will take the form

$$\frac{E}{f_1 r_1} = \sqrt[4]{S_1 A^3}. \qquad \textbf{(119.)}$$

In Table 33, the values of the expression $\sqrt[4]{S_1 A^3}$ are tabulated for various slopes S_1 and areas A.

If the value of the expression $\sqrt[4]{S_1 A^3}$, as thus tabulated, are represented by the letter e, then formula **118** may be written in the form

$$E = f_1 r_1 e, \qquad \textbf{(120.)}$$

in which the value of e is to be taken from Table 33, while the values of f_1 and r_1 will remain as before.

Buerkli fixes the greatest necessary capacity of storm-water sewers at 0.86 of a cubic foot per second per acre. This, however, is merely a general approximation. The required capacity may be greatly affected by the varying conditions relating to different cases, such as the size and shape of the district or the character and slope of its surface. The Buerkli formula is an attempt to rationally formulate an expression for these conditions, and is one of the best formulas that has been proposed. It approximately involves most of the conditions materially affecting the effluent, and, when used intelligently, may be relied upon to give reasonably accurate results. The form of the district is not in any way represented in the Buerkli formula, however. This formula is used at Mannheim with the value of r_1 taken at 1.79.

TABLE 33.

Values of the Expression $\sqrt[4]{S_1 A^3}$ Designated as e.

Acres = A	$S_1 = 2.5$	$S_1 = 5$	$S_1 = 10$	$S_1 = 15$	$S_1 = 20$	$S_1 = 25$	$S_1 = 50$	$S_1 = 100$
40	20.00	23.78	28.28	31.30	33.64	35.57	42.29	50.30
60	27.10	32.24	38.34	42.43	45.59	48.21	57.33	68.17
80	33.64	40.00	47.57	52.64	56.57	59.81	71.13	84.59
100	39.76	47.29	56.23	62.23	66.87	70.71	84.09	100.00
120	45.59	54.22	64.47	71.35	76.67	81.07	96.41	114.65
160	56.57	67.27	80.00	88.53	95.14	100.60	119.63	142.26
200	66.87	79.53	94.57	104.66	112.47	118.92	141.42	168.18
300	90.64	107.79	128.19	141.86	152.44	161.19	191.68	227.95
400	112.47	133.74	159.05	176.02	189.15	200.00	237.84	282.84
500	132.96	158.09	188.02	208.09	223.61	236.44	281.17	334.37
600	152.44	181.28	215.58	238.58	256.37	271.08	322.37	383.37
800	189.15	224.92	267.50	296.03	318.11	336.36	400.00	475.68
1,000	223.61	265.90	316.23	349.96	376.06	397.64	472.87	562.34
1,200	256.37	304.84	362.57	401.24	431.17	455.90	542.16	644.74
1,500	303.08	360.39	428.62	474.34	509.71	538.96	640.93	762.20
2,000	376.06	447.21	531.83	588.57	632.46	668.74	795.27	945.74
2,500	444.57	528.68	628.72	695.79	747.67	790.57	940.15	1,118.03

1452. The McMath Formula; St. Louis.—A formula similar in form to the Buerkli formula was derived from conditions in St. Louis. For the flow per acre, it may be written,

$$F = f_1 r_1 \sqrt[5]{\frac{S_1}{A}}; \qquad \textbf{(121.)}$$

or, for the total effluent,

$$E = f_1 r_1 \sqrt[5]{S_1 A^4}, \qquad \textbf{(122.)}$$

in which all letters represent the same values as in formulas **117** and **118**. As used in St. Louis, r_1 was given a value of 2.75. In that city, sewers having a capacity less than given by this formula, with values of .75, 2.75, and 15 used for f_1, r_1, and S_1, respectively, are known to be overtaxed. It is stated that with the proper values substituted for the coefficients, the Buerkli formula will give results corre-

sponding as well with the conditions [illegible] as [illegible] as the McMath formula.

1453. Baumeister's Formula.—The equation for maximum flow, given by Professor Baumeister, is as follows, the notation being somewhat changed:

$$E = f_2 z r_2 A. \qquad (123.)$$

in which E is the total effluent and r_2 is the rainfall per acre, both in cubic feet per second. A is the number of acres drained, f_2 is the ratio of impervious surfaces to whole area and z is a coefficient relating to the size and shape of the district.

Professor Baumeister gives the following values for f_2 (see also Arts. **1423** and **1431**):

	Min.	Max.	Mean.
In villages..................	.25	[illegible]	[illegible]
In towns...................	[illegible]	[illegible]	[illegible]
In cities....................	[illegible]	[illegible]	[illegible]

As r_2 is the rate of rainfall, it will be equal to the value of y, as given by formula **99**, Art. **1406**, when the length of storm giving the maximum flow has been determined. The length of the storm will depend upon the [illegible] represented by z.

1454. Board of Sanitary Engineers, Washington.—The formula adopted by the Board of Sanitary Engineers, appointed to report on the sewerage of the District of Columbia (Art. **1404**), is as follows:

$$E = 5.886\, A^{\frac{1}{2}}, \qquad (124.)$$

in which E is the total effluent from the district drained and A is the number of acres in the same. This formula is equivalent to the Buerkli formula with values of .75, 3.51, and 25 used for f_1, r_1, and S_1, respectively. It may, therefore, be readily computed from Table 33.

1455. Engineer Department, District of Columbia.—The recent practice in the city of Washington is to

provide capacities for the storm-water sewers sufficient to carry the flow given by the following formulas:

For areas of 10 acres or less:

$$E = 3\,A. \qquad \textbf{(125.)}$$

For areas of from 10 to 60 acres:

$$E = 2\,A. \qquad \textbf{(126.)}$$

For areas of more than 60 acres:

$$E = 5.293\,A^{\frac{3}{4}}. \qquad \textbf{(127.)}$$

In all of which A is the number of acres, and E is the total flow from the district drained in cubic feet per second. Formula **127** is equivalent to the Buerkli formula with .74, 3.20, and 25 substituted for f_1, r_1, and S_1, respectively.

1456. Hawksley's Formula; London.—The formula proposed by Thomas Hawksley, an eminent English hydraulician, is as follows:

$$\log d = \frac{3 \log A + \log N + 6.8}{10}, \qquad \textbf{(128.)}$$

in which d is the diameter of the sewer in *inches*, N is the length in feet, in which the sewer falls one foot, that is, the length of the sewer divided by its fall, and A is the number of acres drained.

This formula is said to be equivalent to that of Buerkli's, with r_1 taken equal to 1.0 and f_1 equal to about .7. It is put in logarithmic form and combined with Eytelwein's formula for flow in pipes.

This formula was used by Sir Joseph Bazelgette and Mr. William Haywood in designing the main drainage works of London. It is sometimes known as the Bazelgette formula, but more commonly as the Hawksley formula. In that city, the method of estimating the flow varies with the size of the district and density of the population, but the coefficient used is generally from $\frac{1}{3}$ to $\frac{1}{2}$, applied to a rainfall of one inch per hour. It is stated that for the intercepting sewers the allowance is from $\frac{1}{8}$ to $\frac{1}{4}$ of an inch of rainfall per hour;

that is, a coefficient of from $\frac{1}{8}$ to $\frac{1}{4}$ applied to a rainfall of one inch per hour. The Hawksley formula was also used in designing the system of sewers for Brooklyn.

1457. Adams' Formula.—The formula proposed by Mr. Julius W. Adams, a well-known sanitary engineer of Brooklyn, is as follows:

$$D = \sqrt[6]{\frac{Q^2 L}{1,542 H}}, \qquad \textbf{(129.)}$$

in which D is the diameter of the sewer in *feet*, L is the length of the sewer, H is the total fall in the same, and Q is the discharge in cubic feet per second.

For a rainfall of 1 inch per hour, $Q = \frac{A}{2}$.

For a rainfall of $1\frac{1}{2}$ inches per hour, $Q = \frac{A}{1.65}$.

For a rainfall of 2 inches per hour, $Q = \frac{A}{1.33}$.

A is the number of acres drained.

Formula **129** may be expressed in the following logarithmic form:

$$\log D = \frac{2 \log A + \log N - C}{6}, \qquad \textbf{(130.)}$$

in which $N = \frac{L}{H}$, or the length in feet in which the sewer falls one foot, and C is a constant which, for different rates of rainfall, has the following values:

For 1 inch per hour, $C = 3.790$.
For $1\frac{1}{2}$ inches per hour, $C = 3.623$.
For 2 inches per hour, $C = 3.436$.

1458. Chicago.—On the older sewers of Chicago one inch of rainfall was provided for, and the flow of storm water was considered to be directly proportional to the area drained. With this allowance for rainfall some districts were frequently flooded, and in the newer parts of the city a larger allowance has been made. From the conditions

which have herein been considered, it is evident that a sewer capacity of one inch of rainfall per hour, or one cubic foot per second per acre, may be sufficient for a large district, and wholly insufficient for a small district of like character.

1459. Boston.—On the main drainage of Boston, sewer capacity is provided for a small portion of the rainfall only, and the excess escapes at overflows. For the laterals, the Buerkli formula is used with a value of r_1 equal to 1.0, though this is considered not to be sufficient in all cases.

1460. Berlin.—For districts having an average population, a coefficient of $\frac{1}{3}$ is applied to a rainfall of 0.91 of an inch per hour, giving an effluent of $\frac{1}{3} \times .91 = .30$ of a cubic foot per second per acre. For districts with parks a coefficient of $\frac{1}{6}$ is used, making the estimated effluent $\frac{1}{6} \times .91 = .15$ of a cubic foot per second per acre.

1461. Paris.—On trunk sewers, a coefficient of $\frac{1}{3}$ is applied to a rainfall of 1.79 inches per hour, giving an effluent of $\frac{1}{3} \times 1.79 = .60$ of a cubic foot per second per acre.

1462. In many cities of England, a coefficient of $\frac{1}{2}$ is applied to a rainfall of one inch per hour, giving an effluent of $\frac{1}{2} \times 1.00 = .50$ of a cubic foot per second per acre.

1463. EXAMPLE.—For the example explained in Art. **1448**, what will be the total flow, or effluent, as given by the Buerkli formula, assuming a value of 2.50 for r_1 and assuming for f_1 and S_1 values corresponding to those assumed for f and S, respectively, in that example ?

SOLUTION.—The number of acres in the district, as given by formula **114**, is equal to $\frac{660 \times (5,000 + 280)}{21,780} = 160$. The slope S is 2 feet per hundred, and, consequently, the slope S_1 in feet per thousand will be $10 \times 2 = 20$. From Table 33, Art. **1451**, the value of c for a district containing 160 acres and having a surface slope of 20, is 95.14. As given in Table 30, Art. **1431**, the value of f_1 for class d is .44. Hence, by formula **120**, $E = .44 \times 2.50 \times 95.14 = 104.65$, or, practically, 105 cubic feet per second. Ans.

EXAMPLES FOR PRACTICE.

NOTE.—The following examples relate to those given in Art. **1448.** When not otherwise stated, they are to be solved by the Buerkli formula for the conditions given in those examples. The values of f_1 will be taken as given for the various classes of districts in Table 30; the value of S_1 will be taken at ten times the value stated for S, and, when not otherwise stated, the value of r_1 will be taken at 2.75.

1. What will be the total flow in cubic feet per second from the district described in Example 1? Ans. 224.5 cu. ft.

2. What will be the required discharge for Example 3 in cubic feet per second? Ans. 145.4 cu. ft.

3. What will be the total effluent for the conditions stated in Example 5 in cubic feet per second? Ans. 58.3 cu. ft.

4. What will be the total effluent from the same in cubic feet per second, assuming the value of r_1 at 1.75? Ans. 37.1 cu. ft.

5. For the district described in Example 7, what will be the total flow per second, assuming a value of 2.50 for r_1? Ans. 309.2 cu. ft.

6. For the district described in Example 9, what will be the total flow per second, taking r_1 at 3.50? Ans. 417.5 cu. ft.

7. For the same district, what will be the total effluent, as given by formula **124,** Art. **1454?** Ans. 526.5 cu. ft.

GRAPHICAL REPRESENTATION OF EQUATIONS.

RECTANGULAR COORDINATES.

1464. General Statement and Definitions.—It will now be expedient to briefly study a branch of mathematics which, while not directly connected with the subject of drainage, is very convenient to use in computations relating to the flow of water. This is the branch of mathematics known as Analytical Geometry.

The **locus** of a point is the line, or geometrical figure, generated by the point when conceived as moving according to some fixed law. The path of the point may be straight or curved, but it is commonly spoken of as a **curve,** the straight line being a special case.

In Analytical Geometry, the loci of points are represented by, and constructed from, *equations* by means of what is called **a method of coordinates.**

1465. Proposition.—*If two straight lines be drawn in known positions and intersect each other at right angles, the position of any point in the plane may be fixed by its respective distances from the two lines.*

Thus, in Fig. 357, $X X'$ and $Y Y'$ are two straight lines intersecting each other at right angles at O. Both lines are assumed to be fixed in position, and to be of indefinite length. If the distances x and y of a point P from $O Y$ and $O X$, respectively, are known, the position of P is easily determined; for, if on $O X$ a distance equal to x is laid off, and at the extremity of this distance a perpendicular equal to y is erected, the point P will be at the extremity of this perpendicular, no other point satisfying the same conditions.

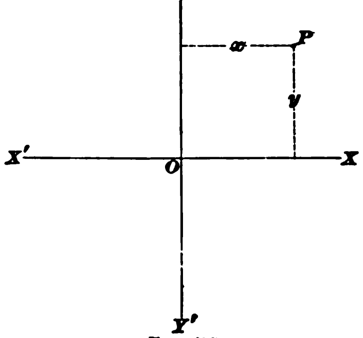


FIG. 357.

1466. The two intersecting lines are called **axes of coordinates.** When, as in Fig. 357, they intersect at right angles, they are called **rectangular axes.**

The horizontal axis is called the **axis of abscissas,** or **axis of** x, and the vertical axis, the **axis of ordinates,** or **axis of** y.

The intersection of the two axes is called the **origin;** it is here designated by the letter O.

The distances of any point in the plane from the two axes are called the **coordinates** of the point; the distance from the point to either axis is always measured parallel to or along the other axis. It is evident that for each point there are two coordinates. When, as in Fig. 357, the axes are rectangular, the coordinates are called **rectangular coordinates.**

The **abscissa** of a point is its coordinate measured parallel to the axis of abscissas; that is, it is its distance

from the axis of ordinates, measured parallel to or along the axis of abscissas. The abscissa, as used here, is the horizontal distance of the point to the right or left of the axis of ordinates. Thus, in Fig. 357, x is the abscissa of the point P.

The **ordinate** of a point is its coordinate measured parallel to the axis of ordinates; that is, it is its distance from the axis of abscissas, measured parallel to or along the axis of ordinates. The ordinate, as used here, is the vertical distance at which a point is situated above or below the axis of abscissas. In Fig. 357, y is the ordinate of the point P.

When using rectangular coordinates, it is customary to measure the abscissa along the axis of abscissas, and then measure the ordinate perpendicularly above or below the point thus obtained. In the usual notation, the abscissas are represented by the letter x, and the ordinates by y.

1467. The four angles formed by the intersection of the axes are designated as follows:

The angle *above* the axis of abscissas and to the *right* of the axis of ordinates is called the **first angle**; that *above* the axis of abscissas and to the *left* of the axis of ordinates is called the **second angle**; the angle *below* the axis of abscissas and to the *left* of the axis of ordinates is called the **third angle**, and that *below* the axis of abscissas and to the *right* of the axis of ordinates is called the **fourth angle.** Thus, in Fig. 358, P, P', P'', and P''' are, respectively, located in the first, second, third, and fourth angles.

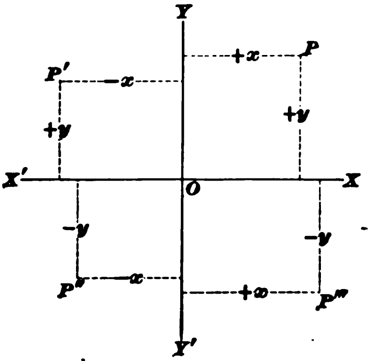


FIG. 358.

In order to indicate in which of the four angles a point is situated, the coordinates are given the signs + or −, in accordance with the following system:

Abscissas measured to the right from the axis of Y are considered $+$, *and those measured to the left from the axis of Y are considered* $-$; *ordinates measured upwards from the axis of X are considered* $+$, *and those measured downwards are considered* $-$. This is clearly shown in Fig. 358. The abscissas x, although usually measured along the axis of X, are, for clearness, shown here as measured from each point directly to the axis of ordinates. Either method is correct.

1468. To Locate a Point by its Coordinates.—When the coordinates of a point are known, the point may be readily located in accordance with the following

Rule.—*On the axis of X, lay off to some convenient scale a distance from the origin equal to the given abscissa; it must be laid off to the right if the abscissa is* $+$, *and to the left if it is* $-$.

Through the point thus located on the axis of X, draw a line parallel to the axis of Y, and on this line lay off to some convenient scale a distance from the axis of abscissas equal to the given ordinate, upwards if the ordinate is $+$, *and downwards if it is* $-$. *The point thus located at the extremity of the ordinate will be the required point.*

1469. Character of Quantities.—The quantities used in equations expressing loci, or curves, are of two classes, *constant* and *variable.*

Constant quantities are those always having the same values in the same equation; they are usually represented by the leading letters of the alphabet, as a, b, c, etc.

Variable quantities are those which may assume any values within the limits established by the nature of the equation; they are represented by the final letters of the alphabet.

NOTE.—The terms *constant* and *variable*, as used here, should not be confused with the terms *known* and *unknown*, as used in algebra. There is no similarity except in the notation employed. Both the known and unknown quantities in algebra are *constant.*

As an example, the equation of uniform motion may be taken. If s is the space passed over during the time t by a body moving with a constant velocity v, we have the equa-

tion $s = v\,t$. (See formula **8,** Art. **859.**) Suppose that v has a definite value, say 4 feet per second, and that we wish to know the different values of s corresponding to different values of t, that is, the different spaces passed over in different times. Suppose $t = 1$ second; then $s = 4 \times 1 = 4$ feet. If $t = 2$, then $s = 4 \times 2 = 8$ feet. If $t = 20$, then $s = 4 \times 20 = 80$ feet. The general equation being $s = 4\,t$, the quantity 4, which is a known number, is the **constant** of the equation; t, to which we give any value we please, is called the **independent variable;** and s, whose values are also variable, but depend upon the values of t, is called the **dependent variable.**

1470. Equation of a Locus.—Every equation between two variables (such as $s = 4\,t$) may be represented by a line whose points are located by referring them to two rectangular **axes.** For we may give t any value, and, having found the corresponding value of s, we may lay off the latter along the axis of x, and at the extremity erect a perpendicular parallel to the axis of y and equal to t. The coordinates of the point thus found will be the values of s and t for that particular case. Other values of t and s will give other points, and the line passing through these points will be the graphical representation of the equation. Conversely, the equation is the algebraical expression of the defining properties of the line; for it tells us that the line is such that, whatever point we take, its abscissa (x or s) is always equal to 4 times its ordinate (y or t).

When the properties of a line, either straight or curved, are known, its **equation,** that is, the general relation between the coordinates of any of its points, may be easily found. In Fig. 359, let $P\,P'''$

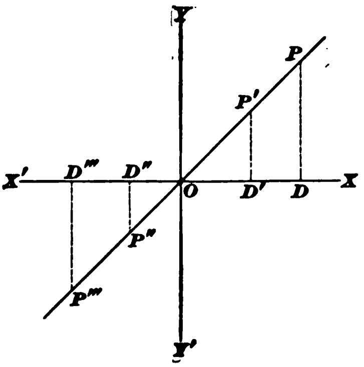


FIG. 359.

be a straight line passing through the intersection of the axes, and making an angle of 45° with each axis. For the point P the abscissa x will be $O\ D$, and the ordinate y will be $D\ P$. But, in the triangle $O\ D\ P$, the angle $D\ O\ P$ is 45° and the angle $O\ D\ P$ is a right angle. Hence, it is known that the angle $O\ P\ D$ is 45°, and the side $O\ D$ is equal to the side $D\ P$; or, for the point P, $x = y$. The same would be true of the point P'; likewise, for the point P'' or P''', it is found that $-x = -y$, which equation at once reduces to the form $x = y$. The equation $x = y$ is thus found to apply to *any* point in the line $P\,P'''$; it is, therefore, the equation of the locus $P\,P'''$.

1471. Construction of Equations.—To construct an equation, or to find the locus of an equation, is to draw the geometrical figure represented by it. An equation may be constructed by locating a sufficient number of points by their coordinates so that the locus may be sketched through the points located.

The coordinates of a point may be determined from an equation by assuming a convenient value for one variable, and solving the equation for the corresponding value of the other variable. If the value of the second variable, as thus obtained, is *real*, the values of the two variables will be the coordinates of the point. In like manner, the coordinates for any number of points may be obtained.

Either variable may be considered as the independent variable, although the x is commonly so taken.

The method of constructing an equation is given by the following

Rule.—*Transpose the equation to a form expressing the value of either variable. Consider this as the dependent variable. Assign any convenient value to the other, or independent, variable, and, by substituting it in the equation, find the corresponding value of the dependent variable. If the quantity thus found for the dependent variable is a real quantity, locate a point in the locus by the values of the two variables used as coordinates. By repeating the process, locate a*

sufficient number of points so that the locus can be sketched through them.

NOTE.—If the value substituted for the independent variable gives an *imaginary* quantity (i. e., an even root of a negative quantity) for the value of the dependent variable, this will indicate that an impossible value has been assigned to the independent variable. It will then be known that this variable can not have the value assigned to it. Consequently, a different value must be assigned to it and the equation solved again.

1472. Examples of the Construction of Equations.—The foregoing principles will now be illustrated by a few simple examples:

EXAMPLE.—Construct the equation $y = -x$. (See Fig. 360.)

SOLUTION.—By substituting numerical values for x in the equation, values of y are obtained as follows:

When $x = 0, y = 0$ (*O*).	When $x = -1, y = 1$ (*a'*).
When $x = 1, y = -1$ (*a*).	When $x = -2, y = 2$ (*b'*).
When $x = 2, y = -2$ (*b*).	When $x = -3, y = 3$ (*c'*).
When $x = 3, y = -3$ (*c*).	etc., etc.
etc., etc.	

By locating the points fixed by the coordinates thus determined, the locus $c\,c'$, Fig. 360, is obtained. Thus, the condition that $y = 0$ when $x = 0$, shows that the locus passes through the origin. When $x = 1$, we measure a distance of 1 unit to the right of the origin along the axis of X; since for this condition $y = -1$, a distance equal to one unit is measured from the axis of X downwards parallel to the axis of Y, thus locating the point a. The points b, c, a', b', and c' are located in a similar manner. Any convenient scale may be used in locating the points, so long as the same scale is used for all abscissas, and the same scale for all ordinates. The entire construction of the figure will be readily understood. It is evident

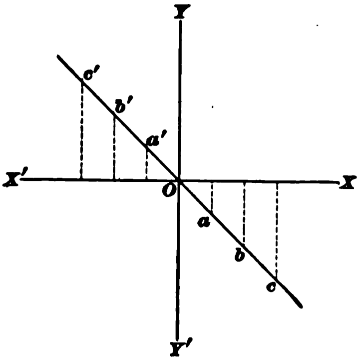


FIG. 360.

that any value substituted for x will give a real value for y, showing that the locus extends indefinitely in each direction.

EXAMPLE.—Construct the equation $x + 2y = 4$. (See Fig. 361.)

SOLUTION.—By transposing and solving for y, we get $y = \frac{4 - x}{2}$, and by substituting values for x, the corresponding values of y are found to be as follows:

When $x = 0, y = 2$, (*a*).	When $x = -1, y = 2\frac{1}{2}$.
When $x = 1, y = 1\frac{1}{2}$ (*b*).	When $x = -2, y = 3$.
When $x = 2, y = 1$ (*c*).	When $x = -3, y = 3\frac{1}{2}$.
When $x = 3, y = \frac{1}{2}$ (*d*).	When $x = -4, y = 4$.
When $x = 4, y = 0$ (*e*).	etc., etc.
When $x = 5, y = -\frac{1}{2}$ (*f*).	
When $x = 6, y = -1$ (*g*).	
etc., etc.	

By locating the points fixed by the above coordinates, the locus *a g*, Fig. 361, is obtained. It extends indefinitely in each direction. The process will be readily understood, and will require no further explanation.

EXAMPLE.—Find the locus of the equation $x^2 + y^2 = 36$.

SOLUTION.—By transposing and solving for y, there is obtained $y = \pm \sqrt{36 - x^2}$. For $x = 0$, $y = +6$ and -6, while for $x = 6$, $y = 0$, and for $x = -6$, $y = 0$. Consequently, as x is zero at the axis of ordinates only, and y is zero at the axis of abscissas only, it follows that the locus cuts both axes at distances of 6 and -6 from the origin. For intermediate positive values of x, the following values of y are obtained:

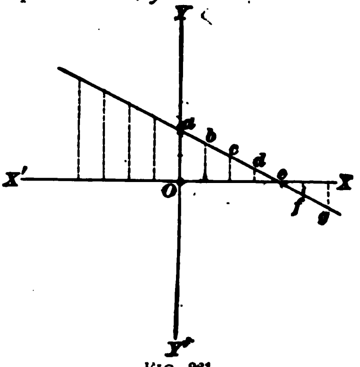


FIG. 361.

When $x = 1, y = \pm \sqrt{35} = 5.92$ and -5.92.
When $x = 2, y = \pm \sqrt{32} = 5.66$ and -5.66.
When $x = 3, y = \pm \sqrt{27} = 5.20$ and -5.20.
When $x = 4, y = \pm \sqrt{20} = 4.47$ and -4.47.
When $x = 5, y = \pm \sqrt{11} = 3.32$ and -3.32.

For negative values of x from 0 to -6, the values of y are the same as for positive values of x. By locating the points whose coordinates are thus obtained, they are found to

be in the circumference of a circle whose radius is 6, and whose center is at the origin, as shown in Fig. 362. It will be noticed that any value (integral or fractional) between $+6$ and -6 substituted for x, will give coordinates locating two points in the circumference of the circle.

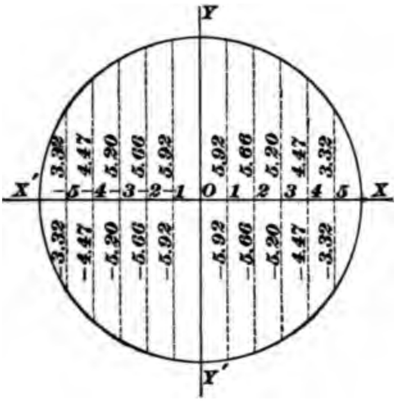


FIG. 362.

But if in the equation $y = \pm\sqrt{36 - x^2}$, we assign the value $x = 7$, there will result $y = \pm\sqrt{-13}$, which is an imaginary quantity, indicating that x can not have this value. The same will be true if x be given any value greater than 6.

EXAMPLE.—Construct the equation $y = \frac{1}{2}x^2$. (See Fig. 363.)

SOLUTION.—By assigning various numerical values to x, the corresponding values of y are obtained as follows:

When $x = 0, y = 0$ (O).	When $x = -1, y = \frac{1}{2}$ (a').
When $x = 1, y = \frac{1}{2}$ (a).	When $x = -2, y = 2$ (b').
When $x = 2, y = 2$ (b).	When $x = -3, y = 4\frac{1}{2}$ (c').
When $x = 3, y = 4\frac{1}{2}$ (c).	When $x = -4, y = 8$ (d').
When $x = 4, y = 8$ (d).	etc., etc.
etc., etc.	

Locating the points by the coordinates thus determined, a curve is obtained lying wholly in the first and second angles, as shown in Fig. 363. This curve is a **parabola** passing through the origin, and symmetrical with reference to the axis of ordinates. In the construction shown in Fig. 363, the same scale is used for the abscissas that is used for the ordinates.

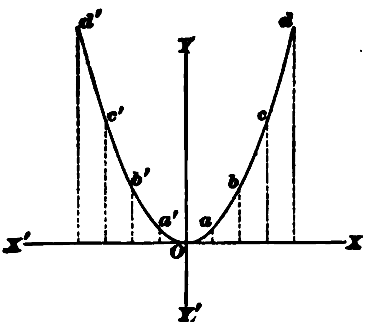


FIG. 363.

EXAMPLE.—Construct the equation $2x = y^2 + 4$. (See Fig. 364.)

SOLUTION.—By transposing and solving for y, we get $y = \pm\sqrt{2x-4}$, and by assigning positive numerical values to x, corresponding values of y are obtained as follows:

When $x = 0$, $y = \pm\sqrt{-4}$ Imaginary.
When $x = 1$, $y = \pm\sqrt{-2}$ Imaginary.
When $x = 2$, $y = 0$.
When $x = 3$, $y = \pm\sqrt{2} = 1.41$ and -1.41.
When $x = 4$, $y = \pm\sqrt{4} = 2.0$ and -2.0.
When $x = 5$, $y = \pm\sqrt{6} = 2.45$ and -2.45.
When $x = 6$, $y = \pm\sqrt{8} = 2.83$ and -2.83.
etc., etc.

Negative values of x substituted in the equation $y = \pm\sqrt{2x-4}$ will always give imaginary quantities. From the coordinates determined above, the curve shown in Fig. 364 is obtained. This curve is also a parabola; it is symmetrical with reference to the axis of abscissas, but does not pass through the origin.

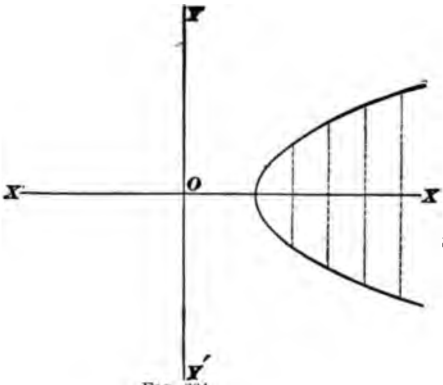


FIG. 364.

EXAMPLE.—Construct the equation $xy + y = 4$. (See Fig. 365.)

SOLUTION.—By solving for the value of y, there will result $y = \frac{4}{x+1}$; and, by assigning arbitrary values to x, corresponding values of y are obtained as follows:

When $x = 0$, $y = 4.00$ (a).
When $x = 1$, $y = 2.00$ (b).
When $x = 2$, $y = 1.33$ (c).
When $x = 3$, $y = 1.00$ (d).
When $x = 4$, $y = 0.80$ (e).
When $x = 5$, $y = 0.66$ (f).
When $x = 6$, $y = 0.57$ (g).
etc., etc.

When $x = -1$, $y =$ infinity.
When $x = -2$, $y = -4.00$ (a').
When $x = -3$, $y = -2.00$ (b').
When $x = -4$, $y = -1.33$ (c').
When $x = -5$, $y = -1.00$ (d').
When $x = -6$, $y = -0.80$ (e').
etc., etc.

From the coordinates thus determined, the points $a \ldots\ldots g$

and $a' \ldots\ldots c'$, Fig. 365, are located, through which the locus may be sketched.

NOTE.—In order to obtain points sufficiently close together so that the locus may be accurately sketched, it is often necessary to assume fractional values for the independent variable.

It will not be necessary to pursue this very interesting branch of mathematics further here, as sufficient has been

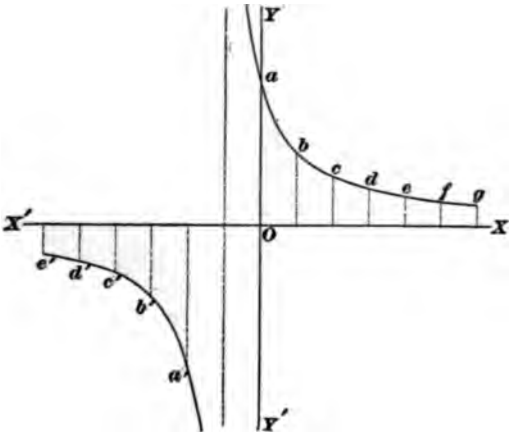


FIG 365.

given for present purposes. Should the student care to continue the study independently, he is referred to one of the various excellent text-books treating upon the subject, among the best of which may be mentioned Olney's General Geometry and Calculus. It will be necessary, however, for him to have a greater knowledge of algebra and trigonometry than is contained in this Course in order to study the book referred to.

1473. Cross-Section Paper.—For the purpose of constructing equations, a special kind of paper, which is ruled in small squares and is known as **cross-section paper**, is very convenient. When this kind of paper is used, the coordinates may be measured, and the required points located without the aid of a scale by simply counting the number of divisions from the axes corresponding to the values of

the respective coordinates. If either of the coordinates is expressed by a fraction, the fractional part of a division may generally be judged with sufficient accuracy by the eye. This facilitates the construction of the locus. In nearly all cross-section paper, every tenth line is made heavy, for convenience in counting the lines. In using the paper, it is customary to simply mark the value represented by each heavy line. It is often convenient to give the divisions greater values in one direction than in the other.

It will be well to notice here that there are two general varieties of cross-section paper. One kind has the lines ruled upon it and is known as **ruled cross-section paper**; the other kind has the lines printed upon it from an engraving, and is known as **engraved cross-section paper.** Ruled cross-section paper, though suitable for many purposes, is not really accurate, the spaces between the lines not being of exactly uniform size. Engraved cross-section paper is very accurate, and is the kind that should be used in the construction of the curves of hydraulic formulas.

EXAMPLES FOR PRACTICE.

1. Construct the equation $y = 2x + 4$.
2. Construct the equation $y = x - 6$.
3. Find the locus of the equation $y = x^2 - 4$.
4. Construct the equation $y = x^2 - x + 4$.
5. Construct the equation $3y^2 + 4x^2 = 12$.
6. Find the locus of the equation $y = x^2 + x$.
7. Construct the equation $2y^2 + 5x^2 = 20$.
8. Construct the equation $y = x^3 - \frac{1}{2}x^2 + 2x + 4$.

APPLICATIONS.

1474. Diagrams and Curves.—In the foregoing examples, the aspect of the locus in all four angles of the plane has been considered; but, in practice, by far the greater number of curves represented by equations are constructed only in the first angle. Such a curve, or locus, will

represent the conditions in which all values of both co-ordinates are positive, which generally includes all practical conditions to which the equation relates. Hereafter, all loci, or curves, considered will be understood to lie wholly within the limits of the first angle, and, consequently, the origin will always be at the lower left-hand corner of the diagram, which will be constructed upon cross-section paper.

1475. Curve for Rate of Rainfall.—A diagram representing graphically the maximum rate of ordinary

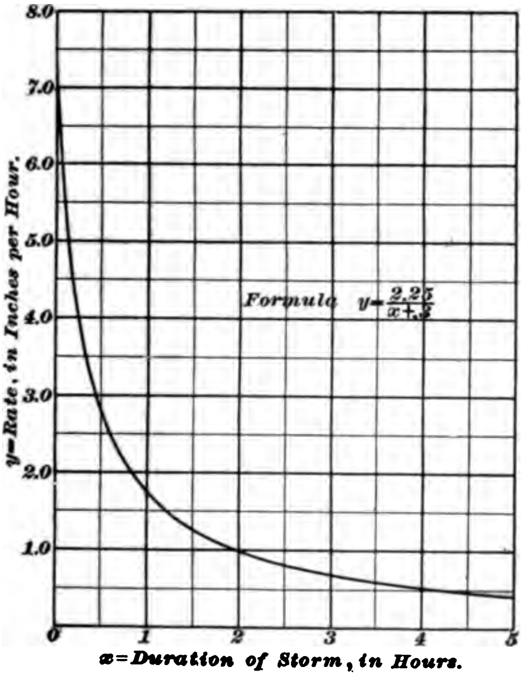


FIG. 366.

rainfall is shown in Fig. 366. The curve, or locus, in this diagram is constructed from formula **104,** Art. **1408,** by taking the time x as the abscissa and the rate per hour y as the ordinate. Therefore, the ordinate to any

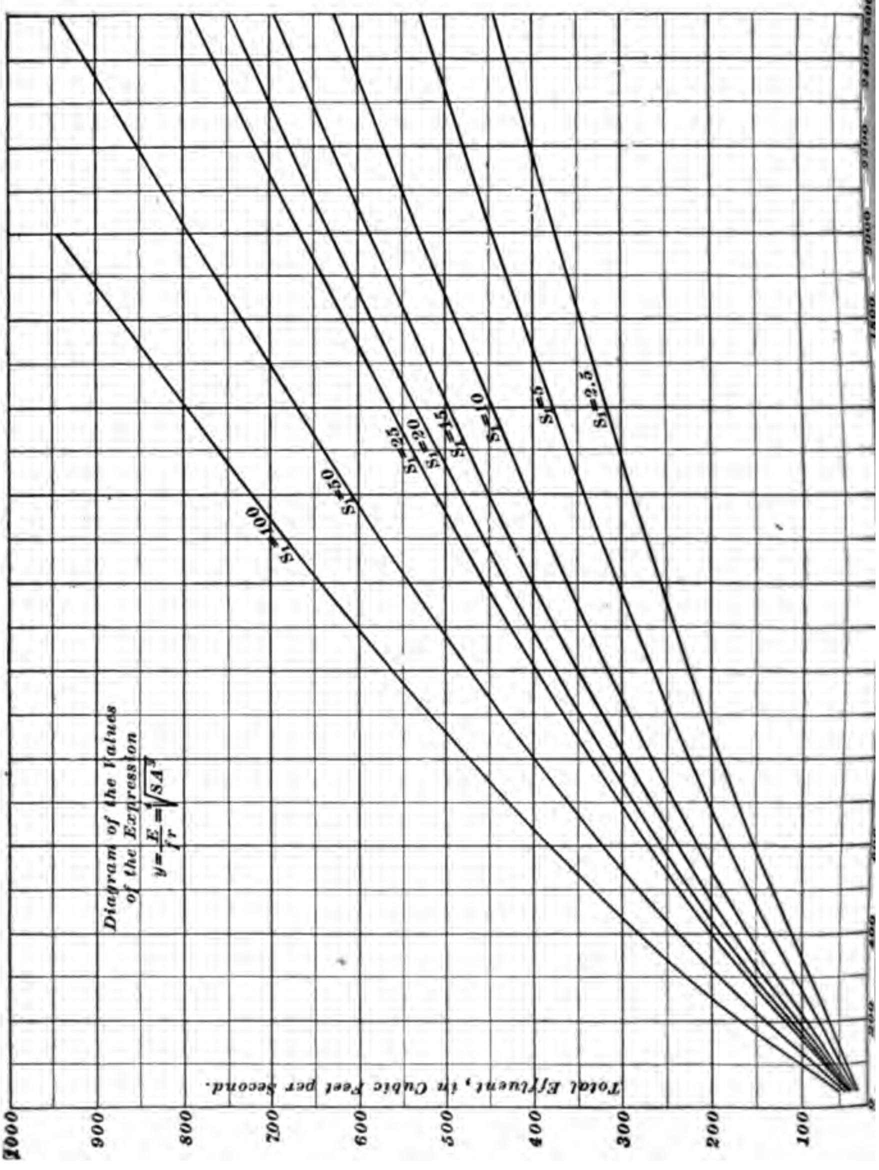


Diagram of the Values
of the Expression
y=E/fr=⁴√SA³
Total Effluent, in Cubic Feet per Second.
S1=100
S1=50
S1=25
S1=20
S1=15
S1=10
S1=5
S1=2.5
1000
900
800
700
600
500
400
300
200
100

point on the curve will represent the maximum rate of rainfall given by an ordinary storm continuing during the period of time represented by the abscissa to the same point. When constructed on cross-section paper, each division on the vertical line of the diagram, or axis of ordinates, should represent one-tenth of an inch of rainfall, and each division on the horizontal line, or axis of abscissas, should represent five minutes of time. Every tenth horizontal line, representing a full inch, will be heavier, while every *twelfth* vertical line, representing a full hour, should be made heavy. As on regular cross-section paper every tenth line in each direction is heavy, every twelfth vertical line may be indicated by drawing a lead-pencil line over it. This diagram, when constructed on accurate cross-section paper, will fulfil the same purpose as Table **29,** Art. **1413,** and is more convenient to use. It gives with sufficient accuracy not only all values given in that table, but also all intermediate values.

1476. Curves for Effluent.—In the diagram shown in Fig. 367 are curves representing the values of the expression $\sqrt[4]{S_1 A^3}$, as tabulated in Table 33, Art. **1451.** In order to construct these curves, y is substituted for $\frac{E}{f_1 r_1}$ and x is substituted for A in formula **119,** Art. **1451,** putting it in the form $y = \sqrt[4]{S_1 x^3}$. Then, for any constant value of S_1, this equation is constructed in the usual manner, y being the ordinate and x the abscissa. In the diagram the curves are shown for the eight different values of S_1 corresponding to the values tabulated in Table 33. The construction of the curves will be readily understood, and no special explanation is required. For any value of S_1 for which a curve is constructed, the ordinate to the curve at any point will represent the value of c, formula **120,** Art. **1451,** corresponding to the number of acres represented by the abscissa to the same point.

FLOW OF WATER IN CONDUITS.

FORMULAS FOR VELOCITY AND DISCHARGE.

NOTE.—Before beginning the study of this subject, the student should carefully review the subjects of *Hydrokinetics* and *Flow of Water in Pipes* in the section on Hydraulics, Vol. I.

1477. Formulas.—Various formulas have been proposed and used for the purpose of expressing in mathematical terms the velocity of the flow of water in pipes and other channels. The three formulas most extensively used for this purpose are the formula of Weisbach, which is much used by English and American engineers; that of Darcy-Bazin, which is used in France, and the Kutter formula. Of these, the Kutter formula is most popular among hydraulicians, on account of its great flexibility and adaptability to varying conditions. It is quite generally used in this country for computing the velocity of the flow of water, and, all things considered, is probably the best formula that has been proposed for the purpose. The Kutter formula is a modification of what is commonly known as the *Chezy formula.*

1478. The Chezy Formula.—A simple and practical fundamental formula for the velocity of the flow of water was suggested by Brahms in 1753. It is, however, commonly attributed to Chezy, and known as the Chezy formula. It is as follows:

$$v = c\sqrt{rs}, \qquad \textbf{(131.)}$$

in which v is the mean velocity of flow in feet per second, r is the hydraulic mean radius in feet, s is the gradient or sine of the slope, and c is a coefficient, i. e., coefficient of mean velocity.

The **mean velocity** v is the average velocity of the flow throughout the cross-section of the channel. The filaments of water near the sides of the channel move more slowly than those near the center of the channel, the flow of the former being somewhat retarded by friction. The mean

velocity is that velocity which, multiplied by the cross-section of the water, will give the discharge.

The **hydraulic radius** r is found by dividing the area a of the cross-section of the water by the wetted perimeter p of the channel. That is, $r = \frac{a}{p}$. By **wetted perimeter** is meant that portion of the boundary of a perpendicular cross-section of the channel which is in contact with the water. In case of a circular pipe flowing *full*, the wetted perimeter is the inner circumference of the pipe. Hence, for this condition, if d is the diameter of the pipe, $r = \frac{a}{p} = \frac{.7854\,d^2}{3.1416\,d} = \frac{1}{4}d$. The value of r must be expressed in foot-units, and will be expressed here to the nearest hundredth of a foot. If the area of the cross-section is expressed in square inches and the wetted perimeter in inches, then the quotient r will be in inch-units and must be divided by 12 to reduce to foot-units. The hydraulic mean radius is also called the **hydraulic mean depth.**

The **gradient,** or **slope,** s is found by dividing the fall of the channel by its actual length, or the length along the channel, whether inclined or horizontal. That is, if h is the fall and l is the length of the channel for which the velocity of flow is to be estimated, then $s = \frac{h}{l}$. (See Art. **1442.**) Consequently, s is the fall of the channel in one unit of its length, in fractional parts of that unit. In other words, s is the sine of the angle made by the grade line with a horizontal line. It is called the **sine of the slope,** or, simply, the **slope.**

The **coefficient of mean velocity** c was originally supposed to be a constant whose value was to be determined by experiment.

The above expression for the velocity of the flow of water has been used for over a century, and for rough approximations is very satisfactory. It has been found, however, that the value of the coefficient c varies with the character

of the interior surface of the channel, with the hydraulic mean radius, and, to some extent, with the slope.

1479. Kutter's Formula.—In order to embrace the effect of these three conditions, Mr. Ganguillet, City Engineer of Berne, together with his assistant, Mr. Kutter, some years ago developed an algebraic expression for the value of c in the Chezy formula involving these three conditions. The resulting expression, known as **Kutter's formula**, is as follows:

$$v = c\sqrt{r\,s} = \frac{41.66 + \dfrac{1.8113}{n} + \dfrac{.002807}{s}}{1 + \left[41.66 + \dfrac{.002807}{s}\right]\dfrac{n}{\sqrt{r}}}\sqrt{r\,s}.$$

This may be reduced to a simpler form by dividing both numerator and denominator by 1.8113, which gives:

$$v = \frac{23 + \dfrac{1}{n} + \dfrac{.00155}{s}}{.5521 + \left[23 + \dfrac{.00155}{s}\right]\dfrac{n}{\sqrt{r}}}\sqrt{r\,s}. \qquad \textbf{(132.)}$$

In both formulas n is a coefficient of roughness depending upon the condition of the interior surface of the channel, while r and s have the same values as in formula **131**.

The value of the coefficient c, though elaborated from gaugings made in open channels, has been found to apply satisfactorily to closed conduits also, when the proper values of n are used. The Kutter formula is very extensively used in this country for computing the flow of water, and is generally considered to be the most reliable, as well as the most flexible, formula that has been proposed for this purpose.

Great care and judgment must be exercised in selecting the value of n, as the results will be materially affected by the values used. In common practice, the values of n may be generally taken as given in the table of Coefficients of Roughness (Tables and Formulas).

1480. Values of *a*, *p*, and *r* for Circular Conduits.—In order to facilitate the operations for various depths of

flow in circular conduits, a table of the relative values of r for the various depths will be found convenient. In the table of Hydraulic Elements of Circular Pipe, given in Tables and Formulas, will be found values of p, a, r, and $\sqrt{r}$ for various depths of flow in a circular conduit whose diameter is unity. The hydraulic mean radius for any other diameter of conduit may be found by multiplying the tabulated value of r by the diameter of the conduit, in feet. Or the square root of the hydraulic mean radius for any other diameter may be readily found by multiplying the values of $\sqrt{r}$, given in the last column, by the square root of the diameter of the conduit in feet.

It will be noticed that the value of the hydraulic mean radius is the same when the conduit is flowing half full, as when flowing full. If d is the diameter of a circular conduit, then, when flowing full, $r = \frac{.7854\, d^2}{3.1416\, d} = \frac{1}{4} d$, and, when flowing half full, $r = \frac{\frac{1}{2} \times .7854\, d^2}{\frac{1}{2} \times 3.1416\, d} = \frac{1}{4} d.$

1481. EXAMPLE.—For an ordinary circular pipe sewer having an internal diameter of 15 inches and laid to a grade of 5 feet per thousand, what is the velocity of flow when flowing full?

SOLUTION.—From the table of Coefficients of Roughness, the value n for this character of conduit is found to be .013. From the table of Values of Wetted Perimeter, Sectional Area, etc., the value of $\sqrt{r}$ for a pipe one foot in diameter flowing full is 0.5. Hence, for a pipe 15 inches (= 1.25 ft.) in diameter and flowing full, the value of $\sqrt{r}$ is equal to $.5 \times \sqrt{1.25} = .559$. The value of s is $\frac{5}{1{,}000} = .005$. By substituting these values in formula **132**, we get

$$v = \frac{23 + \frac{1}{.013} + \frac{.00155}{.005}}{.5521 + \left[28 + \frac{.00155}{.005}\right] \times \frac{.013}{.559}} \times .559 \times \sqrt{.005} = 3.62 \text{ ft. per second.}$$

Ans.

EXAMPLES FOR PRACTICE.

NOTE.—The student may find some difference between the *hundredths* of his solutions and those here given, owing to neglected decimals.

1. For the same conditions as in the preceding example, what will be the velocity when the sewer is flowing half full?

2. For a clean, smooth, cast-iron pipe of 9 inches internal diameter, laid to a grade of 2¼ feet per thousand, what will be the velocity when flowing full? Ans. 2.15 ft. per sec.

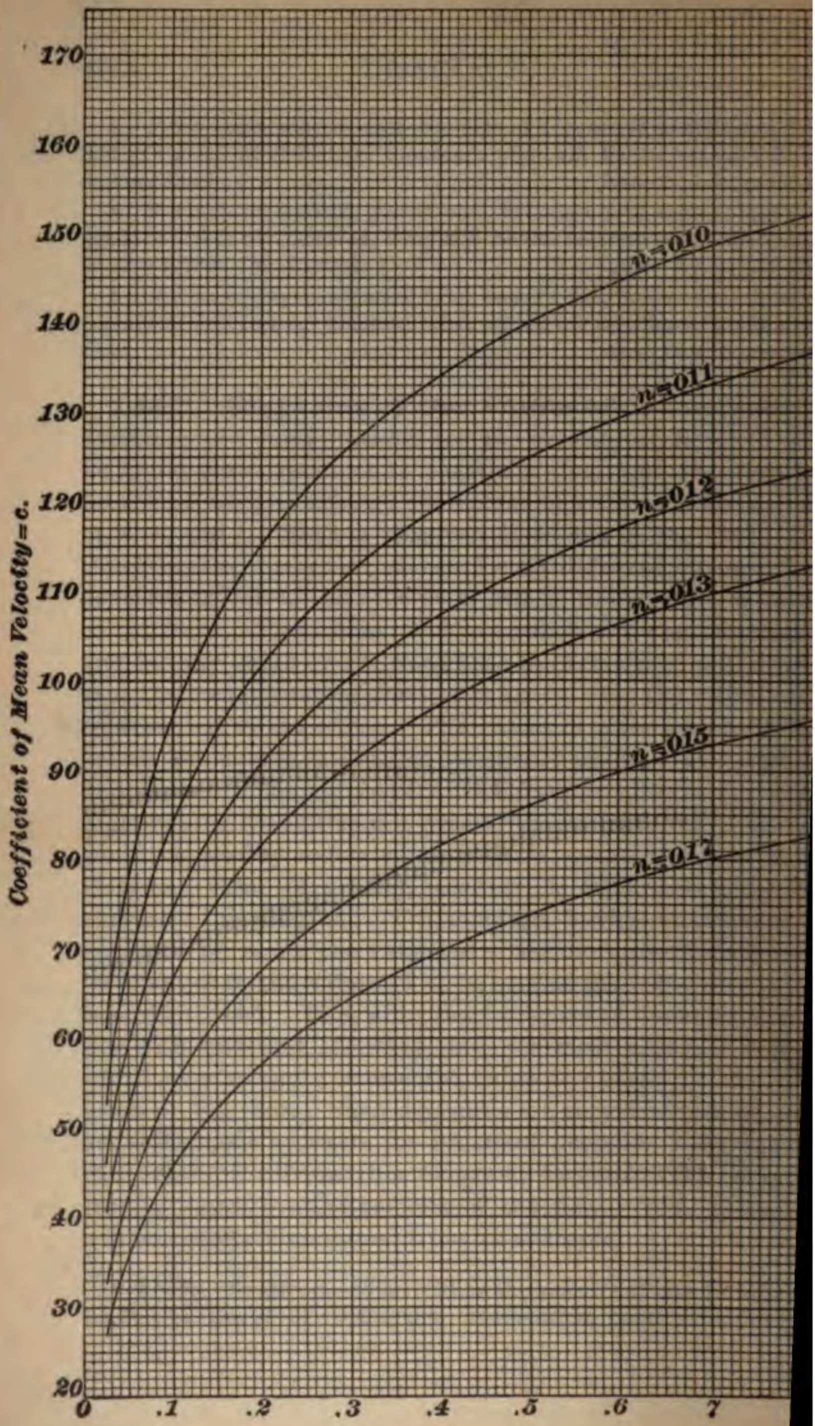
3. For a clean, smooth, glazed pipe 12 inches in diameter, laid to a grade of 1 foot per hundred, what will be the velocity when the depth of flow is .8 of the diameter? Ans. 7.0 ft. per sec.

4. For a roughly constructed brick sewer 4 feet in diameter, having a grade of 2.25 feet per hundred, what will be the velocity when flowing full? Ans. 13.0 ft. per sec.

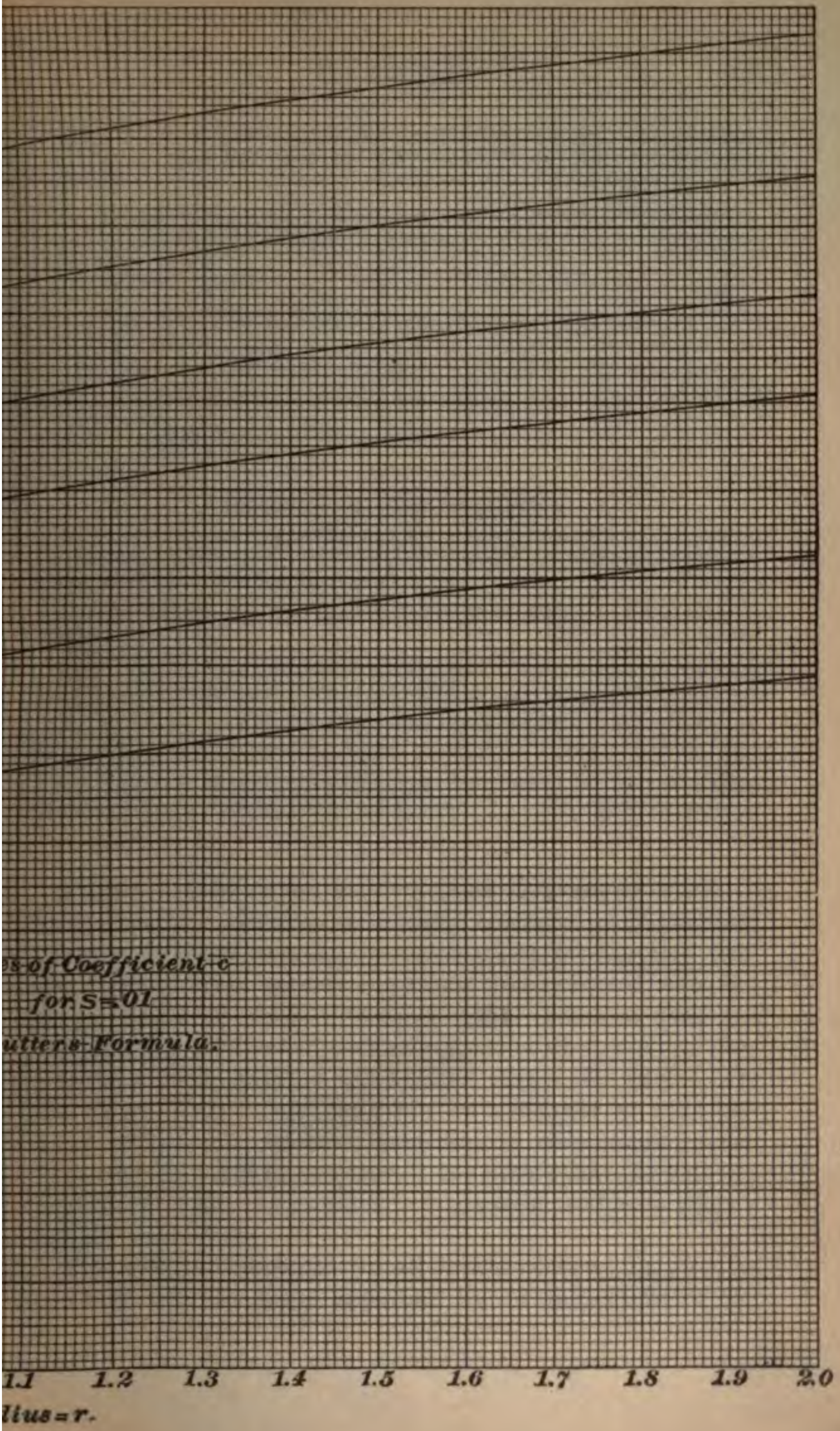
1482. Diagrams of the Values of *c*.—In order to facilitate the computations for the velocity of flow, it will be found very expedient to construct diagrams showing by means of loci, or curves, the values of c for various values of n, r, and s. Such a diagram is shown in Fig. 368. In this diagram, curves are shown for six different values of n, constructed for values of r from .025 to 2.0, and for a value of .01 for s. The values of r are taken for the abscissas, and for any value of r the ordinate to each curve will represent the corresponding derived value of c for the value of n represented by the curve. The values of n for which curves are shown include the values commonly used for water pipes and sewers. The values of r used apply to all circular conduits having diameters not greater than 8 feet; the diagram could be readily extended to include greater values of r. Only one value of s (.01), however, is represented by the curves in this diagram. For conduits having diameters of one foot or more, the values of c are not greatly affected by changes in the values of s, so long as the value of s is not less than .0005. This will cover all cases of ordinary practice with sufficient accuracy, and, for most cases, it will be sufficiently accurate to use the value of c given by the diagram of Fig. 368. Such diagrams are very convenient, and are sufficiently accurate for practical purposes. By taking the value of c from this diagram, formula **131** may be readily solved for the velocity of the flow. The operation will be readily understood from an example.

1483. EXAMPLE.—What will be the velocity of flow in an open conduit or trough of rectangular cross-section, constructed of un-

417



Coefficient of Mean Velocity=c.
170
160
150
140
130
120
110
100
90
80
70
60
50
40
30
20
0
.1
.2
.3
.4
.5
.6
7
n=010
n=011
n=012
n=013
n=015
n=017



s of Coefficient c
for S=.01
utters Formula.
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2.0
lius = r.

planed timber and having an inner width of 1 foot, with vertical sides, **the** depth of flow being 9 inches and the slope being 1 foot per hundred ?

SOLUTION.—The area a of the cross-section of the water is $1 \times \frac{3}{4} = .75$ sq. ft., and the wetted perimeter p is $\frac{3}{4} + 1 + \frac{3}{4} = 2\frac{1}{2}$ ft. Hence, the hydraulic mean radius $r = \frac{a}{p} = \frac{.75}{2.5} = .3$ of a foot. The slope $s = \frac{1}{100} = .01$. From the table of Coefficients of Roughness, the value of n for unplaned timber is .012. In the diagram of Fig. 368, the ordinate c to the curve for $n = .012$, corresponding to a value of .3 for the abscissa r, has a value of 100.4. Hence, the velocity v is equal to $100.4 \times \sqrt{.3 \times .01} = 5.50$ ft. per second. Ans.

EXAMPLES FOR PRACTICE.

NOTE.—In each of the following examples, the value of the coefficient will be taken from the diagram of Fig. 368, the same as though the given slope were 1 per cent.

1. By the aid of the diagram, find the velocity for Example 1 of Art. **1481**. Ans. 3.61 ft. per sec.

2. Find, by the aid of the diagram, the velocity for the conditions described in Example 2 of the same article. Ans. 2.16 ft. per sec.

3. Find, by the aid of the diagram, the velocity for the conditions described in Example 3 of the same article. Ans. 7.0 ft. per sec.

4. Find, by the aid of the diagram, the velocity for the conditions described in Example 4 of the same article. Ans. 12.98 ft. per sec.

1484. Velocities for Various Depths of Flow.—In computing velocities in circular conduits, it is generally more convenient to make the calculations for the pipe running full and then modify the result to correspond with the given depth of flow. Formula **131** indicates that the velocity varies directly as the square root of r, but by referring to formula **132,** it will be seen that r also enters into the value of the coefficient c in such a manner as to somewhat affect this relation, rendering it only approximate. From formula **132,** however, it is evident that the velocity is greatest when the value of r is greatest, and least when r has its least value.

The relative velocities for all depths of flow in a circular conduit will be made apparent by an inspection of the

diagram shown in Fig. 369. The velocity curve is constructed by plotting the depths of flow as ordinates and the corresponding relative velocities as abscissas. The velocity with conduit flowing full is taken as unity, and all other velocities are referred to this unit. For half depth of flow, the velocity is also unity. It will be noticed that the velocity is not so great when the conduit is flowing full as when somewhat less than full. This is due to the retarda-

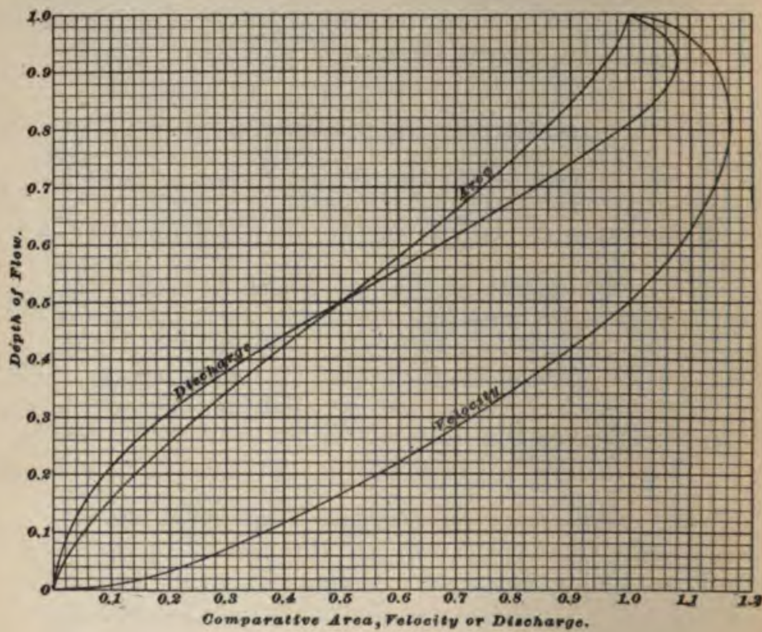


FIG. 369.

tion of friction produced by the relatively greater amount of surface in contact with the water when the pipe is flowing full, or, in other words, to the somewhat decreased value of the hydraulic mean radius. The greatest value of the hydraulic mean radius, and, consequently, the greatest velocity, is attained when the depth of flow is about .81 of the diameter of the conduit. When the velocity has been found for a full depth of flow, the velocity for any other depth

of flow may be obtained by multiplying the velocity for the full depth of flow by the abscissa to the velocity curve in the diagram corresponding to the given depth of flow.

1485. Weisbach's Formula.—As the formula of Weisbach for the velocity of flow is quite extensively used in this country, it will be given here. For circular conduits, it is as follows:

$$v = \frac{\sqrt{2gh}}{\sqrt{1 + e + \frac{cl}{d}}}, \qquad (133.)$$

in which v is the velocity in feet per second when the conduit is running full, g is the acceleration of gravity, h is the head, l is the length, and d is the diameter of the pipe, all in feet; c is a coefficient of friction in pipe, and e is a coefficient of resistance for entrance.

The acceleration of gravity may be taken at 32.16.

The head h is the difference of elevation in the water surface at the upper and lower ends of the pipe when l is the length of the pipe; that is, h is the fall of the water surface in a distance l.

The coefficient of friction c is taken equal to

$$.014392 + \frac{.017156}{\sqrt{v}}.$$

The coefficient of resistance e for entrance is not really applicable to long continuous pipes fed at various points. It is, however, customary to assume an average value of .505 for e.

Formula **133** may, therefore, be written in the following form:

$$v = \sqrt{\frac{64.32\,h}{1.505 + \frac{cl}{d}}}. \qquad (134.)$$

In substituting the value of c in the Weisbach formula, the value assigned to v should be as accurate as it can be arrived at by judgment. If the value of v, as derived from the equation, varies greatly from the value assigned,

it will be necessary to assign a more correct value and again solve the equation. Likewise, as it is necessary also to assume a value for d, it may be necessary to correct its value and again solve the equation.

1486. Discharge.—The quantity of water conveyed by a conduit is usually called its **discharge.** The discharge is commonly and conveniently expressed in cubic feet per second, but it may be expressed in cubic feet per minute, gallons per second, gallons per hour, etc. The quantity of the discharge Q in cubic feet per second is given by the formula

$$Q = v\,a, \qquad \textbf{(135.)}$$

in which v is the velocity in feet per second and a is the area of the cross-section of the water in square feet.

If the values of v and a remain as given here, then the quantity of discharge Q in cubic feet during any given period of time will be given by the formula

$$Q = v\,a\,t, \qquad \textbf{(136.)}$$

in which t is the given period of time in seconds.

A cubic foot contains 7.48 liquid gallons. In sewer computations, however, it is generally taken at 7.5 gallons. Hence, the discharge Q_1 in gallons per second will be given by the formula

$$Q_1 = 7.5\,v\,a. \qquad \textbf{(137.)}$$

The quantity of discharge Q_1 in gallons during a given time of t seconds will be given by the formula

$$Q_1 = 7.5\,v\,a\,t. \qquad \textbf{(138.)}$$

1487. Velocity, Area, and Discharge Curves.—The relative areas of the cross-section of water for varying depths of flow in circular conduits are also shown by a curve on the diagram in Fig. 369.

Also, the relative quantities of discharge given by a circular conduit running at different depths of flow are shown by a curve on the same diagram. Both of these curves are so constructed that their abscissas are unity for a full depth

of flow, the same as the velocity curve; hence, for both curves, the values for all other depths of flow are in parts of the respective values for full flow. It will be noticed that in a circular conduit, the maximum discharge is given when the depth of flow is about .92 of the diameter of the conduit.

As the velocity curve, area curve, and discharge curve represent, respectively, the relative values of v, a, and $Q\,(= v\,a)$, it is evidently not necessary to use all of these curves in reducing the value of Q for a full depth of flow to its proper value for any other depth of flow. A convenient method is to compute the velocity and discharge for the conduit flowing full; then, these results multiplied respectively by the abscissas to the velocity and discharge curves corresponding to the given depth of flow will be the required velocity and discharge. Or, the velocity may be found for the given depth of flow, and this velocity multiplied by the abscissa to the area curve for the same depth of flow will give the required discharge.

THE REQUIRED DIMENSIONS OF STORM-WATER SEWERS.

1488. The Required Capacity.—It is evident that the dimensions of a storm-water sewer must be such that it will discharge the total effluent E, Arts. **1447** to **1455.** Considerations of economical construction, and of the relative discharge at different depths of flow, lead to the conclusion that it is the best practice to calculate the dimensions of the sewers so that *when running full* they will deliver the amount of water given by the total effluent. The actual capacity of the sewer will then be from 6 to 8 per cent. in excess of this; for *the velocity in the sewer will be increased and its capacity augmented by the flow sinking to the depth of maximum discharge*, which is about .92 of the diameter of the sewer. When the depth of flow is about .81 of the diameter of the sewer, the discharge will be the same as when flowing full. Thus it will be seen that the practice

indicated above is safe and consistent, provided the value of E accurately represents the greatest flow for which it is desired to make provision. Judgment must be exercised in determining this condition. When it is impossible to satisfactorily determine the value of E, it may be advisable to give the sewer such dimensions that its discharge will equal the estimated value of E when the depth of flow is materially less than .8 of the diameter. The practice of designing the sewers with capacities sufficient to convey the estimated effluent when flowing full will be followed here, however.

1489. The Effluent.—As the discharge Q in cubic feet per second must equal the total effluent E, the computed value of E may be written for Q in formula **135**, giving the formula

$$E = v\,a, \qquad \textbf{(139.)}$$

in which v is the velocity in feet per second and a is the area of the cross-section of the water in square feet.

1490. The Diameter; Value of *r*.—In a circular sewer flowing full, the area of the cross-section of the water is given by the formula

$$a = .7854\,d^2, \qquad \textbf{(140.)}$$

in which d is the internal diameter of the sewer in feet.

By substituting this value of a in formula **139**, there will result:

$$E = .7854\,d^2\,v. \qquad \textbf{(141.)}$$

$$d = 1.128\sqrt{\frac{E}{v}}. \qquad \textbf{(142.)}$$

It has been stated that for a pipe flowing full the value of the hydraulic mean radius is $\frac{1}{4}\,d$. By substituting $4\,r$ for d in formula **142**, for a circular sewer, we derive the formula

$$r = .282\sqrt{\frac{E}{v}}. \qquad \textbf{(143.)}$$

From this formula it is evident that the total effluent, or required discharge, being known, the value of r may be

approximately determined by assuming a value for v. It is unnecessary to compute the value of r closer than to the nearest hundredth of a foot. As used here, it will be thus expressed.

1491. The Velocity from Total Effluent.—If, after having found the value of r, the character of the interior surface (n) and slope (s) of the sewer is also known, we have all data necessary for calculating the velocity of flow by Kutter's formula, and, consequently, the required dimensions of the sewer. If the velocity is found to vary greatly from the value of v assigned in formula **143,** it will be necessary to correct the value of r.

It is evident that the value of r given by formula **143** will be correct only when correct values are substituted for E and v. Consequently, if the velocity, as computed with a value of r, as given by formula **143,** is found to vary materially from the value assigned to v in that formula, a corrected value must be assigned to v, obtaining a new value for r, and the velocity must be again computed. When the computed value of v varies materially from the value assigned in formula **143,** an approximately correct value of v may be found by the formula

$$v_o = .58\ v_c + .42\ v_a, \qquad \textbf{(144.)}$$

in which v_a is the value assigned to v in formula **143,** v_c is its computed, and v_o its approximately correct, value. The value of v_o thus obtained, though but roughly approximate, will generally be sufficiently correct for substituting in formula **143,** in order to obtain the corrected value of r. Indeed, the value of v_o will, in many cases, very closely approximate the correct value of v, and may be used for it in preliminary calculations. As substituted in formula **143,** the value of v_o will be written here to the nearest unit only.

1492. Example.—What will be the velocity in a circular brick sewer discharging the effluent estimated in the example explained in Art. **1448,** assuming s equal to .02, as given for the surface slope, and using a value of .015 for n?

SOLUTION.—The total effluent, as estimated in the example referred to, is 99.5 cubic feet per second. By assigning to v in formula **143** a value of 4 feet per second, as assumed in estimating the effluent, the value of r is found to be $.282\sqrt{\frac{99.5}{4}} = 1.41$. For a value of r of 1.41, in the diagram of Fig. 368, the ordinate to the curve for $n = .015$ gives a value of 106.2 for c. Hence, on first trial, the computed velocity v_c is equal to $106.2\sqrt{1.41 \times .02} = 17.83$ feet per second. This is so greatly in excess of the assumed velocity that a correction will be necessary. From formula **144**, $v_o = .58 \times 17.83 + .42 \times 4 = 12.02$ feet per second. Hence, the new value of r becomes $.282\sqrt{\frac{99.5}{12}} = .81$, and for this value, the ordinate to the curve $n = .015$ in the diagram gives 95.7 as the value of c. Hence, the correct velocity, as given by Kutter's formula, is equal to $95.7\sqrt{.81 \times .02} = 12.18$ feet per second. Ans.

EXAMPLES FOR PRACTICE.

NOTE.—The following examples relate to the examples of effluent given in Art. **1448.** The slope of the sewer will in each case be assumed to be the same as that given (as a per cent.) for the surface in the example referred to; that is, $s = \frac{S}{100}$. Except when otherwise stated, the value of n will be taken at .015. When the value of r is less than 2.0, the value of c will be found by the aid of the diagram, Fig. 368; otherwise, it will be computed from Kutter's formula. Results obtained by the aid of the diagram may vary, in the second decimal place, from the results given, but they will be near enough for all practical purposes. Three results, v_c, v_o, and v, will each time be found.

1. What will be the velocity in feet per second in a circular brick sewer discharging, under the conditions described in Example 1, the effluent estimated for that example?

$$\text{Ans.} \begin{cases} v_c = 42.76. \\ v_o = 28.16. \\ v\ = 27.94. \end{cases}$$

2. What will be the velocity in feet per second in the sewer at the point described in Example 3?

$$\text{Ans.} \begin{cases} v_c = 36.21. \\ v_o = 24.36. \\ v\ = 24.55. \end{cases}$$

3. What will be the velocity in feet per second in a pipe sewer discharging, under the conditions described in Example 5, the total effluent estimated for that example, assuming the value of n at .012?

$$\text{Ans.} \begin{cases} v_c = 7.07. \\ v_o = 4.94. \\ v\ = 5.18 \end{cases}$$

4. What will be the velocity in feet per second in a circular brick sewer discharging the total effluent estimated for Example 7, under the conditions described in that example?

Ans. $\begin{cases} v_c = 40.89. \\ v_o = 25.40. \\ v = 22.16. \end{cases}$

5. What will be the velocity in feet per second in a circular brick sewer discharging the total effluent estimated for Example 9, under the conditions described in that example?

Ans. $\begin{cases} v_c = 18.67. \\ v_o = 13.35. \\ v = 14.18. \end{cases}$

1493. Error in Velocity Assumed in Estimating Effluent.—It will be noticed that in the above examples the velocities obtained vary materially from the velocities assumed in estimating the effluent by formula **116**, Art. **1447.** This will affect the time required for the storm water to reach the point under consideration, and, consequently, the length of storm giving the maximum flow at this point. As the rate of rainfall varies with the duration of the storm, it is evident that this condition will also affect the quantity of storm water given to the sewer, and the flow in the sewer. It will, therefore, be necessary to recalculate the effluent by substituting in formula **116** the computed value of v (or the value of v_o) expressed simply to the nearest unit.

1494. EXAMPLE.—What will be the total effluent, or required discharge, as estimated for the example explained in Art. **1448**, using the corrected velocity?

SOLUTION.—All values will be the same as used in Art. **1448**, except the velocity, which is given by the example explained in Art. **1492.** By substituting the respective values in formula **112** for the flow per acre, and taking the number of acres the same as obtained in Art. **1448**, we have

$$E = 160 \times \frac{8,100 \times .24}{\frac{660}{.6 \times \sqrt{2}} + \frac{5,000}{12} + 1,080} =$$

$160 \times .855 = 136.8$ cubic feet per second. Ans.

EXAMPLES FOR PRACTICE.

NOTE.—The following examples relate to the examples given in Art. **1448.** In estimating the total effluent, or required discharge, the velocities obtained in Art. **1492** will be used.

1. What will be the total effluent, or required discharge, in cubic feet per second for Example 1, as estimated with corrected velocity? Ans. 322.1.

2. What will be the required discharge in cubic feet per second for Example 3, as estimated with corrected velocity? Ans. 171.0.

3. What will be the required discharge in cubic feet per second for Example 5, as estimated with corrected velocity? Ans. 43.4.

4. What will be the required discharge in cubic feet per second for Example 7, as estimated with corrected velocity? Ans. 405.1.

5. What will be the required discharge in cubic feet per second for Example 9, as estimated with corrected velocity? Ans. 557.2.

1495. Error in Estimated Effluent.—It will be noticed that in each of the above examples the required discharge, or total effluent, is considerably greater than was estimated for the same conditions in Art. **1448.** This is due to the fact that the velocities are found to be much greater than were there assumed. In each of these examples, the slope, or grade, of the sewer has been considered to be the same as the slope of the surface, which, in most of the examples, has been assumed steeper than the grades at which sewers are generally constructed, while the velocities given in Table 32, Art. **1444,** apply generally to sewers having moderate slopes. These velocities also correspond generally to those in sewers of small dimensions, while in most of the examples, the effluents will require sewers of reasonably large dimensions. As the hydraulic mean radius varies directly as the diameter of the sewer, and the velocity varies approximately as the quantity $\sqrt{r\,s}$, it is evident that velocities applicable to small sewers of moderate slope will be quite incorrect for large sewers with steep slopes.

It has been clearly shown by the preceding examples that if the velocities assumed in estimating the effluent do not approximate reasonably closely to the velocities computed according to the condition of the slope and the size and

character of the sewer, considerable error may result in the estimated effluent. The importance of assuming the velocity as nearly correct as possible in estimating the effluent thus becomes evident. When the velocity of the flow in the sewer has been computed, if it is found that the error in the assumed velocity is very great, the effluent should be reestimated, using the corrected velocity, as was done in the preceding article.

1496. The Corrected Computations for Velocity. —Formula **131**, Art. **1478**, shows that the velocity varies approximately as the square root of the hydraulic mean radius r, while formula **143** shows that r varies as the square root of the effluent divided by the velocity. By substituting in formula **131** the value of r as given by formula **143**, it may be shown that the velocity varies as the fifth root of the effluent. Hence, if the velocity has been computed for one value of the effluent, the approximate velocity may be obtained for a different value of the effluent by proportion. If v is the velocity as correctly computed for the effluent E_1, and v_2 is the desired velocity for the corrected effluent E_2 (Art. **1494**), then we have the proportion $v : v_2 :: \sqrt[5]{E_1} : \sqrt[5]{E_2}$, from which

$$v_2 = v \sqrt[5]{\frac{E_2}{E_1}}. \qquad (145.)$$

In applying this equation, v_a may generally be used for v without computing the latter. (See Art. **1491.**) Formulas **145** and **146** may be readily solved by means of logarithms. The value of v_2 thus obtained will be reasonably near to the correct value of v, and, if substituted in formula **143**, in connection with E_2, it will give the value of r with sufficient accuracy. In substituting v_2 in formula **143**, for obtaining the corrected value of r, it will be well to write its value to the nearest tenth of a foot. From the value of r thus obtained, the value of c may be found from the diagram, and the correct velocity computed.

The value of the hydraulic mean radius r_2 for a sewer

having a discharging capacity equal to E_2 may, however, be more expeditiously obtained by the formula

$$r_2 = r\sqrt[5]{\left(\frac{E_2}{E_1}\right)^2}, \qquad \textbf{(146.)}$$

in which r is the value of the hydraulic mean radius corresponding to the discharge, or effluent, E_1, and r_2 is the value of the hydraulic mean radius corresponding to the required discharge E_2.

1497. EXAMPLE.—What is (*a*) the approximate and (*b*) the computed velocity for the corrected effluent E_2 obtained in the example explained in Art. **1494,** the effluent E_1 being as obtained in the example explained in Art. **1448,** and the velocity for the same as obtained in the example explained in Art. **1492?**

SOLUTION.—(*a*) As estimated in Art. **1448,** $E_1 = 99.5$ cubic feet per second, and the velocity v for the same, as obtained in Art. **1492,** is 12.18 feet per second. As obtained in Art. **1494,** $E_2 = 136.8$ cubic feet per second. Hence, by formula **145,**

$$v_2 = 12.18\sqrt[5]{\frac{136.8}{99.5}} = 12.98 \text{ feet per second. Ans.}$$

(*b*) By substituting this value of v_2 (taken to the nearest unit) and the value of E_2 in formula **143,** we get $r = .282\sqrt{\frac{136.8}{13.0}} = .91$ of a foot. From the diagram shown in Fig. 368, the ordinate to the curve $n = .015$, corresponding to a value of .91 for the abscissa r, gives a value of 98 for c. Hence, by formula **131,** Art. **1478,**

$$v = 98\sqrt{.91 \times .02} = 13.22 \text{ feet per second. Ans.}$$

EXAMPLES FOR PRACTICE.

NOTE.—The following examples relate to those having the same respective numbers in Art. **1494** which, in turn, refer to the corresponding examples in Arts. **1448** and **1492.** In each example, E_1 will be taken as obtained in Art. **1448,** v as in Art. **1492,** and E_2 as in Art. **1494.**

1. What is (*a*) the approximate and (*b*) the computed velocity in feet per second for the corrected effluent E_2, as obtained in Example 1? (*c*) What is the value of r and (*d*) the value of the coefficient c, as taken from the diagram of Fig. 368?

Ans. (*a*) 29.48 ft. per sec.
(*b*) 30.01 ft. per sec.
(*c*) .93 ft.
(*d*) 98.4.

2. What are the corresponding values for the corrected effluent in Example 2?

Ans. (a) 25.23 ft. per sec.
(b) 25.32 ft. per sec.
(c) .73 ft.
(d) 93.7.

3. What are the corresponding values for the corrected effluent of Example 3?

Ans. (a) 5.44 ft. per sec.
(b) 5.52 ft. per sec.
(c) .80 ft.
(d) 123.4.

4. What are the corresponding values for the corrected effluent of Example 4?

Ans. (a) 23.44 ft. per sec.
(b) 24.89 ft. per sec.
(c) 1.17 ft.
(d) 102.9.

5. What are the corresponding values for the corrected effluent of Example 5?

Ans. (a) 14.52 ft. per sec.
(b) 14.58 ft. per sec.
(c) 1.75 ft.
(d) 110.2.

NOTE.—It will be noticed that, in the above examples, the correctly computed velocities (*b*) do not differ greatly from the approximate velocities (*a*), as obtained by formula **145.** The latter will, therefore, be sufficiently correct for many purposes. In making the final calculations for the dimensions of sewers, however, the velocities should be correctly computed on the basis of an effluent estimated as nearly correctly as possible.

1498. The Practical Diameter of the Sewer.—Having determined the total effluent, or discharge, and the velocity, the diameter of a circular sewer may be obtained by formula **142.** As, however, the hydraulic mean radius of a circular sewer running full is always one-fourth of its diameter, and as the former quantity will always be used in the computations, and will, therefore, be known, the simplest manner of obtaining the diameter d of a circular sewer is by the formula

$$d = 4\,r. \qquad (147.)$$

Four times the hydraulic mean radius will be the theoretical diameter of the sewer. The actual diameter should be taken at the nearest practical dimension greater than this, which, for brick sewers, may generally be a multiple of 2 inches, although multiples of 4 or 6 inches are more

commonly used; diameters of pipe sewers must correspond with the diameters of manufactured pipe. It will be noticed that the diameter of the sewer may be determined without making the second computation for the velocity, which is very convenient for preliminary calculations and estimates. In the final calculations, however, the correct velocity should be carefully computed as a check upon the other calculations.

1499. EXAMPLE.—What will be the diameter of the sewer required for the example explained in Art. **1497**?

SOLUTION.—The hydraulic mean radius was found to be .91 of a foot. Hence, by formula **147**, the theoretical diameter will be $4 \times .91 = 3.64$ feet. Although, in the usual practice, a sewer 4 feet in diameter would not uncommonly be used for a district having these requirements, a diameter of 3 feet 8 inches (= 3.67 feet) would be sufficient.

EXAMPLES FOR PRACTICE.

NOTE.—The following examples refer to the examples given in Art. **1497**. The diameter will each time be taken at the nearest multiple of 2 inches above the theoretical diameter.

1. What will be (*a*) the theoretical and (*b*) the practical diameter required by the conditions of Example 1?
   Ans. { (*a*) 3.72 ft.
   (*b*) 3 ft. 10 in.

2. What will be (*a*) the theoretical and (*b*) the practical diameter required by the conditions of Example 2?
   Ans. { (*a*) 2.92 ft.
   (*b*) 3 ft. 0 in.

3. What will be (*a*) the theoretical and (*b*) the practical diameter required by the conditions of Example 3?
   Ans. { (*a*) 3.20 ft.
   (*b*) 3 ft. 4 in.

4. What will be (*a*) the theoretical and (*b*) the practical diameter required by the conditions of Example 4?
   Ans. { (*a*) 4.68 ft.
   (*b*) 4 ft. 10 in.

5. What will be (*a*) the theoretical and (*b*) the practical diameter required by the conditions of Example 5?
   Ans. { (*a*) 7.00 ft.
   (*b*) 7 ft. 0 in.

EGG-SHAPED SEWERS.

1500. Variation of Flow in Storm-Water Sewers.—The flow in a storm-water sewer will necessarily fluctuate greatly. During a violent storm, the sewer may be taxed to its full capacity; while, during an extended

drouth, the flow may become very small. It is evident, however, that the sewer must be large enough to have sufficient capacity to carry its greatest flow. When the flow in a large circular sewer becomes very small, the cross-section of the flow will necessarily be very shallow, and, consequently, will have a very small hydraulic mean radius. It follows that, as the velocity varies approximately as the square root of the hydraulic mean radius (see formula **131**, Art. **1478**), the velocity also will be small and in many cases insufficient to prevent the depositing of solid matter carried in the sewage. Hence, it will be readily seen that the circular form of section is not well adapted to sewers in which the flow of sewage varies greatly, as is the case with storm-water sewers.

1501. Other Forms of Cross-Section.—As a result of the preceding conditions, other forms of cross-section having greater values of the hydraulic mean radius for comparatively small cross-sections of flow have been devised for storm-water sewers. A variety of forms have been employed, especially in Europe. The form of cross-section, however, which most satisfactorily accomplishes this purpose is the form known as **egg-shaped** on account of its form, which is shown in Fig. 370. As this form of cross-section is quite extensively used for storm-water sewers in America, its comparative dimensions will be noticed. The following formulas relating to the comparative dimensions of egg-shaped sewers are principally taken from a detailed discussion of the subject, published in the Michigan Engineers' Annual for 1894, by Col. E. W. Muenscher, president of the society.

1502. General Form of Egg-Shaped Sewers.—The general form of the cross-section of an egg-shaped sewer is shown in Fig. 370. The method of drawing the cross-section is also indicated in the figure. The portion of the cross-section above the line $O\,O'$ is a semicircle. That portion of the inner perimeter below the line $O\,O'$ is formed by the three arcs $D\,E$, $E\,G$, and $G\,A$, the arcs $D\,E$ and $G\,A$

having equal radii. It will be noticed that three different lengths of radii are used in constructing the figure, namely,

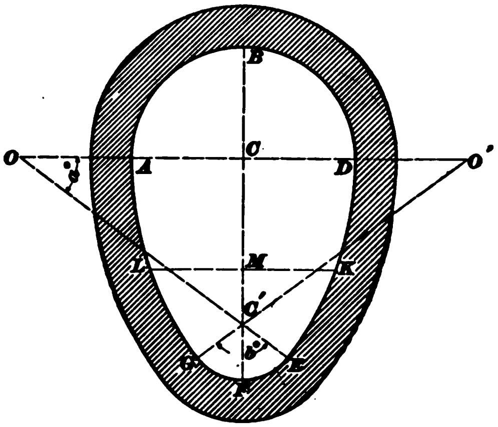


FIG. 370.

$CA = CB = CD$ for the upper semicircle, $O\,D = O\,E = O'G = O'A$ for the two side arcs $D\,E$ and $G\,A$, and $C'E = C'F = C'G$ for the lower arc $E\,F\,G$, commonly called the **invert.**

1503. General Formulas for Egg-Shaped Sewers. —Let letters be taken to represent certain values relating to the internal cross-section as follows:

$r = C\,A = C\,B = C\,D =$ radius of upper semicircle;
$r_1 = C'E = C'F = C'G =$ radius of lower arc;
$r_0 = O\,D = O\,E = O'G = O'A =$ radius of sides;
$d_h = A\,D = 2\,r =$ horizontal diameter;
$d_v = B\,F =$ vertical diameter;
$c = C\,C' =$ distance between centers;
$a° =$ angle $C\,O\,C'$ or $C\,O'\,C'$, subtended by the side arc $D\,E$ or $G\,A$;
$b° =$ angle $E\,C'\,G$, subtended by the lower arc $E\,F\,G$;
$P =$ inner perimeter $A\,B\,D\,E\,F\,G$;
$A =$ area of internal cross-section;
$R =$ hydraulic mean radius.

The following relations are easily obtained from Fig. 370:

$$c = d_v - (r + r_1). \qquad \textbf{(148.)}$$

We may also write $(r_o - r)^2 + c^2 = (r_o - r_1)^2$, from which

$$r_o = \tfrac{1}{2}\left[\frac{c^2}{r - r_1} + r + r_1\right]; \qquad (149.)$$

$$\sin a^\circ = \frac{c}{r_o - r_1}; \qquad (150.)$$

$$b^\circ = 180^\circ - 2a^\circ. \qquad (151.)$$

By adding together the various expressions for the values of the arcs $A\,B\,D$, $D\,E$, $E\,F\,G$, and $G\,A$, we can obtain the expression

$$P = \pi\left[\frac{a^\circ}{90^\circ}(r_o - r_1) + r + r_1\right]. \qquad (152.)$$

Also, by adding together the various expressions for the areas $A\,B\,D\,A$, $A\,C\,C'\,G\,A$, $C\,D\,E\,C'\,C$, and $C\,E\,F\,G\,C'$, we can obtain the expression

$$A = \frac{\pi}{2}\left[r^2 + r_1^2 + \frac{a^\circ}{90^\circ}(r_o^2 - r_1^2)\right] - c\,(r_o - r). \qquad (153.)$$

The value of R is given by the formula

$$R = \frac{A}{P}. \qquad (154.)$$

The above formulas, relating to the elements of the cross-sections of egg-shaped sewers, are perfectly general, and apply to all cases, whatever may be the ratio between the horizontal and vertical diameters. It will be remembered that $\pi = 3.1416$.

1504. Horizontal and Vertical Diameters.—The usual ratio between the horizontal and vertical diameters (d_h and d_v) is that of two to three. In other words, $d_h = \frac{2}{3}\,d_v$, whence

$$d_v = \tfrac{3}{2}\,d_h = 3\,r. \qquad (155.)$$

Hence, from formula **148**,

$$c = 2\,r - r_1. \qquad (156.)$$

1505. Cross-Sections of Flow.—The computations for velocity and discharge of egg-shaped sewers are usually made for sewers running *full*, *two-thirds full*, and *one-third*

full. By two-thirds and one-third full are meant a depth of flow equal to $\frac{2}{3}\,d_v$ and $\frac{1}{3}\,d_v$, respectively.

For the cross-section of a sewer in which the relations between d_h and d_v are as given by formula **155,** if we subtract the semi-circumference $A\,B\,D$, Fig. 370, from the perimeter P, the remainder will be the wetted perimeter for sewer flowing two-thirds full. Hence, for a depth of flow equal to $\frac{2}{3}\,d_v$, the wetted perimeter $P_{\frac{2}{3}}$ will be given by the formula

$$P_{\frac{2}{3}} = P - \pi\,r. \qquad \textbf{(157.)}$$

For the same depth of flow, the area of the cross-section of *flow* may be found by subtracting the semicircle $A\,B\,D\,C\,A$ from the total area A. Hence, for this condition, the area of the cross-section of flow $A_{\frac{2}{3}}$ will be given by the formula

$$A_{\frac{2}{3}} = A - \frac{\pi\,r^2}{2}. \qquad \textbf{(158.)}$$

The point M, Fig. 370, is at one-third the vertical diameter $B\,F$, and $L\,K$ is a horizontal line through that point. With the sewer running $\frac{1}{3}$ full, therefore, $K\,E\,F\,G\,L$ will be the wetted perimeter, and that portion of the internal cross-section below the line $L\;K$ will be the cross-section of the flow. These values, however, can not be expressed by any convenient general formula; but, when applied to certain cases, simple expressions for them will be given below.

1506. Old and New Forms of Cross-Section.—Formerly, it was quite the general practice to use a value of $\frac{r}{2}$ for the value of r_1. The form of cross-section obtained by using this value of r_1 will here be called the **old form.**

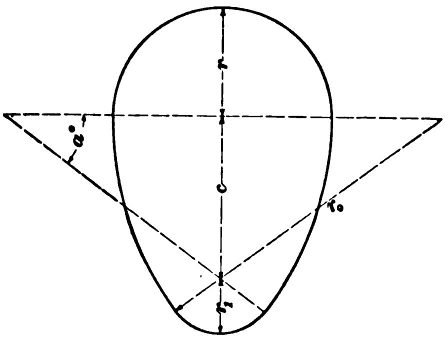


FIG. 371.

In later years, however, it has become a not uncommon practice to use $\frac{r}{4}$ as the

value of r_1. When the flow is light, the latter practice gives a slightly deeper current than the former practice for the same volume of sewage; it is, therefore, a somewhat better practice, especially for large sewers. The form of cross-section resulting from this practice will here be called the **new form.**

1507. Elements of the Cross-Section.—The old form for the interior cross-section of an egg-shaped sewer is shown in Fig. 371, and the new form in Fig. 372. The values of all the elements essential to the design of either cross-section, in terms of the upper radius r, are given in the table of Elements of Egg-Shaped Sewers (Tables and Formulas.)

From that table, it will be noticed that the value of the hydraulic mean radius, for either form of an egg-shaped sewer, is greater when the depth of flow is two-thirds the vertical diameter than when the sewer is flowing full. In the old form the greatest velocity will occur when the depth of flow is approximately .85 of the vertical diameter; and the greatest discharge will occur when the depth of flow is about .93 of the vertical diameter.

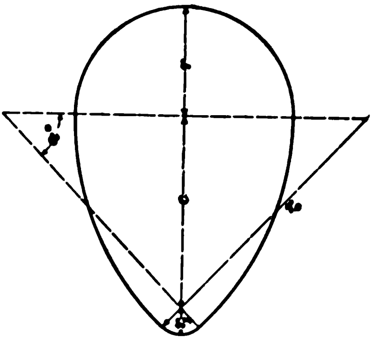


FIG. 372.

With sewer running full, the following approximate formulas will apply to both forms near enough for most practical purposes:

$$P = 2.7\, d_v \qquad \textbf{(159.)}$$

$$A = .524\, d_v^2 \qquad \textbf{(160.)}$$

$$R = .194\, d_v \qquad \textbf{(161.)}$$

1508. Relative Capacities of Egg-Shaped and Circular Sewers.—If a is the internal area of a circular sewer having a diameter equal to the horizontal diameter of an egg-shaped sewer of area A, then,

For the old form,

$$a = .6838\ A, \text{ or } A = 1.4624\ a \qquad \textbf{(162.)}$$

For the new form,

$$a = .7044\ A, \text{ or } A = 1.4197\ a. \qquad \textbf{(163.)}$$

The formulas commonly used for computing the dimensions of a sewer, necessary to give a required discharge or drain a given area, generally apply most expeditiously to circular sewers. Some formulas give the diameters of circular sewers as direct results. (See Arts. **1456** and **1457.**) In computing the dimensions of an egg-shaped sewer, it is generally better to obtain first the required dimensions of a circular sewer, and then find the dimensions of an egg-shaped sewer having an equivalent discharge.

If A is the internal area of an egg-shaped sewer; a, the internal area, and r_c, the internal radius of a circular sewer, then, in order that the areas A and a shall be equal, the relative values of r and r_c must be as given by the following formulas:

For the old form,

$$r = .8269\ r_c. \qquad \textbf{(164.)}$$

For the new form,

$$r = .8393\ r_c. \qquad \textbf{(165.)}$$

Or, approximately, and near enough for most practical purposes,

For either form,

$$r = \tfrac{5}{6} r_c. \qquad \textbf{(166.)}$$

The same relations will, of course, exist between the horizontal diameters as between the horizontal radii, and, therefore, the three preceding equations will apply to the horizontal diameters by substituting d_h for r and by substituting for r_c the diameter d of the circular sewer.

1509. Comparative Values of Hydraulic Mean Radius.—The hydraulic mean radius for a circular sewer running full is equal to one-fourth its diameter, or $.50\ r_c$. (Art. **1498.**) For the egg-shaped sewer of *equal cross-*

section, the hydraulic mean radius, when running full, will be as given by the following formulas:

For the old form,

$$R = .5793 \times .8269\, r_c = .4790\, r_c. \qquad \textbf{(167.)}$$

For the new form,

$$R = .5688 \times .8393\, r_c = .4774\, r_c. \qquad \textbf{(168.)}$$

These values are so nearly the same as the value of the hydraulic mean radius, $.50\, r_c$ of the circular sewer flowing full, that for many practical purposes they may be considered to be the same. Hence, having obtained the diameter of a circular sewer necessary to give a required discharge when running full, the horizontal diameter of an egg-shaped sewer that will give an approximately equivalent discharge may be obtained by applying formula **164** or **165**, or, near enough for most practical purposes, by applying formula **166.** If, however, it is desired that the egg-shaped sewer, when running full, shall have the *same hydraulic mean radius* as the circular sewer when running full, then the horizontal diameter d_h of the former must have the value given by the following formulas, in which d is the diameter of the circular sewer:

For the old form,

$$d_h = .863\, d. \qquad \textbf{(169.)}$$

For the new form,

$$d_h = .879\, d. \qquad \textbf{(170.)}$$

1510. Sewers of Equal Discharge.—In order that the egg-shaped sewer, when flowing full, shall have the *same theoretical discharge* as the circular sewer, its horizontal diameter d_h must have the value given by the following formulas:

For the old form,

$$d_h = .834\, d. \qquad \textbf{(171.)}$$

For the new form,

$$d_h = .847\, d. \qquad \textbf{(172.)}$$

The value of the horizontal diameter given by formula

171 is almost identical with that given by formula **166**, while the value given by formula **172** exceeds that given by formula **166** by about 1.6 per cent. Hence, as the practical diameter will be in even inches, and will generally somewhat exceed the theoretical diameter, formula **166** will be satisfactory for most practical purposes. When great accuracy is required, however, formulas **171** and **172** may be applied. In applying these formulas to obtain the dimensions of an egg-shaped sewer that will have a capacity equal to that required for a circular sewer, the *theoretical* diameter, or radius, of the circular sewer, should, of course, be used.

1511. EXAMPLE.—What should be the upper radius r of an egg-shaped sewer that it may have a cross-section equal to that required for the example explained in Art. **1499**—(*a*) if of the old form, (*b*) if of the new form, and (*c*) if computed by formula **166** for either form?

SOLUTION.—(*a*) The diameter required for the circular sewer is 3.64 feet; consequently, its radius r_c is $\frac{3.64}{2} = 1.82$ feet. Applying formula **164**, the upper radius required for an equivalent egg-shaped sewer of the old form will be $.8269 \times 1.82 = 1.505$ feet. Ans.

(*b*) By applying formula **165**, the upper radius required for an equivalent egg-shaped sewer of the new form will be $.8393 \times 1.82 = 1.528$ feet. Ans.

(*c*) By applying formula **166**, the upper radius required for an equivalent egg-shaped sewer is found to be $\frac{5}{6} \times 1.82 = 1.517$ feet. Ans.

EXAMPLES FOR PRACTICE.

NOTE.—The following examples refer to those given in Art. **1499.** In each case, three values will be found for the upper radius; namely, (*a*) for the old form, (*b*) for the new form, and (*c*) as given by formula **166.**

1. What will be the upper radius r of an egg-shaped sewer having the same cross-section as that of the circular sewer required by the conditions of Example 1?

Ans. $\begin{cases} (a)\ 1.538 \text{ ft.} \\ (b)\ 1.561 \text{ ft.} \\ (c)\ 1.550 \text{ ft.} \end{cases}$

2. What will be the upper radius r of an egg-shaped sewer having the same cross-section as that of the circular sewer required by the conditions of Example 2?

Ans. $\begin{cases} (a)\ 1.207 \text{ ft.} \\ (b)\ 1.225 \text{ ft.} \\ (c)\ 1.217 \text{ ft.} \end{cases}$

3. What will be the upper radius r of an egg-shaped sewer having the same cross-section as that of the circular sewer required by the conditions of Example 3?

Ans. $\begin{cases} (a)\ 1.323 \text{ ft.} \\ (b)\ 1.343 \text{ ft.} \\ (c)\ 1.333 \text{ ft.} \end{cases}$

4. What will be the upper radius r of an egg-shaped sewer having the same cross-section as that of the circular sewer required by the conditions of Example 4?

Ans. $\begin{cases} (a)\ 1.935 \text{ ft.} \\ (b)\ 1.964 \text{ ft.} \\ (c)\ 1.950 \text{ ft.} \end{cases}$

5. What will be the upper radius r of an egg-shaped sewer having the same cross-section as that of the circular sewer required by the conditions of Example 5?

Ans. $\begin{cases} (a)\ 2.894 \text{ ft.} \\ (b)\ 2.938 \text{ ft.} \\ (c)\ 2.917 \text{ ft.} \end{cases}$

NOTE.—From the above examples it will be noticed that, for either form of egg-shaped sewer, the value of the upper radius given by formula **166** is sufficiently accurate for most practical purposes, being nearly a mean between the values for the old and new forms.

1512. Practical Dimensions of Egg-Shaped Sewers.—In practice, the horizontal diameters of egg-shaped sewers are usually multiples of two inches. The practical value of the upper radius r should, therefore, be taken at the nearest full inch above its theoretical value. In case the theoretical value of r obtained for the *new* form should be expressed by exact inches, it will be well to add one inch to this value for its practical value. Having determined the practical value of r, the values of all other dimensions, for either the old or new form, may be obtained from table of Elements of Egg-Shaped Sewers (Tables and Formulas).

1513. EXAMPLE.—For the example (c) explained in Art. **1511**, what will be the value of the following practical dimensions for an egg-shaped sewer of the *old* form, namely, horizontal and vertical diameters, radii of side and bottom arcs, and vertical distance between centers?

SOLUTION.—In the explanation referred to, the upper radius r was found to be 1.517 feet, or somewhat more than 18 inches. Hence, the practical value of r will be 19 inches. By applying the coefficients or

multipliers given in items (1) to (5), inclusive, of the table referred to the following values are obtained:

$d_h = 2 \times 19 = 38$ inches $= 3$ ft. 2 in.
$d_v = 3 \times 19 = 57$ inches $= 4$ ft. 9 in.
$r_1 = \frac{1}{2} \times 19 = 0$ ft. $9\frac{1}{2}$ in.
$r_0 = 3 \times 19 = 57$ inches $= 4$ ft. 9 in.
$c = 1\frac{1}{2} \times 19 = 28\frac{1}{2}$ inches $= 2$ ft. $4\frac{1}{2}$ in.

EXAMPLES FOR PRACTICE.

NOTE.—The following examples relate to the answers (c) obtained for the examples given in Art. **1511.** In each case the practical values of the following dimensions will be obtained; namely, the horizontal diameter (d_h), vertical diameter (d_v), radius of bottom arc (r_1), radius of side arcs (r_0), and vertical distance between centers (c).

1. What are the practical dimensions of an egg-shaped sewer of the *new* form, required by the conditions of Example 1?

Ans. $\begin{cases} d_h = 3 \text{ ft. } 2 \text{ in.} \\ d_v = 4 \text{ ft. } 9 \text{ in.} \\ r_1 = 0 \text{ ft. } 4\frac{3}{4} \text{ in.} \\ r_0 = 4 \text{ ft. } 2\frac{3}{8} \text{ in.} \\ c = 2 \text{ ft. } 9\frac{1}{4} \text{ in.} \end{cases}$

2. What are the practical dimensions of an egg-shaped sewer of the *old* form, required by the conditions of Example 2?

Ans. $\begin{cases} d_h = 2 \text{ ft. } 6 \text{ in.} \\ d_v = 3 \text{ ft. } 9 \text{ in.} \\ r_1 = 0 \text{ ft. } 7\frac{1}{2} \text{ in.} \\ r_0 = 3 \text{ ft. } 9 \text{ in.} \\ c = 1 \text{ ft. } 10\frac{1}{2} \text{ in.} \end{cases}$

3. What are the practical dimensions of an egg-shaped sewer of the *new* form, required by the conditions of Example 3?

Ans. $\begin{cases} d_h = 2 \text{ ft. } 10 \text{ in.} \\ d_v = 4 \text{ ft. } 3 \text{ in.} \\ r_1 = 0 \text{ ft. } 4\frac{1}{4} \text{ in.} \\ r_0 = 3 \text{ ft. } 9\frac{1}{8} \text{ in.} \\ c = 2 \text{ ft. } 5\frac{5}{8} \text{ in.} \end{cases}$

4. What are the practical dimensions of an egg-shaped sewer of the *old* form, required by the conditions of Example 4?

Ans. $\begin{cases} d_h = 4 \text{ ft. } 0 \text{ in.} \\ d_v = 6 \text{ ft. } 0 \text{ in.} \\ r_1 = 1 \text{ ft. } 0 \text{ in.} \\ r_0 = 6 \text{ ft. } 0 \text{ in.} \\ c = 3 \text{ ft. } 0 \text{ in.} \end{cases}$

5. What are the practical dimensions of an egg-shaped sewer of the *new* form, required by the conditions of Example 5?

Ans. $\begin{cases} d_h = 6 \text{ ft. } 0 \text{ in.} \\ d_v = 9 \text{ ft. } 0 \text{ in.} \\ r_1 = 0 \text{ ft. } 9 \text{ in.} \\ r_0 = 8 \text{ ft. } 0 \text{ in.} \\ c = 5 \text{ ft. } 3 \text{ in.} \end{cases}$

LATERAL SEWERS.

1514. General Description.—In previous articles, the computations for the storm-water effluent have been made under the assumption that the storm-water flow is entirely over the surface until it reaches the main sewer. This, however, is not always, nor even usually, the case The storm water from those portions of the district remote from the main, or trunk, sewer is generally conveyed to the main sewer by smaller branch sewers, commonly called **lateral sewers**, or simply **laterals.**

The principal sewer, which conveys the entire drainage from a district and discharges it at the outlet, is called the **main sewer** or **trunk sewer.**

A drainage district is represented in Fig. 373. This district is, for simplicity, represented as perfectly rectangular, although actual drainage districts are generally more or less irregular. The principles apply in substantially the same manner. In this district, $T\,O$ is the main, or trunk, sewer, O being the outlet. The sewers l_1, l_1', l_2, l_2', etc., are the laterals, which discharge into the trunk sewer at m_1, m_2, etc. The main trunk sewer $T\,O$ passes down through the middle of the district, and that portion of the drainage district on each side of the main sewer is divided into five equal sub-districts, $c\,d$, $d\,e$, $e\,f$, $f\,g$, and $g\,h$, which are drained by the lateral sewers. These sub-districts are shown separated by dotted lines. All the sewers are assumed to be laid along streets, as is usual, but in order to avoid confusion, the streets are not shown. It is assumed that the lateral sewer l_1 takes all the drainage from the sub-district $c\,d\,d_0\,T$, the lateral l_2 all the drainage from the sub-district $d\,e\,e_0\,d_0$, etc.

In referring to the sub-districts, those drained by the laterals l_1, l_2, etc., will be designated as *sub-district 1*, *sub-district 2*, etc., and those drained by the laterals l_1', l_2', etc., will be designated as *sub-district 1'*, *sub-district 2'*, etc.

1515. Conditions Governing the Design.—The dotted lines $a\,b$ and $a'\,b'$ are lines located at the same distance

from the main sewer as the lines $c\,c'$, $d\,d'$, $e\,e'$, etc., are from the lateral sewers. Consequently, the drainage from that portion of each sub-district between the lines $a\,b$ and $a'\,b'$ may generally be taken by either the main sewer or the lateral, according to the slope, the location of surface inlets, etc. In fact, it is probable that, in most cases, the drainage from this portion of each sub-district will be taken partly by the lateral and partly by the main sewer. In designing sewers, however, it is, in most cases, the best practice *to consider the entire drainage from each sub-district to be discharged into the main sewer from the lateral.* This practice will be on the side of safety, giving the greatest possible required capacity for the lateral, and, in every ordinary case, sufficient capacity for the trunk sewer. A case might arise in which it would be necessary to somewhat increase the capacity of the trunk sewer over that given by this assumption at certain points just above the points m where the laterals connect. Such cases, however, would be exceedingly rare.

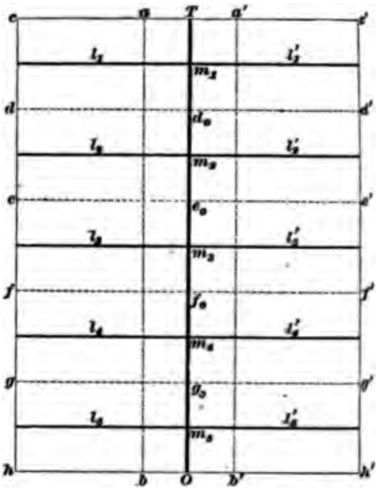


FIG. 373.

1516. Design of the Laterals.—The design of the lateral and trunk sewers for a district similar to that shown in Fig. 373 follows the same general methods that have been explained in the preceding articles. The lateral l_1 is designed to take the entire drainage from sub-district 1, the same as if it were the single sewer of an independent drainage district. In like manner, the lateral l_1' is designed to take the entire drainage from the sub-district 1′. The laterals l_1 and l_1' both discharge into the trunk sewer at m_1.

Hence, from m_1 to m_2 the trunk sewer must have a capacity sufficient to convey the combined flow from the laterals l_1 and l_1'. Also, if any additional drainage enters the trunk sewer directly between the points d_0 and m_2, the trunk sewer must have sufficient capacity at and below such points to take the additional drainage. The capacity of the trunk sewer, however, at any point between d_0 and m_2 will very rarely need to be greater than at m_1, because it will require a longer time for the storm water to flow from the most remote portion of the district (as c or c') to a point below d_0 than will be required for it to flow to m_1. Consequently, the length of storm giving the greatest flow will be greater and the rate of rainfall less. The laterals l_2 and l_2' are designed to convey the drainage for the sub-districts 2 and 2', each as if for an independent district. These laterals both discharge into the trunk sewer at m_2; consequently, between m_2 and m_3, the trunk sewer must have sufficient capacity to convey the combined effluent from the sub-districts 1, 1', 2, and 2', *as given by a storm of sufficient length for the storm water from the most remote portion of the district to reach* m_3. This operation is simply repeated and extended throughout the entire district until the combined effluent from all the sub-districts is provided for. Between m_4 and the outlet O, the discharging capacity of the trunk sewer must be sufficient to convey the effluent from the entire district, which includes all the sub-districts.

1517. Comparative Lengths of Storms.—If the grades of all the laterals and the characters and surface slopes of all the sub-districts are the same, the maximum effluent and the length of storm giving the maximum effluent will be the same for each sub-district and lateral. The length of storm giving the maximum charge in different sections of the trunk sewer will, however, vary materially. For each section of the trunk sewer, the storm giving the maximum charge to the sewer in that section must continue for a length of time sufficient for the storm water from the most remote portion of the district to reach that point, Thus,

the capacity of the trunk sewer at and below m_3 must be sufficient to convey the effluents from sub-districts 1, 1′, 2, 2′, 3, and 3′, given by a storm of sufficient length for the storm flow from the most remote portion of the district (as from c or c') to reach m_3. This effluent will be somewhat less than the combined *maximum* effluents from the six laterals l_1, l_1', l_2, l'_2, l_3, and l_3', because the storm will be of longer duration, and, consequently, will have a lower rate of precipitation. It must also be noticed that, if the grades of the laterals l_1 and l_1' are materially steeper than those of l_3 and l_3', it may require a longer time for the storm flow from the extreme end of l_3 to reach the point under consideration, than that from the extreme end of l_1. The flow from l_3 will then be the governing condition.

CONCLUDING REMARKS.

1518. Recapitulation.—In order that each step of the process might be thoroughly explained, the applications of the formulas relating to the flow of water and the capacities of sewers have been given in a somewhat disconnected manner, and it will be well, therefore, to briefly recapitulate the main features of the process.

In applying formula **116**, Art. **1447**, for estimating the effluent, all conditions should be determined, or assumed, as accurately as possible, in order that the value of the effluent may be estimated as nearly correctly as possible. In estimating the effluent, it is generally well to assume the value for v, the velocity of the flow in the sewer, below, rather than above, what will probably be its actual value; it should, however, be assumed as nearly correctly as possible. To this end, in actual practice, it will be well to use more than one formula and compare the results. As the expression $\frac{k}{v_1\sqrt{S}}+\frac{l}{v}$, in the denominator of formula **112**, **113**, or **116**, expresses the duration of the storm *in seconds*, it will generally be unnecessary to obtain the value of this expression closer than to the nearest unit.

Having estimated the effluent, the approximate value of the hydraulic mean radius required to convey the effluent may be obtained by formula **143,** Art. **1490,** using the assumed value of v. As the diameter of a circular sewer will be, approximately, four times this approximate hydraulic mean radius, the character of the sewer may be decided, thus determining the value of n. The slope, or gradient, of the sewer will have been ascertained in making the surveys and collecting the data for the district. The values n, r, and s being thus known, the velocity may be readily computed.

If the computed velocity is found to vary materially from the velocity assumed in estimating the effluent, it will be necessary to reestimate the effluent, using a corrected velocity. For this purpose, the value of v_0, as given by formula **144,** Art. **1491,** may generally be used.

The velocity obtained by applying formula **144** will be sufficiently correct to use in estimating the effluent. The approximate velocity v_s for the corrected effluent is given by formula **145.** This should agree reasonably well with the velocity v_0, as given by formula **144.** Either of these velocities substituted in formula **143** should give the value of r for a circular sewer close enough for all practical purposes. The theoretical diameter of the circular sewer will then be equal to $4\ r$.

1519. Form of Cross-Section.—If the estimated effluent is so great as to require a sewer of considerable size, it will be well to make the cross-section egg-shaped. If the required cross-section be only moderately large, the old form of cross-section may be used, but if it be exceedingly large, the new form should be used. It will be a quite good rule to use the new form of cross-section whenever the value of r_1 for the old form will exceed 8 inches. When the diameter of a circular sewer of a capacity sufficient to convey the effluent is known, the horizontal diameter of an egg-shaped sewer having an equivalent capacity may be found by applying formula **166.** The horizontal diameter thus obtained will be practically correct for the old form, and, for the new

form, close enough for most practical purposes. When greater accuracy is required, however, formula **172** should be used for the new form.

1520. Final Computations; Conclusion.—When the dimensions of the sewer have finally been determined, the velocity and discharge should be computed by formulas **131,** Arts. **1478,** and **135,** Art. **1486,** respectively. The computed discharge should then be compared with the effluent, or required discharge, in order that there may be no uncertainty as to whether the capacity of the sewer, when running full, will be sufficient to convey the effluent. If no mistake has been made in the computations, the computed discharge should always agree with the estimated effluent, near enough for all practical purposes. The computed discharge will not generally agree exactly with the effluent, however, owing to the fact that the formulas are only approximate.

After some familiarity with the work, the entire process can be materially shortened, and some of the steps indicated here can be omitted.

In conclusion, it will be well to notice that the dimensions obtained for sewers in the examples for practice that have been given are generally large for the sizes of the districts drained. This is in each case due to the large effluent estimated by using formula **104,** Art. **1408,** for the rate of rainfall, which, for most localities, will give excessive values. Had formula **102,** Art. **1407,** which is better adapted to the Northern States, been used for the rate of rainfall, the resulting dimensions would have been materially less. This formula will be used in the questions relating to this section.

SEWERAGE.

GENERAL CONSIDERATIONS.

NECESSITY FOR SEWERAGE.

1521. Beneficial Effects.—Evidence with regard to the healthful effect of the sewerage and drainage of cities is abundant and need not be given here. The experience of many cities could be cited to show the marked reduction in the death rate resulting from the construction of sewerage systems; but the benefits of sewerage are now so well known as to make such proofs unnecessary. There is no longer any question as to *why* sewers and drains should be built; the only question at present is *how* they shall be built. The drainage of the surface and subsoil has been studied in the section on Drainage; the subject of Sewerage, as relating to the removal of waste and refuse matter, will now be considered.

1522. A Requisite to Health.—One of the principal conditions requisite to health is that the air we breathe shall be pure. The air in the vicinity of human habitations may be maintained pure by promptly removing those things that pollute it, such as decaying matter, injurious gases, and all conditions favorable to the development of disease germs.

1523. What Must Be Removed.—Besides the prompt removal of all water from the surface and subsoil, which is accomplished by efficient drainage, all garbage, street sweepings, solid kitchen and factory waste, decaying vegetable matter, and other dry refuse should be promptly collected and removed, and all excrementitious, or human waste, and liquid refuse should be removed by an adequate

For notice of copyright, see page immediately following the title page.

system of sewerage. By such means, the putrefying matter, stagnant water, and dampness, which not only generate disease germs, but also make their continued existence possible, are effectually removed. The soil is thus rendered dry and wholesome, and the air is purified.

DIFFERENT SYSTEMS OF REMOVAL.

1524. Systems Employed.—The different methods employed for the removal of excrementitious and liquid waste may be divided into three general classes, namely: the system by **direct removal,** the **pneumatic system,** and the **water-carriage system.**

1525. Direct Removal Systems.—The principal methods of direct removal are the **pail system** and the **dry-earth closet.** In the former system, the excrement is caught in a pail or other vessel, and, at stated intervals, is transferred to and removed by carts. In the latter system, dry, powdered earth or ashes is added to the excreta in sufficient quantities to absorb the moisture and deodorize the entire mass until it can be removed. There are various modifications to both these systems of direct removal, all of which give results more or less satisfactory, and also involve features more or less objectionable.

1526. The **pneumatic systems** generally consist essentially of systems of air-tight pipes, through which the excrementitious matter is drawn by atmospheric pressure, the air being exhausted from the pipes by large air pumps. Such systems are intended for the removal of only such portions of the wastes as is most valuable for fertilizing purposes. Separate conduits are provided for liquid wastes. These systems, though disposing of only a portion of the refuse, require costly machinery and are also expensive to operate.

1527. The **Shone system,** however, employs compressed air for the purpose of *raising* sewage. The sewage flows through pipes by gravity, in the usual manner, until it reaches the lowest level practicable or desirable for it. At

such a point the sewage flows into a large iron tank, called a **pneumatic ejector,** from which, when full, it is forced into pipes at a higher level by means of compressed air automatically applied. This appliance is especially valuable in cities where sufficient fall to the outlet of the sewers can not be obtained. In such cases, it can often be advantageously substituted for pumps for raising the sewage to the required elevation. This is not of itself a distinct pneumatic system, but is more properly considered as a special expedient of the water-carriage system.

1528. The **water-carriage system** for the removal of sewage is by far the most popular. It is thoroughly efficient and requires only a comparatively inexpensive conduit, which conveys all the sewage. It will not be necessary to discuss here the merits of this system of sewerage; the statement that it is the system universally employed in this country, and quite generally used in all civilized countries, will be sufficient. Only the water-carriage system of sewerage will be considered in this paper.

1529. Classes of Water-Carriage Sewerage.—There are two general classes of water-carriage sewerage systems, respectively known as the *combined system* and the *separate system.* In the **combined system,** the storm-water drainage and the domestic sewage are both conveyed in the same conduit, while in the separate system, an underground conduit is provided for the sewage only, the storm water being conveyed in a different conduit, either on the surface or underground. In the **separate system,** all storm water is usually excluded from the sewage conduits, though the system is sometimes modified so as to admit a portion of the storm water.

THE SEPARATE SYSTEM OF SEWERAGE.

1530. Advantages of this System.—For some conditions, the separate system of sewerage possesses certain material advantages over the combined system.

The advantage of the separate system which most strongly

appeals to the taxpayer is its reduced cost, which is generally from one-eighth to one-half the cost of the combined system for corresponding conditions. This statement relates to the system of sewage conduits. But it is also evident that where it is necessary for the sewage to be pumped, or where it is to be utilized on a sewage farm, or by any process purified, the expense of the process will be very materially reduced by the exclusion of the storm water.

The separate system, if properly constructed, will meet the requirements for the efficient removal of house sewage more perfectly, and under conditions more strictly sanitary, than the combined system. In the former system, the sewers, being of small section, will run comparatively full once every day, in dry as well as in wet weather, and this tends to prevent permanent deposits; the sewers will have more uniform velocities of flow, and, consequently, can generally be constructed with flatter grades than in the combined system. Moreover, when flushing is necessary, the same degree of cleanliness can be obtained with less water in the separate than in the combined system.

1531. Where Adaptable.—The separate system has been found to be peculiarly adapted to the requirements of small towns and to the outlying districts of large cities. The comparatively small cost of this system permits its construction in such towns and suburbs where the greater expense of the combined system would render it impracticable. The population of such districts not being dense, the questions of street refuse and surface drainage are not of an urgent nature. The comparative advantages of the separate system for large and densely populated cities, however, are not so great.

The design of the sewers of a separate system is necessarily founded on the quantity of sewage delivered to each sewer or branch by the district tributary to the same. The method of estimating the quantity of sewage will now be studied.

COMPUTATIONS RELATING TO SEWERS.

QUANTITY OF SEWAGE; WATER CONSUMPTION.

1532. Sewage Discharge and Water Supply.—The available records of sewer gaugings for American cities are not sufficient to indicate accurately the quantity of sewage per capita that must be provided for. Records of water supply, however, are abundant, and, as such sewer gaugings as have been made indicate that the quantity of sewage from a given district is somewhat less than the quantity of water consumed by its inhabitants, the statistics of water supply may be taken as a basis for estimating the sewage discharge. This statement, of course, relates to the sewage proper, and is wholly independent of storm-water or subsoil drainage.

That the amount of actual sewage will be somewhat less than the corresponding water supply will be evident when we consider that all the water used for sprinkling, and a portion of that used for cleaning, either soaks into the ground or evaporates. In manufacturing districts, too, considerable quantities of water are used which do not reach the sewers.

For most American cities, it would probably be sufficient to call the *average* daily discharge of *house sewage* per capita equal to 80 per cent. of the corresponding water supply. It will, in any case, be perfectly safe to *assume the average daily discharge of house sewage per capita as equal to the corresponding water consumption.*

1533. Average Rate of Water Consumption.—In the year 1885, the average rate of water consumption in one hundred and seventy-six American cities, having populations varying from 10,000 to over 500,000, was 89 gallons per day per capita, as averaged upon the basis of population. In many of these cities, however, the average consumption was much greater, being in several cases considerably more

than 50 per cent. above this average figure. The average for large cities is somewhat greater than for small cities.

1534. Increasing Rate of Water Consumption.—It must also be noticed that the per capita rate of water consumption is gradually increasing, and that it has not yet reached its maximum. The average rate of increase at present is probably somewhat less than one per cent. per year. If, then, we make the reasonable assumption that, in the one hundred and seventy-six cities noticed above, the average total increase during the twelve years subsequent to 1885 is 10 per cent., then, for these cities, the average rate of water consumption in 1897 will be nearly 98 gallons per day per capita.

1535. Basis of Rate per Capita.—In this connection it will be well to notice that the above statements and figures are based on the *total population*, as is customary. In most cases, the *rate* of consumption per *consumer* will be found to be materially greater, and the *increase* in the rate materially less, than the respective rate and increase per *inhabitant*. This statement applies with especial force during the first few years subsequent to the establishment of a water-works system, when the number of consumers is comparatively small, but is increasing rapidly. Nothing approaching a definite ratio between the rate of consumption per inhabitant and the rate per consumer can be stated; after a water-works system has been established a number of years, the average rate of daily consumption per consumer may generally be taken at from one-quarter to one-third greater than the average rate per inhabitant. Wherever the rate per capita is herein mentioned, the rate per *inhabitant* is meant.

1536. Variations in the Water Consumption.—The preceding statements relate to the *average* daily water consumption and the corresponding sewage discharge. It is important to notice, however, that the consumption of water, and, consequently, the discharge of sewage, varies greatly. In the design of sewerage systems, two different

periods of fluctuation must be recognized; namely, the daily fluctuations during the year, giving what is called the **day maximum**, and the hourly fluctuations during the day, giving what is known as the **hour maximum**. The hourly fluctuations generally occur with a considerable degree of regularity, while the daily fluctuations are less regular. There are also quite well-defined weekly fluctuations, the consumption generally being greater on Monday than on other days of the week. It is quite probable, however, that the weekly fluctuations may, in most cases, be safely neglected.

1537. The Hour Maximum.—The hourly fluctuations in the water consumption and sewage discharge generally occur at quite regular periods during the twenty-four hours; the maximum rate an hour or so before noon, and the minimum between two and five o'clock in the morning. In the designing of sewers, only the maximum rate must be considered. The hour maximum may attain a rate one and one-third times the mean hourly rate, and in extreme cases, even a considerably higher rate.

1538. The Day Maximum.—There are two periods of excessive water consumption during the year, either one of which may be the maximum. One period occurs during the coldest weather, and the other during the warm, dry period, which generally comes late in the summer.

This is shown quite clearly by the diagrams of Fig. 374, which represents graphically the *relative* monthly rates of water consumption in five of the larger cities of the United States during the year 1884. The actual rates are not shown by the diagrams; the relative rates for the different months, reduced to the same basis, are shown for the purpose of comparison; the average rate is 100 in each case. A study of these diagrams will be found both interesting and instructive.

Diagram *A* shows a high maximum in January and a lower one in August. Diagram *B* shows a single maximum in February, and in the next two months falls to a point

at which a nearly uniform rate is maintained during the remainder of the year. Diagram *C* shows a high rate in January and a maximun in August. From a minimum in May, diagram *D* rises abruptly to a high maximum in June;

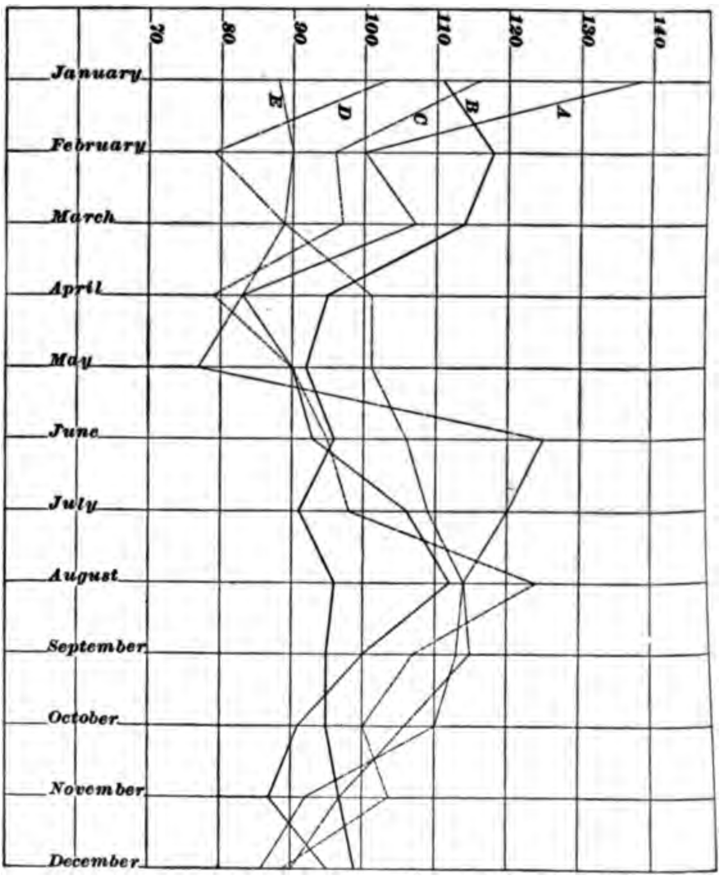


FIG. 374.

it also shows a low rate in February and a high rate in September. Diagram *E* rises quite gradually to a moderate maximum in August. The weather conditions represented by diagrams *A* and *C* are very similar, while those represented by the other three diagrams show great variations

The maximum of the cold weather is the one that principally affects the sewage discharge. In severe weather, water pipes burst, and faucets are left running to prevent freezing, and this is a source of much waste. These and other conditions present during such weather will, in most cases, produce the maximum rate of water consumption during the year. Nearly the entire volume of this cold weather maximum will enter the sewer. During the maximum of the warm and dry weather, much water will be used for sprinkling and similar purposes; but a great deal of it evaporates or is absorbed by the soil, so that a comparatively small quantity will reach the sewer. Hence, the maximum of the cold weather is the one that will to the greatest extent augment the sewage discharge. The day maximum may easily reach a rate 50 per cent., and in extreme cases, more than 75 per cent. above the mean rate.

1539. Probable Absolute Maximum.—The hour maximum occurring on the day of the cold weather day maximum will almost always be the absolute maximum rate of discharge of sewage for the year. The ratio of the probable hour maximum to the hourly average during the day multiplied by the ratio of the probable day maximum to the daily average during the year will, therefore, give the ratio of the absolute hour maximum to the average hourly rate during the year and indicate the probable value of the absolute maximum.

A common practice among American engineers is to assume the maximum daily rate of sewage discharge, i. e., the day maximum, to be one and one-half times the average daily discharge, and to assume the maximum hourly rate of discharge, or hour maximum, to be one-twelfth the average daily discharge. If, then, D_a is the average daily discharge, D_m the day maximum, and H_m the hour maximum, we shall have:

$$D_m = \tfrac{3}{2} D_a. \qquad \textbf{(173.)}$$

$$H_m = \frac{D_a}{12}. \qquad \textbf{(174.)}$$

1540. Actual Rate of Water Consumption.—In nearly all cities where the water supply is pumped, and in many cities where it is not pumped, records are kept of the quantities of water consumed daily, and, in many cases, of the quantities consumed hourly. Consequently, there will generally be no difficulty in determining either the average or the maximum total daily water consumption for any particular city; while in cities where hourly records are kept, the *absolute maximum*, i. e., the greatest total hour maximum, can be readily determined. *This total daily or hourly consumption (average or maximum, as the case may be) divided by the total population will give the corresponding consumption per capita.*

1541. Assumed Probable Rate of Water Consumption.—In cities having no records of their water consumption, it is safe to assume the daily consumption to be from 60 to 150, with a probable average of about 90 gallons per day per capita, according to the size of the city and the habits of its inhabitants.

When not otherwise specially stated, an average daily water consumption of 90 gallons per capita will here be assumed in all problems, and the volume of sewage will be assumed to be equal to the water consumption. (See Art. **1532.**)

1542. Relation Between Cubic Foot and Gallon.—The United States liquid gallon contains 231 cubic inches. Consequently, one cubic foot contains $\frac{1728}{231} = 7.48$ gallons, very closely, or, approximately, $7\frac{1}{2}$ gallons. In computations relating to sewerage, it is more convenient and sufficiently accurate to use the approximate ratio. This practice will be followed here. If, then, F is any number of cubic feet, and G the corresponding number of gallons, we have:

$$\left.\begin{aligned} G &= 7.5\,F, \\ F &= \frac{G}{7.5}. \end{aligned}\right\} \qquad \textbf{(175.)}$$

1543. The Unit of Discharge.—For computing the required dimensions of sewers the total required discharge should be expressed in the compound unit, *cubic feet per second*. This corresponds to the velocity unit, *feet per second*, used in formulas for velocity, and is, therefore, convenient for computing the required dimensions of a sewer from its estimated discharge.

When the absolute maximum H_m of the sewage discharge, expressed in cubic feet per hour per capita, is known, the required discharge of the sewer, in cubic feet per second, will be given by the formula

$$D = \frac{PH_m}{3,600}, \qquad \textbf{(176.)}$$

in which D is the required maximum discharge, in cubic feet per second, P is the total population, or that portion of the total population tributary to the sewer above the point in question, and H_m is the absolute per capita hour maximum, or greatest discharge per hour per capita, expressed in cubic feet. Also, from formulas **174** and **175**,

$$D = \frac{PH_m}{3,600 \times 7.5} = \frac{PD_a}{3,600 \times 7.5 \times 12} = \frac{PD_a}{324,000}, \qquad \textbf{(177.)}$$

D being in cubic feet, and D_a in gallons.

1544. Population per Acre.—In order to intelligently design a system of sewers, it is necessary to know approximately, not only the density of the present population in the various districts tributary to the sewer, but also the probable future increase. In German cities the density of population is from about 50 to 200 per acre. The common American practice in sewer design is to provide for from 30 to 60 per acre, although the density of the population in some places undoubtedly exceeds 60 per acre. The future population is usually taken at from 10 to 50 per cent. greater than the present. It must, in each case, be carefully estimated from a study of the local conditions as compared with the known increase under similar conditions in other cities. Statistics of population are abundant, and the

probable future increase also can generally be estimated with sufficient accuracy from the increase in the past. In many cases it will be safe to assume the greatest density of population in any one district, or in any small portion of the city, as the probable future density of all similar districts. In designing trunk sewers, provision must also be made for the extension of the district and the annexation of new districts. This, of course, applies also to such branch sewers as may be made trunk sewers by the annexation.

The discharge, in cubic feet per acre, may be calculated by substituting the population per acre for P in formulas **176** or **177**.

1545. EXAMPLE.—What will be the discharging capacity, in cubic feet per second, required for the main trunk sewer conveying the sewage from a city of 30,000 inhabitants?

SOLUTION.—The assumed daily average is 90 gallons per day per capita. By formula **174,** the hour maximum H_m for this daily average would be $\frac{90}{12} = 7.5$ gallons, or, by formula **175,** $\frac{7.5}{7.5} = 1.0$ cubic foot per capita. By formula **176,** this would require a discharge of $\frac{30,000 \times 1.0}{3,600} = 8\frac{1}{3}$ cubic feet per second. Ans. The same result may be obtained by applying formula **177.**

NOTE.—From this solution it will be noticed that, according to the rate of consumption assumed in Art. **1441,** the per capita hour maximum is 1 cubic foot. For an average daily consumption of 150 gallons, the per capita hour maximum would be $1\frac{2}{3}$ cubic feet.

EXAMPLES FOR PRACTICE.

1. What will be the required discharging capacity, in cubic feet per second, of the main trunk sewer conveying the sewage from a city having a population of 45,000? Ans. 12.50 cu. ft. per sec.

2. What would be the required discharging capacity, in cubic feet per second, for a population of (*a*) 80,000? (*b*) 100,000?

Ans. { (*a*) 22.222 cu. ft. per sec.
(*b*) 27.778 cu. ft. per sec.

3. If, in Example 1 above, the average daily consumption be assumed to be 120 gallons per capita, what will be the required discharge in cubic feet per second? Ans. 16.667 cu. ft. per sec.

4. With the same assumption for the average daily consumption as in the preceding example, how large a population can be provided for

by a sewer having a discharging capacity of (*a*) 4 and (*b*) 10 cubic feet per second?

Ans. { (*a*) 10,800.
(*b*) 27,000.

5. For an average daily consumption of 100 gallons per capita, what will be the per capita hour maximum, in cubic feet?

Ans. 1,111 cu. ft. per sec.

6. In a city of 60,000 inhabitants, the average daily water consumption is 168¾ gallons per capita. Assuming the quantity of sewage to be 80 per cent. of the water consumption, what will be the total sewage discharge in cubic feet per second? Ans. 25.0 cu. ft. per sec.

7. Assuming an average daily consumption of 90 gallons per capita, what will be the maximum discharge, in cubic feet per second per acre, from a district having a population of (*a*) 30, (*b*) 60, (*c*) 200, and (*d*) 325 per acre?

Ans. { (*a*) .00833 cu. ft. per sec.
(*b*) .01667 cu. ft. per sec.
(*c*) .05556 cu. ft. per sec.
(*d*) .09028 cu. ft. per sec.

8. Assuming the district to contain 200 acres, what would be the discharge from the district in each case?

Ans. { (*a*) 1.667 cu. ft. per sec.
(*b*) 3.333 cu. ft. per sec.
(*c*) 11.111 cu. ft. per sec.
(*d*) 18.056 cu. ft. per sec.

FLOW OF SEWAGE AND DIMENSIONS OF SEWERS.

1546. The **flow of sewage** is, in general, governed by the same laws as the flow of water. When the required discharge of sewage has been estimated, the dimensions of the sewer necessary to give this discharge can be readily computed. For conduits designed to convey sewage only, however, the governing conditions must be rather more carefully considered, and the principles of design more strictly adhered to, than for storm-water sewers or drains. This is necessary principally because of the smaller sizes of the conduits, permitting less variation in the flow, and because of the greater proportion of solid matter held in suspension.

Sewage conduits are generally quite small, as compared with storm-water sewers, and are almost invariably of circular cross-section. The sizes should, so far as possible,

be so proportioned that the depth of the ordinary flow will be sufficient to induce fair velocities and convey the suspended matter, thus preventing deposits. The depth of the flow should, therefore, be sufficient to immerse the solid matter held in suspension, thus lifting and floating it along without additional friction. This can generally be accomplished in the case of conduits conveying sewage only.

1547. Probable Minimum Flow.—In Art. **1539**, the rate of the day maximum, for sewers of the separate system, was assumed to be one and one-half times the average daily rate, and the rate of the hour maximum was assumed to be one-twelfth the average daily rate. The minimum rate is usually taken equal to *one-sixth* the maximum rate, or $\frac{H_m}{6} = \frac{D_a}{72}$. (See formula **174.**)

If a circular sewer be designed to discharge the maximum rate when running full, then, in order to discharge one-sixth of the maximum rate, the depth of flow must be equal to about twenty-eight hundredths of the diameter of the sewer, for which depth the velocity will be equal to about seven-tenths the velocity when flowing full or half full.

Hence, it is seen that if a sewer of the separate system is designed to convey the maximum discharge, estimated as closely as possible, when flowing full, it will not generally become clogged during the minimum flow. If the sewer is designed to convey the maximum discharge when flowing half full, some solid matter may be deposited during the minimum flow, but this will be flushed out each day by the maximum flow.

1548. Formulas for Velocity.—When the required discharge is known, the diameter of a circular sewer necessary to give the discharge may be computed in the manner explained in the section on Drainage, the slope, or grade, of the sewer having been either previously determined or assumed. As the conditions governing the flow of sewage, however, vary through a less wide range than those govern-

ng the flow of storm water, the computations for the former may be correspondingly modified and shortened. Velocities will here be computed by Kutter's formula (Art. **1479**).

Conduits for conveying sewage only are generally composed of vitrified earthenware pipe. For such pipe, when laid under the usual conditions, .013 is the proper value to be used for n, the coefficient of roughness in Kutter's formula. For exceedingly favorable conditions, as for smooth and perfect pipe laid truly to a uniform grade and thoroughly flushed, a value of .012 may be used. But a value of .013 for n applies well to the usual conditions, and is the value commonly used for pipe sewers. This value will here be used exclusively.

If a value of .013 be substituted for n in Kutter's formula the expression *for the coefficient* c will then have the following form:

$$c = \frac{99.9231 + \frac{.00155}{s}}{.5521 + \left[.299 + \frac{.00002}{s}\right] \times \frac{1}{\sqrt{r}}}. \quad \textbf{(178.)}$$

By again substituting various values for s in the above expression for the value of the coefficient c, the following simplified expressions are obtained:

When $s = .1$,

$$c = \frac{99.9386}{.5521 + \frac{.2992}{\sqrt{r}}}. \quad \textbf{(179.)}$$

When $s = .01$,

$$c = \frac{100.0781}{.5521 + \frac{.301}{\sqrt{r}}}. \quad \textbf{(180.)}$$

When $s = .001$,

$$c = \frac{101.4731}{.5521 + \frac{.319}{\sqrt{r}}}. \quad \textbf{(181.)}$$

When $s = .0005$,

$$c = \frac{103.0231}{.5521 + \frac{.339}{\sqrt{r}}}. \qquad (182.)$$

When $s = .0001$,

$$c = \frac{115.4231}{.5521 + \frac{.500}{\sqrt{r}}}. \qquad (183.)$$

1549. Effect of the Slope Upon the Value of *c*.—By substituting any value of r in each of the above expressions for the value of c, it will be found that for grades not flatter than one in a thousand, that is, for values of s not less than .001, the value of the coefficient c is not materially affected by the value of the slope s. Hence, for pipe sewers having slopes not flatter than .001, the values of c may be obtained with sufficient accuracy from a curve similar to those in the diagram of Fig. 368, Art. **1482**, constructed on *accurate* cross-section paper, using a value of .013 for n, and any value materially greater than .001 for s; a value of s equal to .0025 would be very good to use for this purpose. For any slope much flatter than .001, the value of s should be substituted in formula **178**, and the value of c computed.

When r has a value of about 3.28, the value of c will be the same for any value of s. This would be the hydraulic mean radius of a circular sewer about 13⅛ feet in diameter, when flowing full or half full.

It should be noticed that what is said above does not refer to the effect of s upon the mean velocity v, but only upon the *coefficient* c of mean velocity. (See formula **131**, Art. **1478.**)

1550. Approximate Diameter of the Sewer.—The discharge of a sewer is given by the formula

$$D = v\,a, \qquad (184.)$$

in which a is the area of the cross-section of flow in square feet; v is the velocity in feet per second; and D is the discharge in cubic feet per second.

The fundamental formula for velocity is $v = c\sqrt{rs}$. As a fairly approximate value for pipe sewers, when flowing full, we may write $v = 100\sqrt{rs}$, or, remembering that $r = \frac{1}{4}d$, $v = 100\sqrt{\frac{1}{4}ds} = 50\sqrt{ds}$, d being the interior diameter of the sewer. When the sewer is flowing full, the area of the cross-section of flow is $a = .7854d^2$. By substituting the above values of v and a in formula **184,** and solving for d, we get

$$d = \frac{1}{4.34}\sqrt[5]{\frac{D^2}{s}}. \qquad \textbf{(185.)}$$

In order to provide for contingencies and the possible unusual future increase of population, it is sometimes advisable, in designing small sewers, to make the capacity sufficient to convey the required maximum discharge when flowing *half* full. For this condition, the required diameter may be taken at 1.319 times the diameter d obtained by formula **185.**

1551. Example.—If, for the example explained in Art. **1545**, the slopes be assumed to be .001, what will be, approximately, the required diameter of the sewer?

Solution.—The required discharge is $8\frac{1}{4}$ cubic feet per second. By substituting this value for D and the value assumed for s, in formula **185,** we have $d = \frac{1}{4.34}\sqrt[5]{\frac{(8\frac{1}{4})^2}{.001}} = 2.14$ feet. The solution of this equation by logarithms would be

$$\log d = \frac{2 \times 0.92082 - \bar{3}.00000}{5} - 0.63749 = 0.33084,$$

which is the logarithm of 2.1421 feet. Ans.

EXAMPLES FOR PRACTICE.

1. Assuming a slope of .01 for Example 1 of Art. **1545,** what will be the approximate diameter required for the sewer? Ans. 1.590 feet.

2. Assuming a slope of .0025 for Example 2 of the same article, what will be the approximate diameters required for the sewer?

Ans. { (*a*) 2.640 feet.
(*b*) 2.887 feet.

3. For Example 3 of the same, what will be the approximate diameter required for the sewer, assuming a slope of .005? Ans. 2.049 feet.

4. For Example 6 of the same, what will be the approximate diameter required for the sewer, assuming a slope of .01? Ans. 2.097 feet.

5. For Example 8 of the same, what will be the approximate diameters required for the sewers, assuming a slope of .004 in each case?

Ans. $\begin{cases} (a)\ 0.853 \text{ feet.} \\ (b)\ 1.125 \text{ feet.} \\ (c)\ 1.821 \text{ feet.} \\ (d)\ 2.212 \text{ feet.} \end{cases}$

NOTE.—The large diameters obtained for the above examples are due to the fact that each relates to a main trunk sewer from an entire city or district, as will be understood by reference to Art. **1545.**

1552. A Corrected Value of the Diameter.—The diameter of the sewer obtained by formula **185,** though only approximate, will, in many cases, be sufficiently accurate for practical purposes. This will generally be the case for pipe sewers about two feet, or somewhat less, in diameter, having slopes not flatter than about .001. For other cases, after computing the approximate diameter of the sewer by formula **185,** the corresponding velocity and discharge should be computed; and, if the discharge thus obtained does not agree fairly with the discharge as estimated, the diameter should be recomputed.

Having determined the approximate value of the diameter by formula **185,** the value of the hydraulic mean radius can be easily obtained, and the velocity and discharge computed for this diameter. It will not be necessary, however, to compute the discharge, or even to completely compute the velocity, in order to determine, to some extent, the accuracy of the approximate diameter d, and, at the same time, to largely correct it.

In deriving formula **185,** the coefficient c in the fundamental formula for velocity ($v = c\sqrt{r\,s}$) was assumed to have a value of 100, and the accuracy of the resulting value of the diameter will depend upon the accuracy of this assumption for each particular case. The approximate value and the corrected value of the diameter will be nearly inversely proportional to the square roots of the assumed value and the corrected value of c. Hence, by substituting the value of r ($= \frac{1}{4}\,d$) in formula **178,** or in any one of the

formulas **179** to **183**, and computing the value of c, a corrected value for the diameter may be at once obtained.

If d_0 represents the approximate value of the diameter, and d_1 its corrected value, we may then write

$$d_1 = \frac{10\, d_0}{\sqrt{c_0}}, \qquad \textbf{(186.)}$$

where c_0 is the value of c corresponding to $r = \frac{1}{4}\, d_0$.

1553. Example.—For the example explained in Art. **1551,** what will be the value of the coefficient c_0 and of the diameter d_1, as corrected by formula **186?**

Solution.—A value of 2.1421 feet was obtained for the approximate diameter. Consequently, the approximate value of the hydraulic mean radius will be $\frac{2.1421}{4} = .5355$, the square root of which is .7318. As $s = .001$, formula **181** will apply, giving $c_0 = \dfrac{101.4731}{.5521 + \dfrac{.319}{.7318}} = 102.69$. Ans.

Hence, by formula **186,** we have $d_1 = \dfrac{10 \times 2.1421}{\sqrt{102.69}} = 2.114$ feet. Ans.

EXAMPLES FOR PRACTICE.

1. For Example 1 of Art. **1551,** what will be the value of the coefficient c_0 and the value d_1 of the diameter, as corrected by formula **186?** Ans. $\begin{cases} c_0 = 97.20. \\ d_1 = 1.612. \end{cases}$

2. For (*a*), Example 2 of the same article, what will be the value of c_0 and the value d_1 of the diameter, as corrected by formula **186?** Ans. $\begin{cases} c_0 = 108.10. \\ d_1 = 2.539. \end{cases}$

3. What are the corresponding values for (*b*), Example 2 of the same article? Ans. $\begin{cases} c_0 = 110.06. \\ d_1 = 2.752. \end{cases}$

4. What are the corresponding values for Example 3 of the same article? Ans. $\begin{cases} c_0 = 102.75. \\ d_1 = 2.021. \end{cases}$

5. What will be the corresponding values for Example 4 of the same article? Ans. $\begin{cases} c_0 = 103.41. \\ d_1 = 2.062. \end{cases}$

6. For (*a*), Example 5 of the same article, what will be the corresponding values? Ans. $\begin{cases} c_0 = 82.86. \\ d_1 = 0.937. \end{cases}$

7. For (*b*), Example 5 of the same article, what will be the corresponding values? Ans. $\begin{cases} c_0 = 89.14. \\ d_1 = 1.192. \end{cases}$

1554. The Approximately Correct Diameter The degree of error in the value d_1 of the diameter will be reasonably well indicated by the amount which this value varies from the value d_0. The true value of the diameter will lie *between* the values d_0 and d_1, and nearer to the latter than the former. This fact will often be sufficient for determining the practical diameter of the sewer for the difference between d_0 and d_1 will generally be small.

Where greater accuracy is desired, however, an approximately correct value d' will be given for the diameter by the following formula:

$$d' = \frac{1}{5}(d_0 + 4\,d_1), \qquad \textbf{(187.)}$$

in which d_0 and d_1 have the values given by formulas **185** and **186,** respectively.

The value d' obtained by the above formula, though really only approximate, will be very close to the true theoretical diameter of a circular sewer that will give the required discharge when running full. The slight error in the value d' of the diameter will be indicated by computing the velocity and discharge given by this diameter and comparing the latter with the required discharge.

1555. EXAMPLE.—For the example explained in Art. **1553,** what will be (*a*) the value d' for the diameter, and (*b*) the velocity and discharge for this diameter?

SOLUTION.—(*a*) In Art. **1551,** the value d_0 of the diameter was found to be 2.1421 feet, and in Art. **1553,** the value d_1 of the same was found to be 2.114 feet. Hence, by formula **187,** the value $d' = \frac{1}{5}(2.1421 + 4 \times 2.114) = 2.1195$ feet. Ans.

(*b*) The hydraulic radius for a sewer of this diameter, when flowing full, is $\frac{2.1195}{4} = .5299$, the square root of which is .7279. The sine of the slope is .001, and from formula **181,** we have, for the coefficient of mean velocity, the value $c = \dfrac{101.4731}{.5521 + \dfrac{.319}{.7279}} = 102.47$. Hence, the velocity $v = 102.47 \times \sqrt{.5299 \times .001} = 2.358$ feet per second, and the discharge $D = 2.358 \times .7854 \times 2.1195^2 = 8.32$ cubic feet per second. Ans.

EXAMPLES FOR PRACTICE.

NOTE.—For convenience of reference, the number of the corresponding example in Art. **1545** and Art. **1551** is given in each of the following examples. The required discharge is given in each corresponding example in Art. **1545**; and in each corresponding example in Art. **1551** the sine of the slope is given.

1. For Example 1 of Art. **1553**, what will be the value d' of the diameter as corrected from the values d_0 and d_1? What will be the corresponding velocity v' and discharge D'? [Ex. 1.]

Ans. $\begin{cases} d' = 1.608. \\ v' = 6.179. \\ D' = 12.545. \end{cases}$

2. What will be the corresponding values for Example 2 of the same article? [Ex. 2 (*a*).]

Ans. $\begin{cases} d' = 2.560. \\ v' = 4.296. \\ D' = 22.107. \end{cases}$

3. What will be the corresponding values for Example 3 of the same? [Ex. 2 (*b*).]

Ans. $\begin{cases} d' = 2.779. \\ v' = 4.552. \\ D' = 27.603. \end{cases}$

4. What will be the corresponding values for Example 4 of the same? [Ex. 3.]

Ans. $\begin{cases} d' = 2.026. \\ v' = 5.159. \\ D' = 16.641. \end{cases}$

5. What will be the corresponding values for Example 5 of the same? [Ex. 6, Art. **1545,** Ex. 4, Art. **1551.**]

Ans. $\begin{cases} d' = 2.069. \\ v' = 7.417. \\ D' = 24.948. \end{cases}$

6. What will be the corresponding values for Example 6 of the same? [Ex. 8 (*a*), Art. **1545,** Ex. 5 (*a*), Art. **1551.**]

Ans. $\begin{cases} d' = 0.920. \\ v' = 2.566. \\ D' = 1.7060. \end{cases}$

7. What will be the corresponding values for Example 7 of the same? [Ex. 8 (*b*), Art. **1545,** Ex. 5 (*b*), Art. **1551.**]

Ans. $\begin{cases} d' = 1.179. \\ v' = 3.096. \\ D' = 3.378. \end{cases}$

1556. A Safe Value for the Diameter.—It will be noticed that in each of the examples of the preceding article, the value D' computed for the discharge agrees closely with the value of the required discharge as given in the corresponding example in Art. **1545.** In practical sewer computations, however, such a degree of refinement is not

generally necessary, owing to more or less uncertainty in the amount of the estimated required discharge.

Having computed, by formula **185,** the value d_0 for the diameter, and having computed the coefficient of mean velocity c_0 from this diameter, if the value of the coefficient is found to be greater than 100, then the value d_0 may, with perfect safety, be taken as the required diameter of the sewer. For, if the value of c_0 is greater than 100, the value of d_1 given by formula **186,** and, consequently, the value of d' given by formula **187,** will be less than the value of d_0. In actual practice this will often be as far as it will be necessary to carry the computations for diameter.

1557. The Practical Diameter.—The actual diameter of a pipe sewer must correspond to some size of sewer pipe obtainable in the market, and will generally vary considerably from its computed, or theoretical, diameter. Hence, excessive refinement in computing the theoretical diameter is generally time and labor wasted. The practical diameter of any pipe sewer should be the diameter of commercial sewer pipe nearest to and *not less than* the required theoretical diameter.

In this country, sewer pipe can generally be obtained of the following internal diameters:

Inches,	3	4	5	6	8	9	10	12
Feet,	.25	.3333	.4167	.50	.6667	.75	.8333	1.00
Inches,	15	18	20	22	24	26	28	30
Feet,	1.25	1.50	1.6667	1.8333	2.00	2.1667	2.3333	2.50

It will be well to notice, however, that the preceding sizes can seldom be all obtained from one manufacturer; a diameter of 9 inches is not common.

Sizes of sewer pipe larger than 30 inches in diameter are not commonly made. The sizes manufactured are, however, sufficient for all requirements of the separate system, except the most extreme cases. Sewers of diameter greater than 30 inches, or $2\frac{1}{2}$ feet, are usually constructed of brick. Unless plastered on the inside with cement, as they should be for the separate system, such sewers will require a

greater value for the coefficient of roughness *n* than pipe sewers.

1558. Storm-Water Conduits.—In nearly all small cities, and in many cities of moderate size, the storm water can, without serious inconvenience, be carried in surface gutters or road ditches until it reaches the nearest water-course. In large cities, however, the water, if carried in surface conduits, would necessarily run long distances through the streets before reaching a natural watercourse or body of water. In such cases, underground conduits should be provided for the storm water.

The storm-water conduits of the separate system can, as a rule, be constructed at much less expense than the storm-water sewers of the combined system. As they are for the purpose of surface drainage only, they need not be at great depth below the surface, thus saving an important item in the cost of construction. They are sometimes constructed directly above the sewers proper, although this practice is not to be commended. As they do not convey sewage, they can discharge into the nearest waterway, and, consequently, are not required to be of so large a section as the trunk sewers of the combined system leading to a distant outlet. As they are constructed at a high level, they can almost invariably discharge into a waterway without pumping.

THE COMBINED SYSTEM OF SEWERAGE.

1559. General Considerations.— The combined system of sewerage is more popular and more extensively employed than the separate system, although the latter is the more scientific and strictly sanitary system. Unfortunately, things do not commonly become popular by reason of their scientific merit. The separate system has been employed for the sewerage of many small cities in this country, mainly because for such cities it is the cheapest system.

The combined system is well adapted to the requirements of large and densely populated cities; where no system of sewage purification is employed, it is probably the more

economical system; it is employed almost universally in large cities, in almost all cities of average size, and in many small ones.

1560. The **computations** for the required capacities of sewers of the combined system are based upon the estimated maximum storm-water effluent, as treated in the section on Drainage. The volume of sewage proper is so small in comparison with the maximum volume of storm water given by a severe storm, that the former may be neglected in the computations for the required discharge.

In Example 7 of Art. **1545,** it was found that for a population of 60 per acre, which may be taken as representative of ordinary conditions, the maximum discharge of sewage proper was found to be .01667 of a cubic foot per second per acre. A rainfall of one inch per hour, which is a moderate estimate for a maximum rate, would give a discharge of 0.5 of a cubic foot per second per acre, assuming one-half of the total rainfall to reach the sewer. This storm-water effluent would be equal to $\frac{.5}{.01667} = 30$ times the maximum discharge of sewage. It is thus seen to be safe to base the required capacities of sewers of the combined system wholly upon the estimated storm-water effluent.

CONDITIONS AFFECTING THE SLOPE.

1561. Velocity Necessary to Prevent Deposits.—The slope of a sewer should be steep enough to give a velocity sufficient to prevent the deposit of solid matter. The velocity necessary to prevent deposit will depend upon the size and condition of the sewer and somewhat upon the nature of the sewage. It is generally taken at from two to three feet per second, according to the size of the sewer.

Ordinary domestic sewage will generally flow freely and without deposit, if the velocity is not less than given by the formula

$$v_0 = 2.0 + \frac{6.0}{d}, \qquad \textbf{(188.)}$$

in which v_0 is the velocity in feet per second, and d is the diameter of the sewer in inches.

A sewer of any diameter should have a velocity equal to that given by the above formula, whenever such velocity is attainable, as the depositing of solid matter will thereby be prevented and the satisfactory working of the sewer will be ensured.

1562. Minimum Permissible Velocities.—If sewers are properly constructed and maintained, however, they will work satisfactorily with theoretical velocities slightly less than given by formula **188.** If a sewer is laid to a true grade and is reasonably clean, the tendency to deposit will be less than if it were poorly laid or badly fouled. This is due not only to the greater actual velocity in the former case, but also to the fact that the solid matter that is not lifted entirely free from the bottom will be more easily carried along. Well-constructed sewers will generally prove satisfactory if their theoretical velocities are not less than given by the formula

$$v_0 = 2.0 + \frac{3.0}{d}, \qquad \mathbf{(189.)}$$

in which v_0 and d represent the same values as in formula **188.**

The difference in the corresponding velocities given by formulas **189** and **188** represents about the difference of *theoretical* minimum velocities permissible in thoroughly well constructed and in ordinarily constructed sewers, respectively, the same value of the coefficient of roughness n being used in each case. The difference in the *actual* minimum velocities permissible for these two classes of sewers would probably be less than the difference given by the two formulas.

In a well-constructed sewer, the actual velocity might somewhat exceed its theoretical value, while, in an ordinarily constructed sewer, it would probably not attain it. This is on account of the difficulty of deciding accurately the value of the coefficient of roughness n, used in determining the

theoretical velocity. Unless special means are provided for preventing deposit, the theoretical velocities should never be materially less than given by formula **189.** When not otherwise specially stated, formula **188** will here be used for the minimum velocity to prevent deposit, and formula **189** for the minimum *permissible* velocity.

1563. The Minimum Slope.—For a circular pipe sewer of any diameter, the inclination corresponding to a given velocity, when flowing full or half full, may be determined approximately by the formula

$$s = \frac{5\,v^2}{1{,}000\,d}, \qquad \textbf{(190.)}$$

in which d is the diameter of the sewer, in inches; v is the velocity in the same, in feet per second; and s is the sine of the slope, or fall per foot, expressed in fractions of a foot. If the second term of this formula is multiplied by 100, giving it the form $\frac{v^2}{2\,d}$, then s will represent the fall in feet per *hundred* feet.

The results given by formula **190** are accurate or closely approximate only for cases in which the value of c is about 98. But as this is about an average value for c, the formula can be applied to any case with fairly approximate results.

When the value of c, *as computed from the value of* s *obtained by formula* **190,** *is found to be materially greater or less than 98, the value of* s *may be corrected by multiplying it by the quotient obtained by dividing 9,600 by the square of* c.

The value of s as thus corrected will be near enough for most practical purposes.

It is very desirable that a pipe sewer of any diameter should have a fall not less than determined by formulas **188** and **190,** where such a fall is obtainable. This is especially true in the upper levels of a system, where the volume of sewage is small and quite fluctuating. In the lower levels and mains of the separate system, where the flow of sewage is more constant, the sewers will generally work satisfactorily with slopes as flat as given by formulas **189**

and **190.** But if laid at materially flatter slopes, the sewers will not be satisfactory unless regularly flushed; for the minimum velocities will not be sufficient to prevent deposit, nor the maximum velocities sufficient to remove the deposits occurring during the period of minimum velocities.

1564. Minimum Slope for the Combined System.—It must be noticed that the above formula and remarks relate to the pipe sewers of the separate system, in which the flow of sewage is reasonably uniform. In sewers of the combined system, velocity sufficient to prevent deposit must be maintained during periods of dry weather, when the stream of sewage is very shallow, requiring considerably greater slope. Moreover, the coefficient of friction n is greater for brick sewers, and, consequently, the slope necessary to give the same velocity under corresponding conditions will be greater than for pipe sewers.

The following formula is sometimes used for the minimum slope of sewers of the combined system:

$$s_o = \frac{1}{5\,d + 50}, \qquad \textbf{(191.)}$$

in which s_o is the minimum slope and d is the diameter of the sewer, in inches.

1565. Diminished Flow in Upper Levels.—The above formulas and remarks apply to that portion of a sewer in which the maximum flow of sewage is practically equal to the required capacity of the sewer, that is, to that portion of the sewer in which the maximum discharge is practically equal to the total estimated discharge, from which the capacity of the sewer is determined. This will be that lower portion of each main or lateral sewer through which passes all the sewage that it is designed to convey.

For convenience, this point will here be designated as the **point of maximum discharge.** If the size of the sewer varies, the following remarks will apply to that portion of the sewer, above the point of maximum discharge, that is of uniform cross-section.

From the point of maximum discharge, the volume of sewage will diminish toward the upper end of the sewer. If the population tributary to the sewer above the point of maximum discharge is evenly distributed along the sewer, then the volume of sewage will diminish uniformly, or by a constant arithmetical ratio, from maximum at this point to zero at the extreme upper end. Upon this assumption, the volume Q_x of sewage at any given point between the point of maximum discharge and the upper end will be given by the formula

$$Q_x = \frac{Q_m x}{m}, \qquad \textbf{(192.)}$$

in which Q_m is the discharge at the point of maximum discharge, and x and m are the distances from the upper end to the points where the discharges are Q_x and Q_m, respectively.

If the sewer has a uniform inclination above the point of maximum discharge, the diminished flow of sewage will give a diminished velocity, and, unless the sewer is regularly flushed, stoppages will be likely to occur in the upper levels. This trouble is not uncommon in the higher levels of sewage systems.

1566. Relative Velocities Along Sewer for Uniform Slope.—For obtaining the velocities at points above the point of maximum discharge, it is well to assume the sewer to be flowing half full at the latter point, for which depth the discharge is .5 of the discharge with the sewer flowing full. On this assumption, the *relative* discharge q_x for any point at a distance x below the upper end will be given by the formula

$$q_x = \frac{x}{2m}, \qquad \textbf{(193.)}$$

in which x and m represent the same values as in formula **192.** The value of q_x given by this formula will never exceed .5, and will represent the discharge at the given point as the fractional part of the discharge when the sewer is flowing full, the latter being represented by unity.

1567. Relative Inclinations for Uniform Velocity.—In order to maintain a velocity in the upper portion of the sewer equal to the velocity at the point of maximum discharge, the inclination of the sewer must be very much increased in the upper levels. Indeed, at the extreme upper end of the sewer, the theoretical inclination necessary to give such velocity would be so great as to be impracticable.

If the velocity be assumed uniform, then the area of the cross-section of the flow at any point will be directly proportional to the discharge at that point; the discharge will, of course, be proportional to the distance of the given point below the upper end of the sewer, as shown by formula **192.** If a sewer having any inclination is assumed to be flowing half full at the point of maximum discharge, the inclination s_x of the sewer at a distance x above this point, necessary to induce a uniform velocity, will be given approximately by the formula

$$s_x = \frac{s}{v'^2_x}, \qquad \textbf{(194.)}$$

in which s is the slope of the sewer at the point of maximum discharge, s_x is the slope at the given point necessary to induce a uniform velocity, and v'_x is the *relative* velocity for the same point. The results given by this formula are really only approximately correct, but they are sufficiently close for almost all practical purposes.

1568. Diagram for Relative Inclinations.—If we put $y = \frac{s_x}{s}$, then, from formula **194,** we shall have the equation

$$y = \frac{1}{v'^2_x}. \qquad \textbf{(195.)}$$

By substituting in this equation the values of v_x, corresponding to various values of x, we may construct the curve of this equation, using values of $\frac{x}{m}$ as abscissâs (see formula **192**) and corresponding values of y as ordinates.

The curve is shown in Fig. 375. The ordinates to the curve in this diagram represent the theoretical relative inclinations of the sewer above the point of maximum discharge necessary to give a uniform velocity.

In this diagram, the zero point of abscissas corresponds to the point of maximum discharge, that is, the point for which the required capacity of the sewer is determined, while the

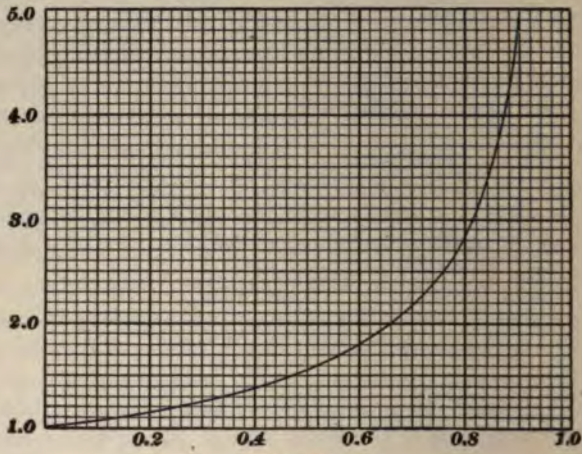


FIG. 375.

abscissa corresponding to the upper end of the sewer is unity. Abscissas intermediate between zero and unity correspond to values of $\frac{x}{m}$ intermediate between $x = o$ and $x = m$.

For the point of maximum discharge, the ordinate is unity, and for any other point it is greater than unity. The portion of the curve for abscissas greater than .9 is omitted, the inclination becoming so great as to be wholly impracticable.

1569. Values of Slope from Diagram.—At the beginning of the previous article, y was assumed equal to $\frac{s_x}{s}$. Hence, at any distance x above the point of maximum discharge, the theoretical slope of the sewer s_x, necessary to

nduce a velocity equal to that at the point of maximum ischarge, will be given approximately by the formula

$$s_x = s\,y. \qquad \textbf{(196.)}$$

It should be stated, however, that it is not customary nor enerally practicable to increase the inclination of sewers n the upper levels sufficiently to give a uniform velocity. 'he available slope is limited by the inclination of the urface and other conditions. Where stoppages occur, lushing is resorted to.

1570. Increased Resistance on Curves.—On ccount of the increased friction due to angular change in lirection, and also on account of the increased roughness of he interior of the conduit, the resistance to flow is considerably augmented on curves. The effect is that of an ncreased value of the coefficient of friction n, and the elocity is correspondingly diminished. Consequently, in rder that the velocity around a curve may be maintained uniform with the velocity on the adjoining straight lines, he inclination should be greater on the curve than on the straight lines. As a rough approximation, the inclination or an ordinary curve should generally be about one and one-half times, and on a sharp curve about two and onequarter times, the inclination on the straight line.

The degree of resistance on a curve will depend upon the adius of the curve and also upon the diameter of the conluit; that is, it will depend somewhat upon the relation of he one to the other. For pipe sewers, the following is a very simple and reasonably satisfactory formula for the *additional* inclination around curves necessary to give a velocity approximately uniform with that of the adjoining straight lines:

$$s_1 = \frac{s}{2}\left(1 + \frac{d}{r}\right), \qquad \textbf{(197.)}$$

n which s is the slope, or fall per foot, on the adjoining straight lines, s_1 is the *additional* slope around the curve, d is the diameter of the sewer, in inches, and r is the radius

of the curve, on center line of sewer, in feet. The total required slope s_o on the curve will, of course, be equal to $s + s_1$.

1571. Example.—For a pipe sewer 15 inches in diameter, what will be (*a*) the minimum velocity that will prevent deposit, and (*b*) the corresponding slope at the point of maximum discharge? (*c*) If the sewer is 4,000 feet long above this point, what will be the slope necessary to induce a uniform velocity at a point 1,850 feet above? (*d*) If a curve of 45 feet radius occurs at the latter point, what will be the slope necessary to give approximately the same velocity on the curve?

Solution.—(*a*) By formula **188,** the minimum velocity that will prevent deposit will be $2.0 + \frac{6}{15} = 2.4$ feet per second. Ans.

(*b*) By formula **190,** the corresponding slope will be $\frac{5 \times 2.4^2}{1,000 \times 15} =$ .00192. Ans.

(*c*) The value $\frac{x}{m}$ will be equal to $\frac{1,850}{4,000} = .4625$. On the diagram of Fig. 375, at the point where the abscissa to the curve has this value, the ordinate has a value of 1.5; hence, by formula **196,** the slope s_x necessary to induce a uniform velocity at the given point is equal to $1.5 \times .00192 = .00288$. Ans.

(*d*) By formula **197,** the additional slope necessary to give approximately the same velocity around a curve of 45 feet radius will be $\frac{.00288}{2} \times (1 + \frac{15}{45}) = .00192$, and, therefore, the total required slope on the curve will be $.00288 + .00192 = .0048$. Ans.

EXAMPLES FOR PRACTICE.

Note.—When the slope involves an extended decimal fraction, the decimal is not, in practice, carried out further than the fifth and, usually, not further than the fourth decimal place, taking the next higher figure in the last place used, if there are figures beyond.

1. What will be the practical diameter d of a pipe sewer that will give the theoretical diameter obtained for Example 1 of Art. **1555?** For this sewer, what will be the maximum velocity v, in feet per second, that will prevent deposit, and also the corresponding slope s at the point of maximum discharge?

$$\text{Ans.} \begin{cases} d = 20 \text{ inches.} \\ v = 2.3 \text{ ft. per sec.} \\ s = .00132. \end{cases}$$

2. In order to induce a uniform velocity in the sewer, what will be the slope s_x required at a point situated at one-fourth the length of the sewer above the point of maximum discharge? If a curve of 50

:t radius occurs at the point so situated, what will be the slope s_c ›und the curve necessary to give, approximately, the same velocity?

Ans. $\begin{cases} s_x = .00159. \\ s_c = .00270. \end{cases}$

3. What will be the practical diameter d of a pipe sewer corre- onding to the theoretical diameter obtained for Example 6, of Art. **555?** For this sewer, what will be the minimum velocity v, in feet :r second, necessary to prevent deposit, and also the corresponding ope s at the point of maximum discharge?

Ans. $\begin{cases} d = 12 \text{ inches.} \\ v = 2.5 \text{ ft. per sec.} \\ s = .00260. \end{cases}$

4. If the sewer is 3,000 feet long above the point of maximum dis- iarge, what will be, approximately, the value of the slope s_x neces- ıry to induce a uniform velocity of 2.5 feet per second at a point 600 :et above? If a curve of 22 feet radius occurs at a point 1,000 feet bove the point of maximum discharge, what will be the slope s_c neces- ıry to give, approximately, the same velocity on the curve?

Ans. $\begin{cases} s_x = .003. \\ s_c = .006. \end{cases}$

MATTERS RELATING TO THE DESIGN AND CONSTRUCTION OF SEWERS.

DIFFERENT PLANS FOR SEWERAGE SYSTEMS.

1572. General Considerations.—The sewers of a own should be laid out in such a manner and according to uch a system as will best conform to the topography of the urface, and fulfil the requirements for efficient sewerage. n most cases, the grades of the sewers should, in a general vay, conform to the slope of the surface. Although this ›ractice can not be rigidly adhered to, yet, if followed as learly as practicable, the minimum amount of excavation vill thereby be attained in the construction of the sewers, ınd they can also be placed at the most satisfactory depths 'or house connections.

In dividing up the territory of a city into sewer districts, ınd laying out the main sewers, therefore, no definite plan :an be rigidly followed, but the direction, position, and rade of the sewer must be such as to best accommodate he surface conditions in each particular case. Certain

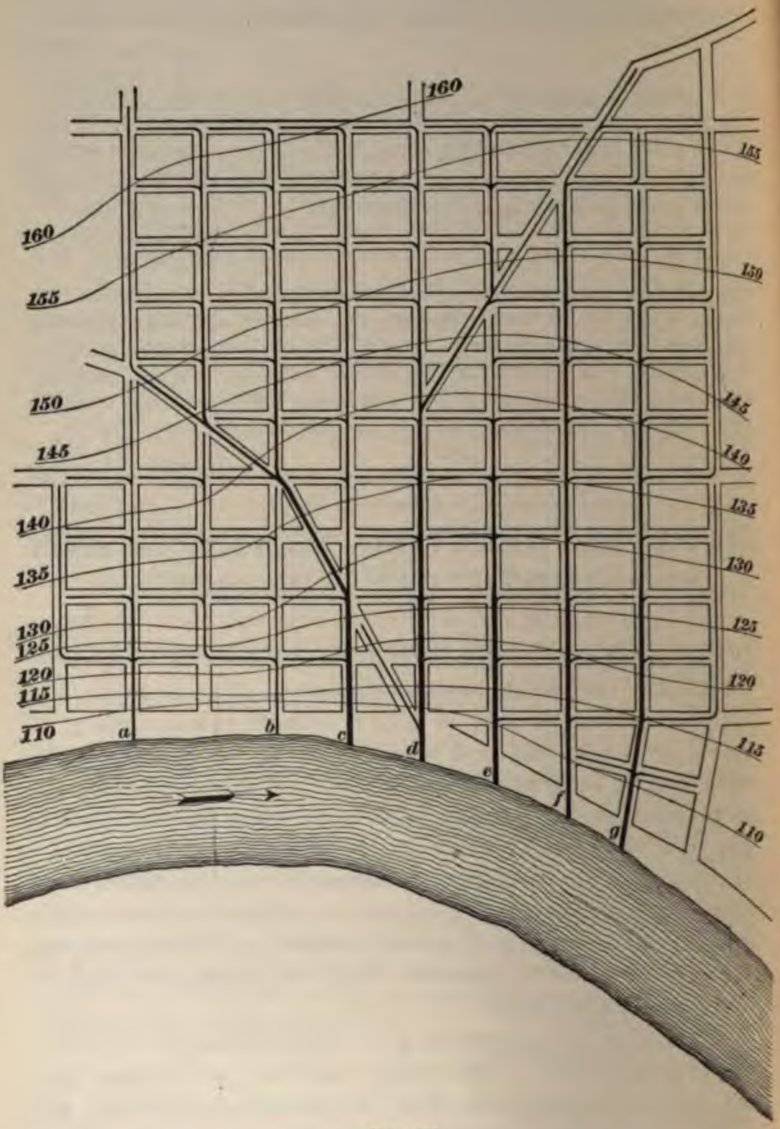


FIG. 376.

ferent methods of dividing the territory and laying out e systems of sewers, however, may be recognized by their neral and characteristic features. The five more promi- nt of these will now be noticed. They will here be desig- ted as the **perpendicular, intercepting, fan, zone, d radial** plans, and may be considered as applicable to her the combined or the separate system.

1573. Perpendicular Plan.—Where a city is bounded one side by a body of water, or is divided by a stream of iter flowing through it, the area is generally divided into number of districts having entirely distinct systems. ich district has its trunk sewer, which has a direction ap- oximately perpendicular to the body of water into which discharges, and may have a number of branch, or lateral, wers discharging into the trunk sewer.

The plan is shown in Fig. 376. It is assumed that the neral surface of the town here shown slopes rapidly wards the river and also gently in the direction of the rrent of the river, as shown by the contour lines. The wn is shown completely sewered by means of trunk sewers nning down nearly every street leading to the river. The stricts tributary to some of the trunk sewers are of con- lerable extent, while those tributary to others are quite iall. The course of the sewer is, in each case, gov- ned by the inclination of the surface and other local nditions. Although towns are not generally sewered as osely and completely as here shown, the figure will serve a fair illustration of a quite complete perpendicular stem.

This plan has the advantage of giving sewers of the short- t length and smallest section possible to any given locality which it is adaptable. It is usually not only the cheap- t, but also the most convenient plan, and is the one nerally adopted by a town before any complete system is signed. The principal disadvantage of the plan is the llution of the stream, or other body of water, within the nits of the town. If the sewers are of the combined

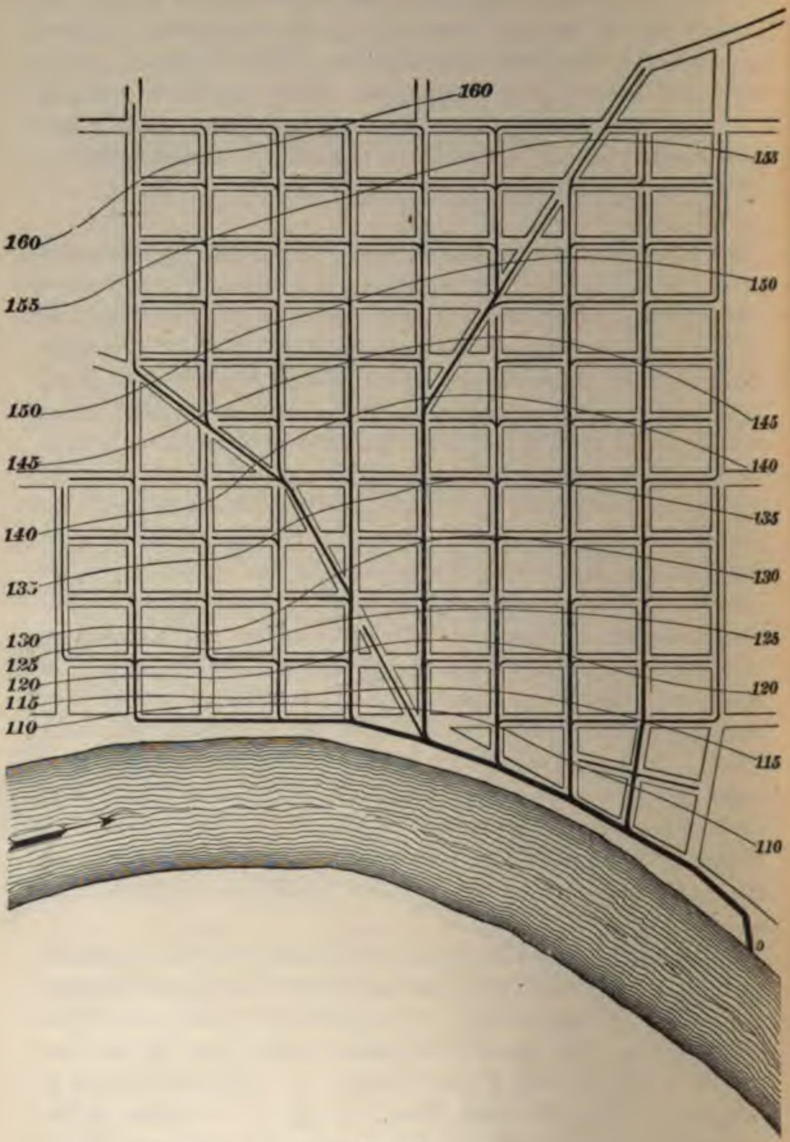

160
160
155
150
145
140
135
130
125
120
115
110
155
150
145
140
135
130
125
120
115
110
0

FIG. 377.

system, however, this plan may, in some cases, also involve the possible overflow of the lower portions of the city from the stream augmented by the storm-water discharged from the upper portions during severe rain storms.

1574. Intercepting Plan.—This plan is similar to the preceding, except that large sewers, called **intercepting sewers,** are constructed along the banks of the river or body of water to intercept the sewage discharged by the sewers of the perpendicular plan and convey it to an outlet below the city, or to suitable filtration beds. This removes both disadvantages of the perpendicular plan mentioned in the preceding article.

After the disadvantages of the cheaper perpendicular plan have been felt, it is often modified by the construction of intercepting sewers. While the intercepting plan is not always the best plan possible to a given district where the entire system can be designed and constructed new, it is almost always the best plan to adopt where portions of the system have been previously constructed, or as a modification of the perpendicular plan. The perpendicular plan shown in Fig. 376 is, in Fig. 377, shown modified to the intercepting plan.

1575. Fan Plan.—In some cases, the sewage from an entire city may be conveyed to a single outlet by means of a number of converging trunk sewers and their various branches. The form of the entire system will approximate the form of a fan, or the skeleton of a leaf. This plan is especially adaptable to a city having a surface contour somewhat of the general form of a basin. In such a case, the district comprising the center of the city is usually much larger than the others. The plan is also adaptable to other forms of surface.

The fan plan is, as a rule, a somewhat more direct system than the intercepting plan and leads to practically the same results. It is the plan most generally available for the average locality, although it has the disadvantage of concentrating the entire volume of sewage in a single outlet

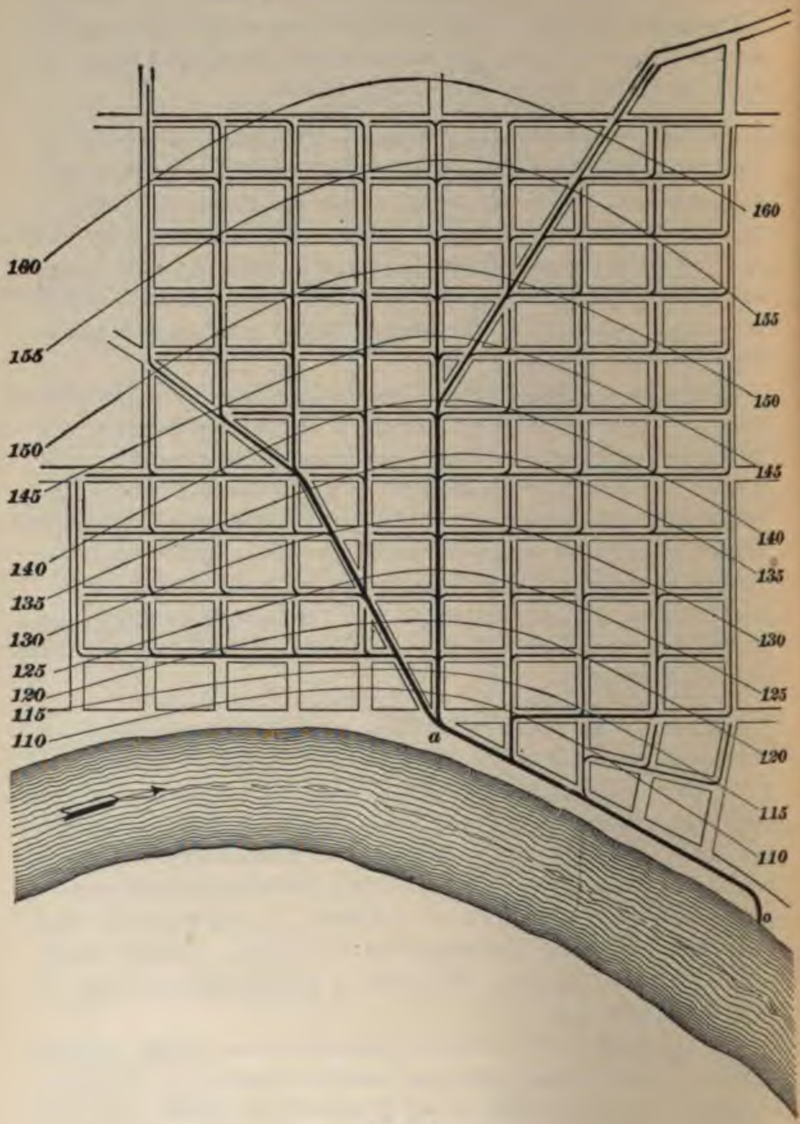

160
155
150
145
140
135
130
125
120
115
110
a
160
155
150
145
140
135
130
125
120
115
110
o

FIG. 378.

sewer, which may occasionally be [illegible] or surcharged, flooding the lower portions of the city. Moreover, as the city grows and the branch sewers are extended, the trunk sewers will need to be enlarged to accommodate the increased volume of sewage.

In Fig. 378, the fan plan is shown applied to the same town to which the perpendicular and intercepting plans are shown applied in Figs. 376 and 377. In this plan, however, the general slope of the surface is assumed to take the form of a half-basin or amphitheater, sloping from every direction toward the point a, as shown by the contour lines.

1576. Zone Plan.—This plan is adaptable to a city in which the surface consists of a series of plateaus; these commonly rise in successive steps, the highest being the one most remote from the watercourse. Each plateau has its own distinct system, which may be either on the fan or intercepting plan. The different systems may be connected for flushing purposes.

A great advantage of this system consists in diverting the water and sewage of the upper plateaus away from the lower portions of the city, thereby obviating the danger of flooding. This also often permits the sewers to be of small section, and affords a very advantageous arrangement when filtration beds are employed.

In Fig. 379, this plan is shown applied to the same town to which the preceding plans have been applied in the three preceding figures. The general form of the surface assumed in this case is shown by the contour lines.

1577. Radial Plan.—In this plan, the city is divided into a number of sectors, corresponding somewhat to sectors of a circle, and the sewage from each sector is carried from the center outwards. The position of the trunk sewer in each sector is approximately that of a radius dividing the sector of a circle. This plan is adaptable to a city whose greatest elevation is in the central portion, and, for a large city having such form of surface, it is, probably, the best possible plan.

One great advantage of this plan is that the sewers in the center of the city are all small, and they become larger only

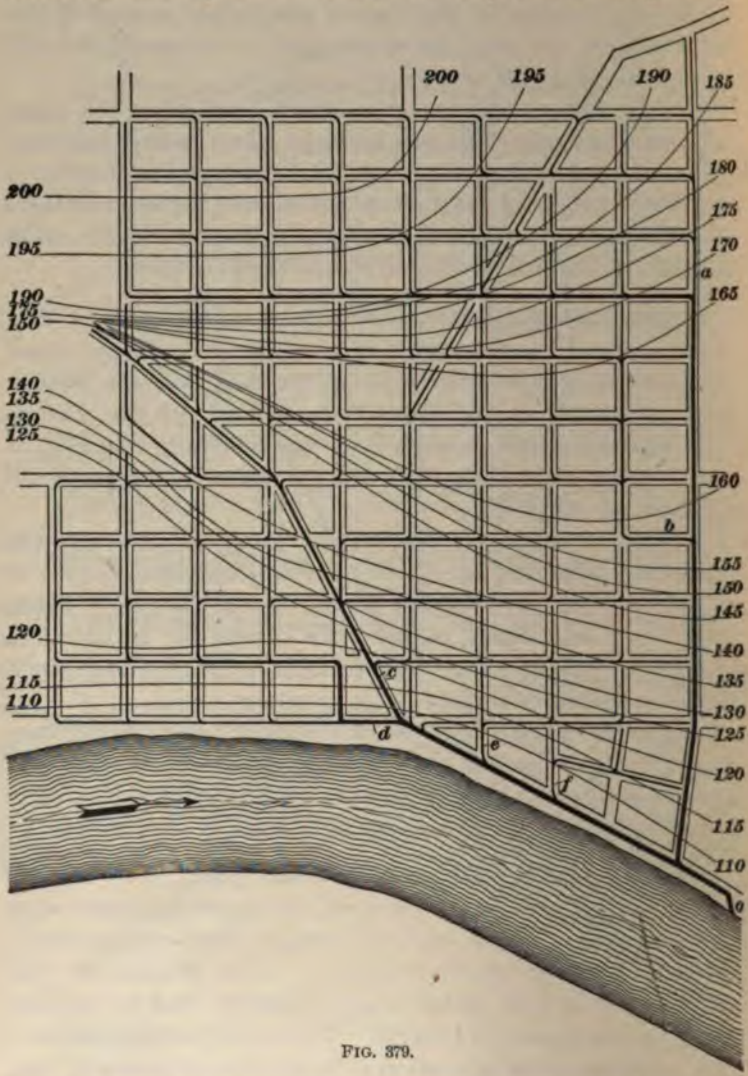


FIG. 379.

as their distances from the center become greater. Moreover, the sewers in the center of the city will be of as great

capacity as will probably be required for at least a very ıg time. As the city grows, it will only be necessary to tend the sewers outwards with enlarged sections. The l sewers, comprising the upper measures in the center of e city, will generally be of sufficient capacity, as there will no important extensions of branch sewers above.

In each of the preceding plans, the trunk sewers must her be designed with a capacity sufficient for future quirements, which it is very difficult to estimate with any asonable degree of accuracy, or must be from time to ne enlarged to provide for the sewage from the new lines tributary branches that must be added as the city grows d new districts are annexed.

If a trunk sewer in one of the preceding systems is signed with a capacity sufficient for the estimated future quirements, it will be much more expensive than required r present purposes, and, moreover, the growth of the city ay even be such as to materially exceed the estimated quirements, necessitating the construction of new sewers. ese new sewers, being trunk sewers, and usually in or near e center of the city, will be expensive in their construcon; for each of the preceding plans involves the conveying more or less sewage from the outskirts directly through e center of the city. The limits of a city may be extended definitely, and the volume of sewage from the outskirts rrespondingly increased.

These difficulties are obviated in the radial plan, which especially advantageous for large cities. The chief difficulty of this plan consists in obtaining suitable outfalls for e sewers, and, for this purpose, pumping must often be sorted to. The system is advantageous for filtration, owever, as the filtration beds may be located at various oints without and around the city, and often in the direct nes of the sewers.

1578. Remarks Concerning the Different Plans. -In a great many cases, the requirements will best be fulled by a combination of two or more of the preceding

plans. In general, it may be stated that the greatest degree of economy is obtained by concentrating the systems as much as possible. A few reasonably large sewers can be constructed more cheaply than a large number of small sewers having the same capacity.

It is, therefore, quite probable that, if all considerations of future requirements are neglected, the fan plan is, for the greater number of cases, the most economical. The intercepting plan can often be made to most economically serve the same purpose as the fan plan in cities which have previously been partly sewered. In cities where the surface somewhat approximates a series of plateaus, the zone plan gives the best results with the least outlay. For large cities having a nearly level topography, or whose surface slopes towards the outskirts, the radial plan is by far the best, all things considered.

PRELIMINARY SURVEYS AND DESIGN.

1579. General Considerations.—In designing a system of sewers for a town or district, all prevailing conditions must be carefully investigated and considered, before deciding upon the plan to be adopted. The conditions and requirements must be carefully studied, and the systems of sewers must be so designed as to meet the requirements and prevailing conditions most effectually and with the greatest economy. Each condition should, so far as possible, be considered wholly upon its merits and in the light of past experience, but not from previously formed opinions.

It must be borne in mind that each small branch, with its house connections, forms a small system that is subject to the same general laws and conditions as the large system of which it forms a part, and must be carefully planned in detail as a distinct system, and, as a whole, must be taken as tributary to, and forming a part of, the larger system.

1580. Preliminary Survey.—Before a definite plan of the sewerage system for a town can be decided upon, a

careful topographical survey must be made. This preliminary survey should include such horizontal and vertical measurements (i. e., measurements of distances and heights) as will enable the engineer to make a reasonably accurate map of the town and of the profiles of the streets.

The lengths and directions of street lines should be measured, and levels should be taken along them. The levels should be taken at stations one hundred feet apart, and also at every street intersection and at every material change in the inclination of the surface. Reference points for the lines and bench marks for the levels should be established at all important street intersections.

If the city is to be sewered with the combined system, or if storm-water conduits are to be provided with the separate system, the survey should also include such outlying territory as may belong to the same natural drainage basin.

The transit and level notes of this preliminary survey should be preserved for reference during the entire construction.

1581. Bench Marks.—Before the levels are taken, a **datum,** or **base-line,** should be chosen at a convenient distance below some permanent and well-defined point in the town. The datum should be lower than the lowest point in the proposed sewers. It is generally satisfactory to assume the elevation of the permanent reference point at 100.00.

The first step in taking the levels should be the establishment of the bench marks. This should done carefully and independently of the surface levels. The levels for the bench marks should always be closed on starting points for a check, and the bench marks should also be checked by a system of cross levels, that is, by a system of level lines carried across the level lines originally run.

The elevations of all bench marks should thus be accurately determined, and the locations and descriptions of the bench marks, with their elevations, recorded in the field

note books. The bench marks should be points suffici permanent for reference during the entire process of struction; the water tables of houses will usually serve purpose satisfactorily.

1582. Preliminary Map and Profiles.—From notes of the preliminary survey, a map of the city or trict should be made. On this may be marked the el tions of all street intersections, and a sufficient numbe other points to show the inclinations of the surface. what is still better, the contour lines may be drawn up throughout the entire area to be sewered, as in the preceding figures.

A careful study of such a map will enable the enginee determine approximately what grades can be given to sewers along the different streets, to select the best a able routes for the main sewers and laterals, and, in a gen way, to plan the entire system. The routes of the se should, where practicable, follow the natural drair channels, as by so doing the best grades will usually be tained. As the route of each sewer is decided upor should be drawn upon the map in pencil, but should be inked in until the entire system is planned; for it often be necessary to make various modifications before entire plan is completed. Even after the entire syste planned, it is almost always necessary or desirable to n some modifications.

Profiles of the streets should also be made, as aid deciding upon the proper grades for the sewers. Upon t profiles the grade lines should be drawn as determined.

1583. Grades.—The systems of main and lat sewers should be so planned as to conduct the sewage tc outfall by the best available route. The most direct r in each case is ordinarily the best route, although, in s cases, where the grades are very steep, it is better to cor the sewage by somewhat circuitous routes, in order to a too high velocities in certain parts of the system. S any retarding of the current will cause deposit of the

the outfall is governed by the elevation of the body of water into which the sewer discharges. The elevation of the outfall and the grade of the trunk sewer being fixed, the elevation of the junction of a branch or lateral sewer may be readily determined, which will also determine the fall available for the branch sewer.

Where one sewer joins another of different size, or where the size of the sewer is changed, considerations of proper

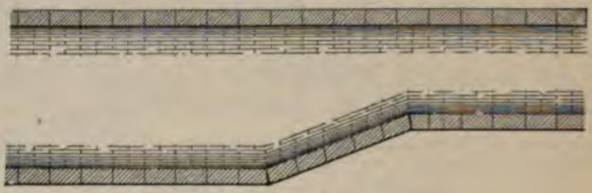


FIG. 380.

ventilation require that the inclination of the sewer shall be continuous along the *crown*, in order not to obstruct the upward passage of the air-currents. Hence, the variation giving the change in size must be made wholly in the *invert*, as indicated in Fig. 380. This will usually cause considerable loss of the available fall, and allowance must be made

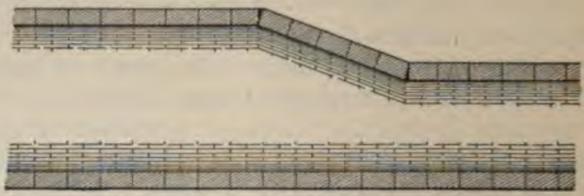


FIG. 381.

for such loss in determining the grades. The loss of the available fall will be not less than the difference between the diameters of the two sizes of sewer. With reference to the cross-section of a sewer, the upper arch is called the **crown**, and the lower or inverted arch is called the **invert**.

It will be noticed that, when the change of size is made in the manner shown in Fig. 380, an unobstructed passage is afforded, not only to the downward flow of the sewage,

much more satisfactorily for sewers that are to convey only sewage proper than for those that are to convey storm water also.

The aggregate yearly discharge of house sewage from built-up residence areas is greater than the entire volume of storm water that ordinarily reaches the sewers during the same period. Yet the sewer capacity required for ample house drainage is about one-fortieth that commonly given to storm-water sewers for like areas. This is due to the fact that the discharge of storm-water sewage may be very excessive for short periods of time and nothing at all for quite long periods, while the discharge of house sewage is reasonably constant. The capacity of the sewer must be sufficient to provide for the maximum rate of discharge.

The maximum rate of discharge must always be carefully considered. This maximum rate is so great for storm-water sewers that it is not considered necessary to provide additional capacity for the domestic sewage in sewers of the combination system. For sewers of the separate system, the required capacity may be materially greater in a manufacturing than in a residence district. This is due to two reasons: First, the daily quantity of sewage from a manufacturing district may be considerably greater than from a residence district of the same size, and, second, the entire daily sewage from the manufacturing district may be discharged during a few hours, while that from the residence district will be distributed through the greater part of the twenty-four hours.

The waste of water is more nearly constant than its legitimate use. Consequently, the greater the percentage of water wasted, the more nearly uniform will be the discharge of sewage and the less will the maximum exceed the average discharge. These and all similar conditions must be carefully investigated and considered, as relating to each particular case, and the required capacities of the sewers must be determined accordingly.

When the required capacities have been determined, the sizes of the sewers necessary to give the required capacities

may be found by computation, as has been fully explained. These computations may be greatly facilitated by diagrams or tables prepared for the purpose. The student can construct for himself, on accurate cross-section paper, diagrams of the values of the coefficient of mean velocity c, or, what will be of still more value, diagrams of the velocity and discharge for different sizes of pipe sewers laid at various grades.

THE CONTRACT.

1587. Advertising for Bids.—When the engineer has finally determined the plan for a system of sewers, including the grades, the sizes required for the different lines, and the details of the accessories, and has determined the depths to be excavated at the different points along the sewers, as well as, approximately, the quantities and general nature of the material to be excavated, it will be next in order to arrange for the construction of the sewers.

Sewers are commonly constructed under contract. The usual procedure is to advertise for bids for constructing the entire work, complete, according to the plans and specifications prepared by the engineer. In some cases, however, the town furnishes the material and lets the contract for the work of construction only. When the work is advertised, a general description of the work to be performed and the approximate quantities (subject to change by the engineer) should be either given in the advertisement or in an estimate which, with the plans, profiles, and specifications, is filed for inspection in some office designated in the advertisement.

All drawings should be carefully made to scale, and full and clear descriptions of the work should be written out, so as to make all conditions plain and easily understood. The specifications should fully and clearly describe the quality of work and material required and the general methods of construction that must be adhered to. Nothing that could affect the cost or quality of the work should be left to be inferred if it can possibly be shown or stated.

Not only should the information given be full, explicit, and unmistakable, but the bids should be called for in such manner that all bidders will bid upon exactly the same basis, to the end that there will be no ambiguity or uncertainty as to who is the lowest bidder when the bids are opened. Printed forms of proposal should be furnished to the bidders, and no bid should be considered that is not written on the blanks furnished, and otherwise in strict conformity with the advertisement.

The bids asked for should state a price per lineal foot for each size of sewer, and a price for each manhole, lamp-hole, flush-tank, catch-basin (for the combined system), or other accessory. The price per lineal foot should generally include the furnishing of all material for the sewer, including Y branches or slants, and may include all excavation and back-filling. Or the prices for excavation and back-filling may be separate, and may be different prices, based upon the depth of the trench, with a separate price for rock excavation. The bids may be upon any system or basis that will best suit the local conditions, but they should be *all upon the same basis.* The basis upon which the bids are received should either be explicitly and fully stated in the advertisement or shown in the blank proposals.

THE FINAL SURVEYS.

1588. Location of Works Previously Constructed.—Before staking out the line of a sewer, preparatory to construction, it will be necessary to ascertain the location of whatever gas, water, and sewer pipes, and conduits of any kind, that may have been previously laid in the street, in order that they may be avoided in the construction of the sewer. This is not always an easy matter. To obtain reliable information concerning the location of these previously constructed works is often very difficult, and sometimes quite impossible, until they are met with in the excavations for the sewer.

Where work of this nature has been constructed without the preparation and preservation of maps or other efficient

s, the subsequent construction of a sewer will gener- ›e attended with very annoying difficulties; pipes, .its, and similar works previously constructed will often countered in the excavation of the sewer trenches in manner as to require material change in the alinements ;rades of the sewers.

map of these previously constructed works should be ned when possible; when not possible to obtain such a a rough map or sketch should be made from such mation as is obtainable.

589. Importance of Record.—The difficulty ex- enced in locating and constructing the sewer among :s of which no record has been made, emphasizes the ›rtance of keeping an accurate record of the exact loca- , laterally and vertically, of every part of a sewerage em. This record should include the position of the :r line with reference to the street lines, the grades of sewer, the elevations of all changes of grade, junctions other important points, the exact location of all catch- ns, flush-tanks, manholes, lamp-holes, and handholes, and positions of all Y branches or slants for house connec- s. It will also be well to keep a record of the position ll gas, water, or other underground conduits encountered ie excavation. This will involve considerable work, and, efore, such methods should be followed in laying out the k and in keeping the construction notes as will be rapid, ırate, and least liable to mistakes.

590. Locating the Lines.—The center line of the er should be carefully located on the ground with a ısit, giving it such a position as to avoid all gas, water, other pipes, so far as their positions can be ascertained, s location should start at the lower end or outfall of the er and proceed upwards along the principal trunk sewer. ; lines for the branch sewers should be similarly located, inning at their junctions with the main sewer. For the pose of checks, and to facilitate the construction of the p, the different lines should be tied together by cross lines

wherever convenient. The position of each line thus run, with reference to the street lines, and all distances along the line, should be measured with a steel tape. All measurements and notes should refer to the center line of the sewer as thus run, but as this line will be within the limits of the excavation for the trench, it can not be preserved and should not be marked by stakes.

Stakes should be set, however, on an offset line at a uniform distance to the right or left of this center line. In order to avoid confusion, the offset line should, if possible, be always on the same side of the center line.

The stakes should generally be about one inch square, with well-squared tops; they should be of such lengths as to be driven flush with the surface of the street without destroying the form of their tops. Where extreme accuracy is required, the point of exact measurement can be indicated by a tack in the top of the stake, but such exactness will seldom be required. The stakes should be set at uniform distances along the offset line; twenty-five feet is a good distance between stakes. In running the center line, it will be necessary to set a few temporary stakes for transit hubs and, possibly, for the purposes of measurement, but no stakes should be left permanently on the center line, as it might lead to confusion in the construction.

1591. Curves.—Where any material change occurs in the direction of the line, it should be made by means of a curve; a circular curve is generally most convenient. In order to avoid as much as possible the additional resistance and consequent loss of head on curves, their radii should be as great as the conditions will permit, as may be readily shown by applying formula **197.** Where the sewer turns from one street into another at right angles, however, as is very commonly the case, the radius of the curve will necessarily be short, probably seldom exceeding 50 feet.

When the change in direction is considerable, the curve may be run in with a transit, by chord deflections, in chords of 25 feet. Intermediate points on the curve can be

located by ordinates from the chord by stretching a tape between the stations located by the transit. This is the best method to employ where the curve is of a comparatively long radius.

Where the curve is of short radius, which will commonly be the case, it can generally be most expeditiously located by running the tangents to an intersection; then, with the tape line, locating the center of the arc and describing the arc from this center, in substantially the same manner that the curve would be laid out on paper with the compasses. This is a simple, accurate, and expeditious method for curves of short radius.

Where the change in direction is slight, simply an angle may be made in the line, and the laying out of the curve omitted until the actual construction. The curve can then be readily located by offsets from the point of intersection and points on the adjacent tangents, the offsets having been previously calculated in the office.

1592. Transit Notes.—Sufficient notes should be made of all the field work to preserve a record of the location of all important points, and such other information as may be of value. In keeping the notes, the starting point of the line, at the outfall, should be recorded as *Station O*, and all measurements along the line should be recorded by stations 100 feet apart, numbered successively, with *pluses* to intermediate points.

The notes should give the position of the sewer line with reference to the street lines, and the position of the offset line with reference to the sewer line. The points where both lines of each street crossed intersect the sewer line should be noted by the stations and pluses on the latter line, and, in most cases, the offset distances to the street lines at such points should be also given. Offset distances to the street lines should also be given for all points where angles occur in either the street lines or the sewer line.

The distances from the sewer line to all buildings, on either side, that stand back from the street lines should be

indicated approximately; these can generally be judged by the eye with sufficient accuracy. Notes should also be made of the crossing of all streams, of the character of the surface, and of all other matters necessary to completely indicate the physical characteristics.

All measurements should be recorded to the *sewer line* and *not* to the offset line.

1593. Reference Points.—The position of the sewer line, as located, should be so fixed by measurements to permanent objects that its exact position may, at any subsequent time, be readily determined. This is generally best accomplished by observing the points where the line of the sewer is intersected by the prolonged lines of the sides of buildings and other well-defined lines of permanent objects, and also measuring along the prolonged line the distance from the nearest corner of each object to the center line of the sewer.

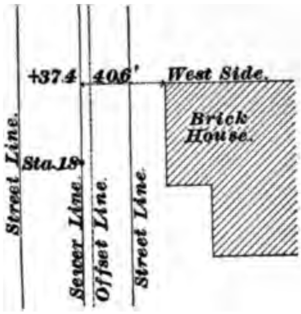


FIG. 382.

The buildings selected should be of a permanent character, such as brick buildings, and the measurement of both the offset and the plus should be taken to the nearest tenth of a foot. This method of reference is clearly shown in Fig. 382. As there shown, the line of the west side of the brick house, prolonged to the sewer line, intersects the latter line at Station 18 + 37.4, and the center line of the sewer is 40.6 feet from the southwest corner of the house.

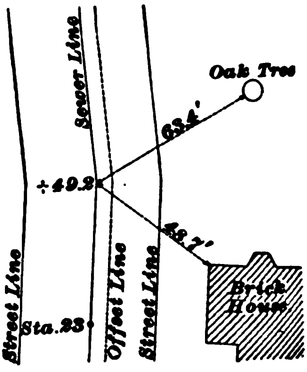


FIG. 383.

The same method can be easily employed to locate the points where the sewer line is intersected

by street lines, the prolongation of the street line being made by simply placing the eye in range with the fence. Each point where an angle occurs in the line of the sewer should also be located by reference points. As the angle will often occur at a point where there is no line of a permanent object in range, the location of the point will generally be best fixed by measurement to two or more permanent objects, as shown in Fig. 383. This method of reference, however, is not as satisfactory as that shown in Fig. 382.

1594. Leveling and Level Notes.—When the sewer lines have been finally located, the levels should be taken over the lines of all the sewers. This can be most expeditiously done by a second party following the transit party. The elevations of the surface should be taken along the center line of the sewer, at all full stations, street intersections, and all material changes in the inclination of the surface. The positions of the true line of the sewer can be readily obtained from the stakes set on the offset line, by measuring the offset distance with a leveling rod, or, after a little practice, simply by the eye. Also, if the location of about every other station is known or found, the intermediate stations and pluses can be located with sufficient accuracy by pacing. As a man ordinarily walks, he will take from about thirty-six to forty steps per hundred feet, depending somewhat on the character and slope of the surface.

The levels should be carefully checked on each bench mark; this will not only check the levels, but will also serve as an additional check upon the bench marks. All important conditions relating to the topography of the surface should be carefully noted. Where the surface is found to slope sharply transversely to the street, or where the buildings on one side of the street are considerably elevated above or depressed below the street level, the fact should be noted, together with the depths of basements, and all other matters which may have a bearing upon the efficient sewerage of each building. The elevation of the surface of the

water in all streams should be taken, and, in order to ascertain the approximate level of the ground water, the height of the water surface in wells along both sides of the street should be observed, when any are to be found.

1595. Working Map and Profiles.—When all the surveys are completed and the plan of the sewer system definitely decided upon, a working map of the system should be made, showing the location of all proposed main and lateral sewers and all catch-basins, manholes, lamp-holes, flush-tanks, and other accessories. This map is for convenient reference during the construction and need not be at all elaborate or finished. It can often be made from the preliminary map by simply making the necessary alterations and additions in red ink. If the changes are too numerous and far reaching to permit this, a tracing of the streets may be made from the preliminary map, on which the sewer system may be drawn as finally planned.

Working profiles of the several lines should also be made. These should be to a rather large scale, and for each sewer should show the surface line and grade line, together with all intersecting streams and such other important matters as can be conveniently and concisely shown. A scale of 4 feet to the inch, vertically, and 20 feet to the inch, horizontally, will be found suitable for such profiles. Much annoyance will be avoided, and the work will be greatly facilitated by furnishing the contractor with a duplicate copy of these profiles. In many cases the preliminary profiles can be so so modified as to serve as construction profiles also; in some cases, very little, if any, modification will be required.

In cases where the physical character of the territory is such that the best routes for the sewers will be easily apparent upon inspection, so that the problem of planning the system is not complicated, one survey may be made to serve all the purposes of the preliminary and final surveys. The same will be true of the map and profiles. This will permit a considerable saving in cost and should be the method followed where practicable. It should not be pursued, how-

ever, in cases where it would result in *insufficient* surveys, as this would be found to be very poor economy in the end. When deciding in regard to the surveys, the engineer must be governed by the local conditions attending each case, deciding upon the basis of common sense and his own past experience.

SEWER CONSTRUCTION.

LAYING OUT THE WORK.

1596. General Considerations.—Generally, it is best to commence the construction at the lower end of the sewer and work towards the higher levels. This will permit the ground water to flow away through the constructed portion of the sewer, which will keep the trench free from water and is not objectionable when the volume of water is not so great or the current so swift as to injuriously wash the cement before it has set. Pipe sewers should always be laid with the socket ends of the pipe towards the summit, and, when the work is begun at the lower end and proceeds upwards, the spigot of each pipe is easily inserted in the socket of the pipe previously laid.

In some cases, however, where the ground water is encountered in such quantities as to render the construction difficult, the work may be prosecuted more advantageously by working downwards, or towards the outlet. This will permit the water to be drained away from the sewer into the lower levels of the trench and then pumped out. This will keep the sewer, where being constructed, comparatively dry, which is in all cases desirable and very essential when the excavation is in certain kinds of material.

A careful record of all matters pertaining to the location of the sewer, of all junctions, and accessories should be kept; and a complete record of everything pertaining to the progress of the work will be of great value for future reference.

1597. Lines and Grade Stakes.—It is important that the sewer be constructed accurately to the grade line

determined upon, in order that the velocities in the various part of the sewer may approximate the computed velocities. Also, in order that the sewer may be in the exact position shown on the map, it should be constructed truly along the center line as surveyed, unless it is necessary to deviate from this line on account of water pipes or other obstructions encountered in the excavation of the trenches, in which case complete notes of the deviations should be made.

The line of the sewer may be determined by measuring horizontally the offset distance from the stakes set on the offset line and dropping a plumb line from the point thus determined. The position of the grade line is sometimes determined by leveling over, with an ordinary mason's level and straight-edge, from the top of each stake set in the offset line, the elevation of which had been previously taken by an engineer's level, to a point about over the middle of the trench and then measuring down a distance equal to the difference between the elevations of the stake and the grade line. This practice is not to be commended, however, and should neither be followed nor permitted. The method described below will give much better results, and is probably the most satisfactory practice that can be followed.

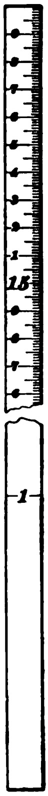


FIG. 384.

The line and grade should be indicated by stakes set in the bottom of the trench, their tops being to the exact line and grade. With reference to the sewer, the grade line represents the lowest line along the *interior* of the sewer invert or the bottom of the stream of sewage. The grade stakes should preferably be set to this line, but may be set the thickness of the invert below it. These grade stakes should be set opposite all stakes in the offset line and in advance of the final shaping of the bottom of the trench. Each stake should be set in line by measuring the offset distance from the stake in the offset line, on a rod laid horizontally across the trench, and plumbing down to the bottom of the trench with a plumb-

bob. The stake should then be set to grade by means of a self-reading rod held upon it, the rod being long enough to be read directly from the leveling instrument. The height of the instrument should be carefully checked at each bench mark, and, it is needless to state, the rod should be held truly vertical.

The rod for this purpose may be made out of a piece of well-seasoned pine; sixteen feet is generally a convenient length for it. The face of the rod should be painted white, and laid off in feet, tenths, and hundredths, as shown in Fig. 384. The graduations and figures should be in black, except the figures for feet, which should be in red. It will not generally be necessary to lay off the lower portion of the rod, say for five or six feet, except in feet.

CONSTRUCTING THE SEWERS.

1598. Brick Sewers.—For the construction of the sewer, the ground should be excavated in open trenches, except in cases where the depth is so great as to make tunneling necessary or advisable. The center line of the trench is, of course, given by measuring the offset distance from the stakes set on the offset line. The trench should be, at the bottom, at least one foot wider than the exterior diameter of the sewer to be constructed within it, and the bottom should so shaped as to conform to the grade and to the form of the sewer, so that the entire exterior surface of the lower half of the sewer may have an even bearing throughout. In the trench thus formed, cement mortar should be spread to a thickness of not less than one inch, upon which to lay the brickwork of the invert; the mortar should be spread only as the brickwork is laid. The bricks of the invert should be laid with their edge, or thinnest side, upon the mortar thus spread in the trench. It is very important that the bricks should be thoroughly wet before laying; otherwise, they will absorb the moisture from the mortar so that it will not set properly.

The upper arch of the sewer should be laid upon timber

centers or templates which, for the straight sewer, should not be less than ten feet long. Branch pipes, commonly called **slants**, or **branches**, forming openings into the sewer for house connections, should be built into each side of the upper arch of the sewer just above the springing line, or line along which the upper arch and invert join. These branches should generally be located about twenty-five feet apart along each side of the sewer. Their ends should be closed by caps or stoppers made especially for the purpose, and a record of their exact positions should be kept. The upper half of the sewer should be covered with a coating of cement mortar not less than one-quarter of an inch thick.

The sewer, as thus constructed, should be well backed in by carefully ramming and packing the earth, free from hard lumps or stones, under and around the sewer. The material filled in above the sewer is called **back-filling**. The back-filling should be in layers and thoroughly rammed or wetted. More or less controversy is always likely to arise with regard to the amount of ramming or packing necessary for the back-filling. The question may usually be settled by requiring that all material from the trench shall be put back into it. To do this will ordinarily require a sufficient amount of ramming, while less will not be sufficient.

1599. Required Thickness of Brickwork.—The most common size of ordinary brick is about $8\frac{1}{4} \times 4 \times 2\frac{1}{4}$ inches. The brickwork of sewers is constructed in **rings**, or courses. The bricks are laid with their longest dimension lengthwise of the sewer and their edges towards the center, so that the intrados of the arch is formed by the edges of the bricks, while their widths extend radially outwards. The thickness of each ring is, therefore, equal to the width of a brick, or about four inches. Hence, the thickness of the brickwork can not correspond very closely to any theoretical thickness, and, in practice, is most conveniently expressed simply by the number of rings required.

The number of rings necessary for a sewer will depend principally upon its diameter, but also upon its depth.

Where the conditions are not unusual, the following empirical formula will be found satisfactory for indicating the number of rings required:

$$R = .4 + \frac{D(H - D)}{25}, \qquad \textbf{(199.)}$$

in which D is the internal diameter of a circular sewer, or the internal horizontal diameter of an egg-shaped sewer, and H is the total depth of the trench, both in feet, while R is the number of rings, or courses, required. Any fraction greater than 0.25 in the value of R should be considered as one.

1600. Number of Bricks Required.—The number of bricks required can only be determined approximately, owing to the fact that the sizes of the bricks, as well as the thickness of the mortar joint, will vary somewhat. The formulas here given are probably as satisfactory as any that can be proposed. They are based upon a size of brick $8\frac{1}{4} \times 4 \times 2\frac{1}{4}$ inches, laid with $\frac{1}{4}$-inch joints on inner face and $\frac{3}{8}$-inch joints between the different rings, or courses.

For a circular sewer, the number of bricks n necessary to go around the first, or inner, ring will be given approximately by the formula

$$n = 1.26\, d, \qquad \textbf{(200.)}$$

in which d is the internal diameter of the sewer, in inches.

For each additional ring, increase the result by 11.0, 22.0, etc. Any fraction in the final result greater than 0.5 should be taken as a one, and any smaller fraction may be dropped.

For an egg-shaped sewer of either the old or new form, the number of bricks n necessary to go around the inner ring will be given approximately by the formula

$$n = 1.58\, d, \qquad \textbf{(201.)}$$

in which d is the internal horizontal diameter of the sewer, in inches.

For each additional ring, increase the result by 13.8, 27.6, etc.

Having obtained the number of bricks necessary to go around the sewer, that is, in a section of the sewer having a length equal to the length of one brick, the total number of bricks in the sewer will be given approximately by the formula

$$N = 1.41\, n\, l, \qquad (202.)$$

in which N is the total number of bricks, n is the number necessary to go around the sewer, as obtained by either of the two preceding formulas, and l is the length of the sewer, in feet.

1601. Pipe Sewers.—For pipe sewers, the trench can be excavated as deep as about the center of the pipe by common laborers, below which depth it should be shaped to receive the pipe by men trained to the work. It should be so shaped that the pipe will be supported entirely upon its cylindrical part (where it is of uniform cross-section), a recess being formed to receive the socket and cement joint for each length of pipe, which recess should be afterwards filled with well-packed sand. The center line of the trench and its width are determined the same as for brick sewers.

The pipe should be carefully laid with socket end towards the summit; this should be done by one trained man, who should have a helper when laying large pipe. The joints should also be cemented by one man, who should be well trained to the work. The earth should then be carefully packed around the pipe, previous to the back-filling, which is done substantially the same as for brick sewers; it should be packed with especial care around all Y branches, the ends of which should be temporarily closed by special caps. The Y branches should generally be located about twenty-five feet apart along both sides of the sewer, and a record of their exact positions should be carefully kept.

1602. Bracing and Sheet Piling.—In most cases, where the depth is not great, the sides of the trench will stand without protection. In some soils, however, there will be great tendency to caving, and it will be necessary to protect the sides of the trench from caving by means of

ıberwork and braces. This is a matter to be looked after
ıncipally by the contractor, as he takes the responsibility

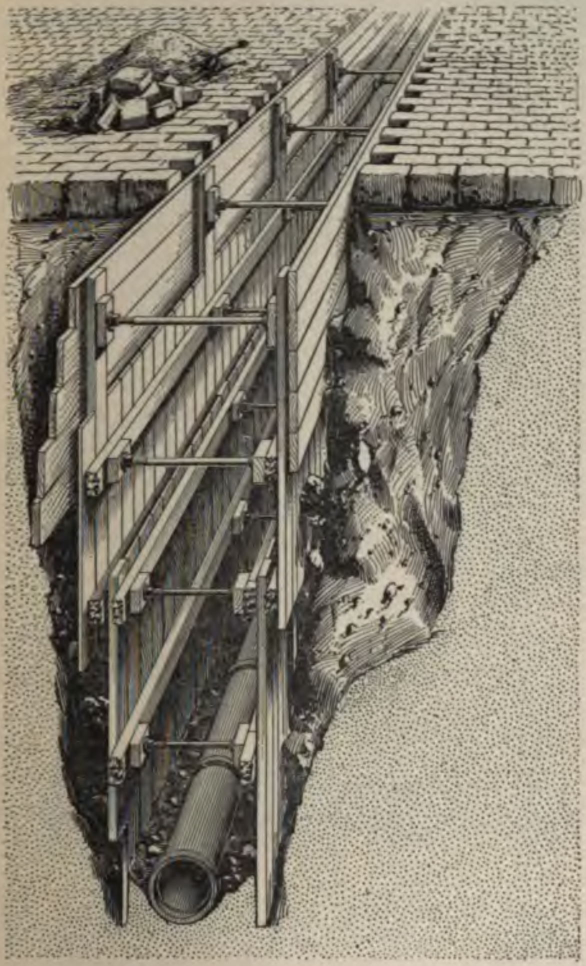


FIG. 385.

all the risks of the undertaking. It is, nevertheless, a
ter over which the engineer should keep a general over-
ıt, as the lives of the workmen may be endangered, and

the work greatly delayed, by accidents due to lack of, or insufficient, protection to the banks of the trench during the construction of the sewer.

The banks of sewer trenches are commonly protected by means of a temporary framing of planks and timbers, known as **sheet piling.** Sheet piling consists, essentially, of a row of planks, having their lower ends sharpened, driven vertically along each bank; these are braced by means of braces extending across the trench. If the trench is deep, the planks which sustain the bank can be placed horizontally with advantage for about the upper four feet of the trench, then driven vertically for the portion below. This is quite plainly shown in Fig. 385.

The horizontal planks may be in the ordinary marketable lengths, sixteen feet being commonly a convenient length. These planks should be two inches thick. A length of about seven feet is to be preferred for the vertical planks or **piles**, which may be one inch thick, if sufficiently supported. One row of such piling, in connection with the horizontal planking, will be sufficient for a depth of eight or nine feet. For greater depths, two or more rows of piling must be driven, each row being on the inner side of the next row above, as shown in Fig. 385. The planking and piling is held in position by horizontal and vertical timbers, usually of about three by six inches cross-section, which are, in turn, held in place by the cross braces.

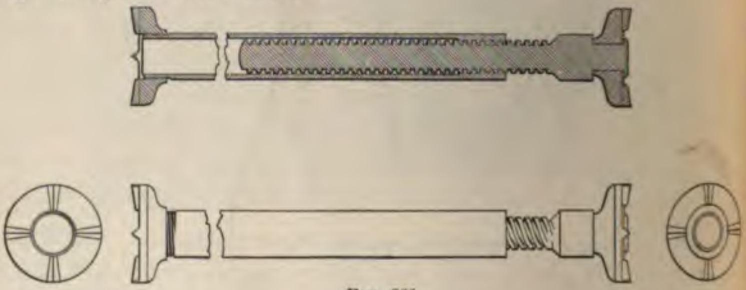


FIG. 386.

The cross braces used are sometimes timber shores. These, however, must be cut to length and driven into

. They often become loose, when they must be wedged ;placed by longer shores, while shores used once can m be used a second time. Much better cross braces fforded by the iron screws shown in Fig. 385, and also n in detail in Fig. 386. These screws can be used any ver of times, can be adjusted to fit any width of trench n reasonable limits, quickly put in place, and removed ghtened without jarring. Considerable experience is red to put in place and remove sheet piling quickly and out damage to the material.

103. **Artificial Foundations.**—It sometimes hap- that the material encountered in excavating sewer hes is so unstable that special expedients must be re- d to in order to support the sewer during the construc- and until the back-filling can be tamped around it. n the material encountered is very treacherous quick- it will generally be necessary to support the sewer on or timbers, while in other and less treacherous soils it be sufficient to excavate somewhat below the grade ınd fill in to grade with gravel. A tile underdrain laid ath the sewer is very advantageous in quicksand and r-bearing strata.

304. **Underdrains.**—Sewers should, in most cases, constructed as to be practically impervious to water. should neither allow the sewage to escape into the ounding subsoil nor admit subsoil water into the sewer. e this condition is seldom, if ever, attained perfectly in :ice, considerations of good sanitation require that it be attained as nearly as practicable. Consequently, ı the sewer is constructed in water-bearing strata, it be very desirable that a tile drain be laid below the r, either directly beneath it or somewhat to one side of rench. This will also be found very advantageous for emoval of subsoil water during the construction of the r. It may be constructed of ordinary drain tile, two, :, or four inches in diameter, according to the amount bsoil water.

MATERIAL AND INSPECTION.

1605. Materials Used for Sewers.—For sewers having diameters not greater than thirty inches, salt-glazed, vitrified earthenware pipe is the best material thus far produced. This pipe forms a smooth, impervious conduit which is not affected by the sewage; it is indestructible except by breakage, and is, at the same time, very strong. It is manufactured in sizes of from two to thirty inches internal diameter (see Art. **1557**), and the pieces or sections are usually two, two and one-half, or three feet long. These pieces are made sometimes with a **bell,** or **socket,** upon one end into which the **spigot** end of the adjacent piece is inserted, and sometimes as simple cylinders with separate collars for making the joints. The socket pipe is generally preferred. For standard sewer pipe, the thickness is made about one-sixteenth the diameter. The pipe is made not only as straight pipe, but also in the form of **Y** branches, tees, curves, and other forms convenient for special purposes. In laying, all joints of the pipe should be cemented with hydraulic cement.

For sewers having diameters greater than thirty inches, brick is the material commonly used. The sewers are constructed by laying to a suitable form one or more rings of brick in hydraulic cement mortar.

1606. Quality of Sewer Pipe.—The pipes used should be of first quality vitrified, salt-glazed sewer pipe. They should be perfectly sound, well burned throughout their thickness, impervious to moisture, with smooth and well-glazed surfaces and free from cracks, flaws, blisters, or fire checks. They should be truly circular and of true form longitudinally, whether straight or curved, of the exact specified internal diameter, and uniformly of standard thickness. They should preferably be socket pipe, and generally two and one-half feet long.

1607. Quality of Brick.—The bricks used in the construction of a sewer should be the best quality of whole bricks; they should be made from good clay, should be

smooth, well formed, strong, and burned hard entirely through. Bricks are sometimes tested for transverse and crushing strength, abrasion, and absorption. For sewer purposes, however, all these tests will seldom be necessary.

The hardness of bricks will be sufficiently indicated by the absorption test and by their sound when struck together. Two bricks that have been made from good material and have been well burned will have a firm, metallic ring when struck together, whereas two inferior bricks will have a dull, heavy sound. This is one of the best indications of the quality of bricks. The capacity of a brick to absorb water is inversely as its density and hardness; hence, the absorption test will indicate the hardness of the brick. While the absorbing capacity of bricks will vary considerably, it may be stated that a well-burned brick should not absorb more than about six per cent. of its own weight of water.

When broken, the bricks should show a uniform texture and thorough burning throughout. They should be well formed and straight; if badly warped, this will indicate a lack of uniformity, either in the composition or burning. The color of bricks is no reliable indication of their quality; bricks of a deep brownish red are generally preferred, however, and salmon-colored bricks are looked upon with suspicion.

By its hardness as indicated by its sound, by its imperviousness and density as indicated by its absorption and weight, by its texture when broken, and by its freedom from warps, the quality of a brick may be quite accurately judged.

The invert of a sewer should properly be constructed of the best vitrified brick, although this is not commonly done.

1608. Quality of the Cement.—The cement used should be an excellent quality of hydraulic cement; it may be either Portland or natural cement, as most suited to

each particular case. As a rule, the best American Portland cement will be found the most satisfactory. In cases where it is desirable to use a quick-setting cement, however, certain brands of natural cement will be satisfactory. The cement should be fresh, finely ground, and heavy.

Portland cement should have a dull greenish-gray color; any material variation from this color may be considered to indicate the presence of some impurity. An excess of lime, iron, or clay will be indicated, respectively, by blue, dark green, or brown colors, while a yellowish shade will indicate under-burned material. Almost all natural cements are of a brownish gray and are light or dark, according to the nature of the rock from which they are made. The color of natural cement is no indication of its quality, although the nearer it approaches to the color of Portland cement, the better its quality will generally be.

The degree of fineness of cement will, to some extent, be indicated by feeling; when rubbed between the fingers, it should feel smooth and soapy, not granular. Cement should be so finely ground that not less than 98 per cent., or, preferably, not less than 99 per cent., will pass through a sieve having 2,500 meshes to the square inch (No. 50); and that not less than 80 per cent., or, preferably, not less than 90 per cent., will pass through a sieve having 10,000 meshes to the square inch (No. 100).

The tensile strength of cement, in pounds per square inch, should *never* be less than the minimum given in table of Tensile Strength of Cements (Tables and Formulas). The requirements here given should be easily filled by good ordinary cements. The tensile strength of cements of good quality will seldom be as low as these minimum values. The figures given in parentheses may be taken as a reasonable and moderate standard for the minimum values of good cement. The maximum values given indicate the general range of the tensile strength, which varies greatly.

1609. Quality of the Sand.—The cement should be mixed in the proper proportion with clean, sharp sand. Too great importance can not be placed upon the quality and condition of the sand used for cement mortar. It should be of a silicious nature, sharp or angular, rather coarse, and free from all loamy or clayey material. Great stress is placed upon the statement that the sand should be *clean*, and to attain this end, it should be washed if necessary. It should, of course, be sifted. For this purpose, two sieves should be used, one sieve to take out the larger stones and gravel and one to take out the very fine sand and dirt.

The mortar should be composed of one part cement mixed with two parts sand when natural cement is used, and of one part cement mixed with three parts sand when the best Portland cement is used. No mortar should be used after it has begun to set.

1610. Inspection of Material.—All pipe, brick, and other material for sewers should be inspected as fast as delivered upon the ground, and all rejected material should be plainly marked with an indelible substance, such as paint, and required to be *immediately removed* from the work. All pipe that is broken, cracked, warped, or in any way imperfect should be rejected, as well as all broken, poorly burned, soft, or in any way imperfect brick.

The engineer should require samples of the cement which it is proposed to use to be submitted to him in time at least to make the twenty-four-hour and the seven-day tests before the work is begun. As the work progresses and the material is being used in the construction of the sewer, it should be carefully scrutinized, and any material found to be defective should be rejected and required to be removed, even if the defects escaped notice in the former inspection. The engineer should never mark material as *accepted*; subsequent and more careful inspection may detect defects not noticed in the first inspection.

DETAILS AND ACCESSORIES.

1611. Y Branches. — House connections are made to pipe sewers by means of a special detail called a **Y branch.** This detail is shown in Fig. 387 by top view and side elevation. It consists essentially of a length or cylinder of sewer pipe intersected by a cylinder of smaller diameter, the angle of intersection towards the upper, or socket, end being about thirty degrees. The axes of the

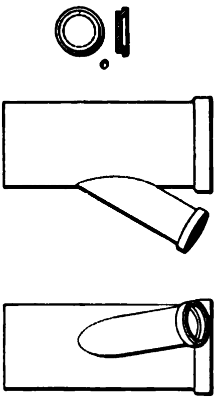


FIG. 387.

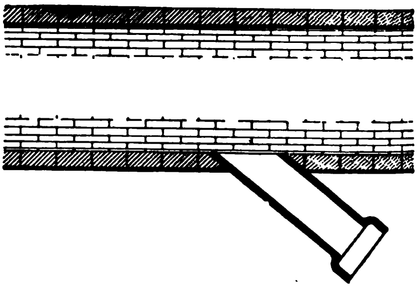


FIG. 388.

two intersecting cylinders meet. The branch can, therefore, be turned either to the right or to the left. Until a house connection is made to a Y branch, the end of the branch is closed by a **cap,** or **stopper,** the general form of which is shown at *c* in the figure. For making the house connections to brick sewers, a piece of pipe corresponding to the smaller cylinder of Fig. 387 is used. This piece of pipe, called a **branch,** or **slant,** is built into the upper arch of a brick sewer just above the springing line, or line along which the upper arch and invert join. The form of the branch, and how it is built into the side of a brick sewer are shown in Fig. 388, which is a longitudinal section and bottom view of both sewer and branch.

1612. Curves. — House connections entering sewers are often required to turn quite sharply. Special curves are required for this and for other purposes. In Fig. 389 a Y branch and connecting curve are shown in plan, eleva-

and section. This figure shows clearly not only the form of the connection but also the proper position for making it.

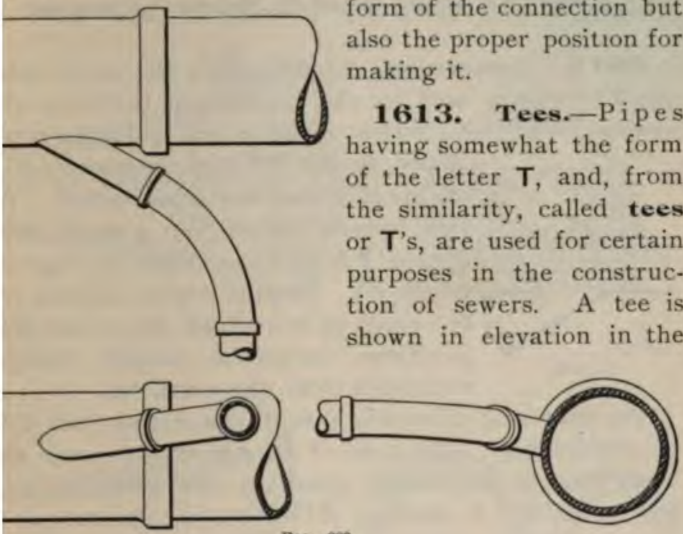


FIG. 389.

1613. Tees.—Pipes having somewhat the form of the letter T, and, from the similarity, called **tees** or T's, are used for certain purposes in the construction of sewers. A tee is shown in elevation in the r portion of Fig. 394. It is not a proper connection for ches, as it produces too abrupt a change in the cur-, causing eddies and producing deposit. It is, however, od detail for certain purposes, some of which will be after explained.

614. Handholes.—Sections of sewer pipe having a chable piece, as shown in Fig. 390, are sometimes laid tervals along a pipe sewer. Such pipes are commonly d **handholes,** although the name is more properly ap- l to the detachable ce. They afford a ns of removing ob- ctions from the sewer out breaking the . When used, they laid at intervals of t one hundred feet g the sewer. As, however, house connections are not lly made to all the Y branches laid for that purpose, the

FIG. 390.

unused **Y** branches may be made to serve the same purpose as handholes by simply removing the cap, or stopper.

1615. Junctions.—Substantially the same detail as the **Y** branch is used for the junction of two lines of pipe sewers. The relative sizes of the branch and main pipes are varied to suit different requirements, and the form is also somewhat varied. Where two branch sewers join a main sewer, a double **Y** branch, as shown in Fig. 391, is employed. In pipe sewers and all sewers too small to be entered, the curves for the junctions should be entirely within, or accessible from, the manholes.

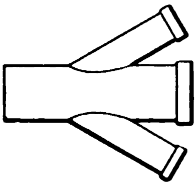


FIG. 391.

The junctions of brick sewers are sometimes quite difficult of construction. The radii of the connecting curves should be as great as practicable, generally from about ten to fifty feet, according to the size of the sewer, the longer radii being for the larger sewers; the sewers should, when possible, connect tangentially. The inclinations on the curves should be increased as noticed in Art. **1570.** Where small sewers connect with sewers of larger size, the bottoms of the sewers should not be at the same level, in order that the small sewer may not become choked by the flow of the larger sewer. So far as possible, sewers should so connect that the surface of the sewage during the *ordinary* flow will be on the same level in all. In most cases, this condition will obtain approximately if the *center lines* intersect in the same horizontal plane.

The junction of three egg-shaped sewers of different size is shown in Fig. 392. It will be noticed that, in the inverts, the forms of the three tributary channels are preserved until they merge into the single larger channel. In order to give sufficient strength to the upper arch, however, the arch of the outlet sewer is continued up stream, flaring or trumpet-shaped, until the three tributary sewers become distinct and separate. Such a flaring arch is sometimes called a **trumpet arch.** At the extreme upper end of the trumpet

arch is shown the lower end of a vertical pipe forming a lamp-hole. The usual and much better practice,

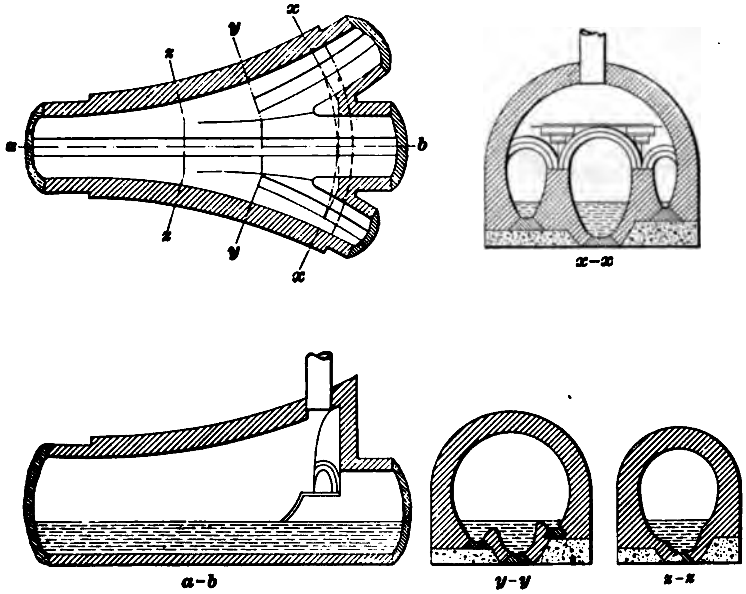


FIG. 392.

however, is to build a manhole over the point of intersection.

1616. Manholes.—Where two or more sewers unite, an opening leading to the surface of the street should be constructed for the purpose of affording access to the sewer. Such openings are called **manholes.** A manhole is shown in Fig. 393. It is here shown as constructed at the junction of three-pipe sewers, but the construction will be substantially the same for any case. Manholes are constructed of brick; they may be directly over the sewer or somewhat to one side, as circumstances may require. In this country, however, they are usually constructed directly over the sewer, in the general form of a truncated cone. The walls should be eight inches thick and plastered on the outside with cement mortar. The foundation and bottom of the

manhole should be formed of concrete, which should be brought up to a level with the bottom of the sewer. The bottoms of manholes should be carefully made to conform to the shape of the sewer. The top of the manhole should be covered with a perforated cast-iron cover, the top of which should be on a level with the surface of the street. In some cases, a dust pan is hung below the cover.

FIG. 393.

Manholes should be constructed at all junctions (except house connections), at the upper ends of curves, and, where the distance between such points is great, at occasional street intersections. In general, they should be constructed at all points where it may be desirable to have access to the sewer for the purpose of inspection, repairs, the removal of obstructions, or for any other purpose. They afford a means for making the connections between pipe sewers which can not be satisfactorily connected by the ordinary **Y** branch, as is often the case with large pipe sewers. In such cases, all the tributary sewers discharge into the manhole, to which an outlet is afforded by the main sewer. Manholes also serve to ventilate the sewers. It should be stated, however, that the construction of manholes adds materially to the cost of a sewer, and they should not be constructed more frequently than necessary.

1617. Lamp-Holes and [illegible].— [illegible]

[illegible] **lamp-holes** [illegible]

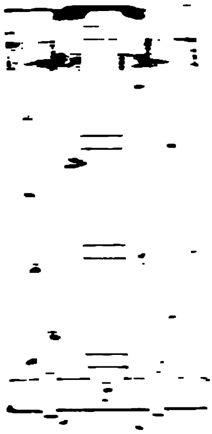


Fig. [illegible]

[illegible] **fresh-air inlets** [illegible]

[illegible] the advantage [illegible] of the sewer [illegible]

1618. Catch-Basins.— [illegible] ers, and sometimes [illegible] s also. They are for the purpose [illegible] r to the sewers, and, therefore, [illegible] m only. Catch-basins are built in various forms, most hich, however, consist of a chamber, or basin, into which storm water flows directly from the street gutters, hav an outlet into the sewer from a point at some distance e the bottom. By this arrangement, a large proportion

of the coarsest and heaviest of the matter suspended in the storm water will not enter the sewer, but will settle to the bottom of the catch basin and be there retained. In order to prevent sewer gas from entering the catch-basin (from which it would escape in the vicinity of the sidewalk and be very objectionable), the outlet to the sewer is given such a shape as to form a trap.

The form of catch basin most common in this country is shown in Fig. 395. It is generally built of brick, but is sometimes built of concrete. When built of brick, it should be lined with cement and plastered on the outside with cement mortar, so as not to leak. In the figure, *o* is the opening for admitting the storm water from the gutter, *t* is the trap to the outlet leading to the sewer *s*; *w* is the surface of the water, and *m* is the deposit of mud and sedimentary refuse; *c* is a cast-iron cover to an opening in the top of the catch-basin, through which the deposited mud and refuse may be removed.

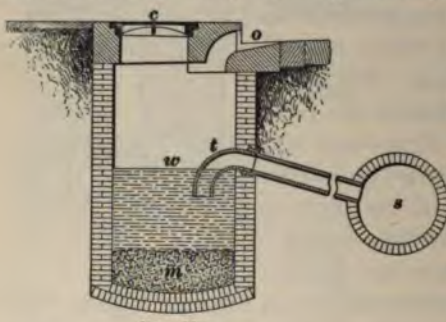


FIG. 395.

It is important that catch-basins should be perfectly water-tight, in order to maintain the water surface at or very near the level shown in the figure. This is necessary, in order both to cover the deposit of mud and prevent it giving off disagreeable gases, and to seal the trap against the escape of sewer gas. The water-level in street catch-basins should be from two to three feet below the street surface, and the total depth of the basin should generally be from six to eight feet, in order to avoid freezing. The construction may be varied in detail to suit circumstances.

1619. The Shone Ejector.—In order that the sewers shall not become clogged, the sewage must flow with sufficient velocity to keep the sewers clean. This requires that

the sewers shall be laid to grades having sufficient inclination to induce the required velocity. In low-lying districts, such grades are not always available without requiring the sewage to be pumped or otherwise lifted; in other words, the available fall to the outlet is not enough to convey the sewage, and the expedient of allowing the sewage to flow to a low point in the system, and there lifting it to a higher level, from which it will flow to the outlet, must be resorted to.

For this purpose a mechanism is used, which is known as the **Shone ejector,** named after its inventor, Mr. Shone.

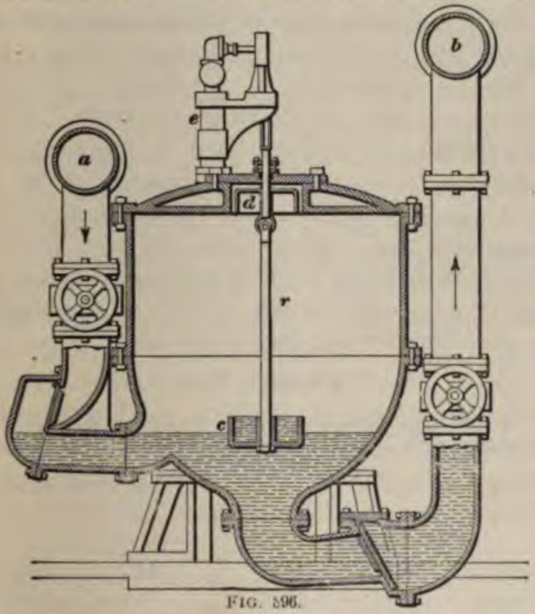


FIG. 396.

In this mechanism, compressed air is applied to lift the sewage. A Shone ejector is shown in Fig. 396. Through the inlet *a*, sewage is admitted to the large chamber, or reservoir, *r*. When full, the pressure of the sewage lifts the small bell *d*, which operates a valve and admits compressed air into the reservoir through the pipe *e*, forcing the sewage out through the outlet *b*. When the chamber is empty, the valve is closed by the weight of the small bucket *c*, which is always full of sewage.

By means of these appliances, the difficulties attending insufficient fall and flat grades may be, to a great extent, overcome. They are used principally in England. Two Shone ejectors have, for some time, been in use in Chicago, however, each having a capacity of eighty cubic feet per minute lifted fifteen feet. Two Shone ejectors are also used at Winona, Minn., for the purpose of lifting the sewage during high water in the Mississippi River, in order to allow the sewage to be discharged into the river.

1620. Cost of Sewers.—The cost of a sewer will depend chiefly upon the price of sewer pipe (or brick, as the case may be), the cost of labor, the depth of the excavation, and the kind of material to be excavated. All these, and especially the last two, may vary considerably. The cost of sewers is generally somewhat higher in large than in small cities. Sewers are commonly constructed under contract at a stated cost per lineal foot. Table 34 gives the minimum, maximum, and average cost per foot, including excavating and back-filling for different sizes of both pipe and brick sewers, as obtained from records of the construction of thirty-five different sewerage systems.

TABLE 34.

COST OF SEWERS PER LINEAL FOOT.

Pipe Sewers.				Brick Sewers.			
Diameter in Inches.	Minimum.	Maximum.	Average.	Diameter in Inches.	Minimum.	Maximum.	Average.
8	$0.25	$2.00	$0.79	30	$1.00	$4.40	$2.83
10	0.35	1.75	0.89	36	1.25	5.83	3.48
12	0.30	2.00	1.01	48	1.50	9.00	5.13
15	0.65	2.39	1.31	60	2.00	16.47	6.91
18	0.70	3.05	1.61				
24	1.37	3.70	2.31				

For these sewers the depth of the excavated trench varied from $4\frac{1}{2}$ to 20 feet, the average being about 10 feet. The cost of manholes will generally vary from about twenty to sixty dollars; lamp-holes, from about three to ten dollars; and flush-tanks, from about twenty-five to eighty-five dollars.

SANITARY REQUIREMENTS OF SEWERAGE.

FLUSHING AND VENTILATING.

1621. Necessity for Flushing and Ventilating.—In order to form some idea of the great amount of gas constantly generated in a sewer, it is only necessary to observe the column of vapor rising from a manhole on a frosty morning, or to get a smell of the same on a warm day. This gas is given off by decomposing sewage, which gets stranded in considerable quantities during the shallow flow of dry weather, and more or less of which adheres to the sides of the sewer at all times. Fresh sewage is not very offensive, but after rapid decomposition has begun in sewage, immense quantities of sewer gas are liberated. It is, therefore, evident that, in order to prevent the generation of sewer gas as much as possible, the sewers should be frequently and thoroughly flushed, washing out the accumulations of decomposing sewage. The generation of sewer gas is largely prevented by keeping the sewers *clean*.

Sewer gas commonly contains sulphureted hydrogen, carbureted hydrogen, nitrogen, ammonium sulphide, and fetid organic vapor. It also carries great quantities of disease-producing micro-organisms, commonly called **bacteria**, which develop and flourish in the warm, moist air of the sewers. Hence, it is readily seen that sewer gas is very dangerous to health, and that it must be constantly removed by efficient ventilation.

1622. Means of Ventilation.—The efficient ventilation of a system of large sewers is a difficult problem, and,

up to the present time, it has not been satisfactorily solved. Some of the highest authorities who have investigated the matter have concluded that the most effective expedient is to ventilate through manholes and air inlets in the middle of the street. This is highly objectionable on account of the disagreeable odors, but is, perhaps, as free from danger as any cheap method of ventilation that can be employed.

The question of sewer ventilation is extremely complicated, and this method possesses the advantage of somewhat simplifying the matter. Sewer gas is usually, but not always, lighter than air. The air currents in sewers are generally towards the summits, but are often towards the outlets. Sometimes, in the same sewer, the air flows towards the outlet in summer and towards the summit in winter, being cooler than the outer air in summer and warmer in winter. Ventilation by manholes is reasonably effective with air currents in either direction.

Various methods of ventilating sewers and treating sewer gas have been proposed. Some of the methods of ventilation proposed are reasonably efficient, but their expense prevents their adoption on any extensive scale. Sewer gas may be purified by passing it through loosely packed charcoal; this method has proved fairly satisfactory on a large scale, but is quite expensive.

Probably, the best practical and economical method of sewer ventilation that has yet been devised is to carry an untrapped pipe from each house connection upon the *outside* of the house up to a considerable height above the roof. If this were done in all houses connected with a sewer, it would afford a reasonably efficient and economical system of ventilation, and, probably, the best practical solution of the problem. Such method of ventilation will also permit the air currents to flow in either direction.

1623. Methods of Flushing.—The flushing of sewers may be accomplished in various ways. One of the simplest methods is to dam up the sewage by gates in the sewer until it is nearly or quite full, then, by opening the

es, allow the sewage to escape with a strong, full cur-
t, washing out the accumulations of solid matter. In
er to avoid holding the
age until it backs up
cellars and basements,
gates used should not
ch quite to the top of
cross-section of the

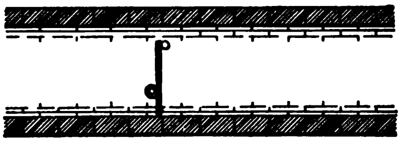


FIG. 397.

er. Automatic gates may be used which turn on a
izontal axis placed somewhat below the center of the
e, as shown in Fig. 397.

Vhen the sewer becomes nearly full, the pressure of the
fined sewage on that portion of the gate above the axis,
oming greater than the pressure against the smaller
ion below the axis, tips the top of the gate outwards and
ases the confined sewage. The confined sewage, how-
r, will have a tendency to deposit a large amount of solid
ter, which the current of the liberated sewage may not
olly flush away.

'he above-described method can not be applied at the
er ends of sewers, where other expedients must be re-
ed to. The sewage may be collected in tanks which
charge automatically when full, but the best method
lushing yet devised is by means of automatic flushing-
ks which are slowly filled with water from a convenient
rce of supply, and discharge automatically when full.
e water so used may be water from streams, bodies of
ter, or springs situated at higher elevations, rain water
m roofs, water collected in cisterns by pumps or pipes, or
ter from the public water supply. The last is the source
supply most commonly adopted.

1624. **Automatic Flush-Tanks.**—Sewers may be
ot reasonably clean by regular flushing with clean water.
is may be accomplished by means of automatic flush-
iks located at all dead ends and various other points
ng the sewers. The requisites for an automatic flush-
k are certainty of action, rapidity of discharge,

simplicity, ease of inspection, durability, and economy, both in first cost and maintenance. Many forms of automatic flush-tanks have been invented, not all of which will meet the above requirements for the usual conditions. As an example we shall describe the tank known as **Van Vraken's flush-tank,** which is shown in Fig. 398. It consists of a large chamber *c* into which water is slowly admitted by an ordinary faucet *f*. The tank empties through the siphon *s*, at the bottom of which is hung a small cast-iron tilting basin *b*. As the tank fills, the water gradually rises through

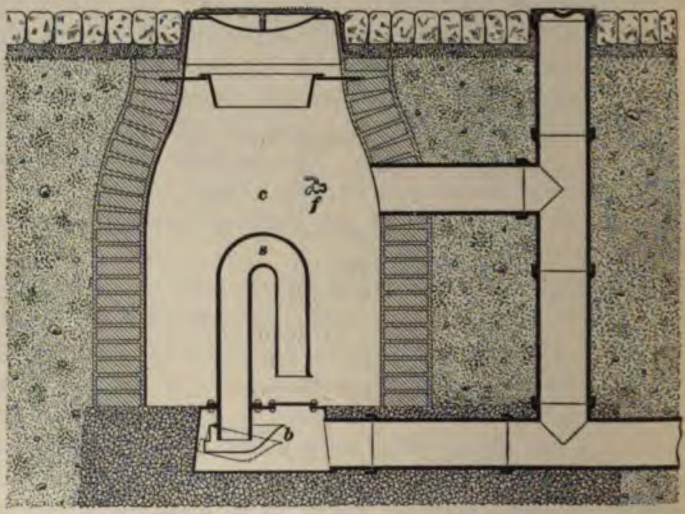


FIG. 398.

the free or ascending leg of the siphon until it overflows down the descending leg, filling the tilting basin. As the basin fills, its center of gravity becomes changed until it tilts over to the position shown by the dotted lines, lowering the surface of the water in the basin about one inch. This produces sudden rarefaction in the siphon and brings it promptly into full action. When the discharge ceases, the basin tilts back to its former position.

Flush-tanks should have discharging capacities equal to those of the sewers into which they discharge; for the sep-

rate system, they are commonly constructed with capacties ranging from about 125 to 250 gallons. They should lischarge automatically once, or, at most, twice during ach twenty-four hours. For thorough flushing, the tanks hould be so located as to give a head, in feet, equal to from ne-half to three-quarters the diameter of the sewer in nches. The head required by this rule could not be readily btained for the large sewers of the combined system, howver. It may be stated that the problems of both ventiting and flushing are much simplified in the separate ystem.

SEWAGE DISPOSAL.

1625. Composition of Sewage.—The character of wage varies considerably, according to the relative prortions of rain and waste water, refuse matter, and excreent that it contains. The composition of sewage given in e following table represents an average of seventeen nglish cities, including London, in which the total amount human excrement is taken by the sewers.

In the table, the dissolved matter may be taken as about e-third organic and two-thirds inorganic matter.

TABLE 35.

COMPOSITION OF SEWAGE.

uspended matter, inorganic, grains per cubic foot	130.2
uspended matter, organic, grains per cubic foot	101.2
issolved matter, grains per cubic foot	298.7
otal solids, grains per cubic foot	530.1
itrogen, grains per cubic foot	36.1
ubic feet of sewage per capita daily	6.71

1626. Sludge is the name given to the residue from wage purification works where the process of sedimentation chemical precipitation is employed. When analyzed in a y condition, sludge has been found to contain from 17 to per cent. of organic matter, from 0.5 to 3.3 per cent. of trogen, and from 0.8 to 2.5 per cent. of phosphoric acid. In

different cities, where sewers are constructed, the amount of excrement entering the sewers will generally vary from 20 to 100 per cent. of the total excrement.

1627. Methods of Sewage Disposal.—In designing a system of sewers, a difficult problem to solve is the final disposal of the sewage. In many cases the engineer is not required, or may not be permitted, to deal with this problem, as limited resources or the decision of those vested with authority may require that the sewage be simply discharged into the nearest watercourse. This is the most common, and, until recent years, almost the only practical, method of sewage disposal. In some cases, the conditions are such that this practice is quite permissible and possesses no very objectionable features, but as the population increases, it becomes more and more objectionable. If, as the population becomes dense, we are to continue to have pure drinking water in sufficient quantities, the pollution of streams and lakes by sewage will need to be prohibited, or at least regulated, by law. In many cases, no watercourse into which the sewage may be discharged is available, and the question of sewage disposal must necessarily be considered.

Various methods of sewage disposal have been proposed and tried; those that have been tested on any considerable scale may be classified as follows:

First. The sewage is simply discharged, without treatment, into a stream or large body of water. This method is sometimes called **natural disposal.**

Second. The solid matter is removed and the sewage is, to a considerable extent, clarified by subsidence, by mechanical filtration, by chemical processes, or by some combination of these different methods, and the clarified effluent then discharged into a stream or body of water.

Third. The sewage is applied to the soil, by intermittent downward filtration, by broad irrigation, or by subsurface irrigation.

1628. Natural Disposal; Dilution and Oxidation.—When sewage is discharged into a stream of water,

e water immediately below the outfall of the sewer will be ndered very impure. But if the stream of water is of con-
derable size, as compared with the stream of sewage, and is considerable current, a change, usually called **self-urification,** will take place and the water will again come pure, so that at a distance of a few miles below the wer, no traces of sewage can be found in the water.

The disappearance of the sewage, or self-purification of e water, is due to several causes. The sewage is highly luted by the comparatively large amount of water into hich it is discharged, but this is simply dilution, and not urification. Actual purification does take place, however. ome of the organic matter becomes food for aquatic plant nd animal life; some combines chemically with the oxygen f the water and air, forming inorganic compounds; con-derable chemical change is due to the presence of micro-rganisms as ferment; while much of the solid matter is eparated and deposited in particles along the bed and banks f the stream.

The water being exposed to the air, the greater its com-arative volume, and the more rapid its current, the more apid and thorough will the purification be. Where the mount of sewage is small in comparison with the volume of ater, and, especially, where the distances between towns re great and the stream has a swift current, this method of ewage disposal may be employed with reasonably satisfac-ory results. In such cases, it is probably the best method f sewage disposal that can be adopted, as it is by far the heapest method. Where the opposite conditions prevail, owever, the streams or lakes may become so polluted by ewage as to not only render the water absolutely unfit for lomestic use, but also become a serious menace to the health of the cities on their banks. It is safe to state that this nethod of sewage disposal is employed in very many cases where it ought not to be.

1629. Clarification.—By this method of sewage dis-posal, the greater portion of the solid matter, and, by some

processes, a portion of the dissolved matter is removed from the liquid sewage before it is discharged into the stream or body of water. The effluent thus obtained, though by no means approximating pure water, is less objectionable than the original sewage. Three general processes of clarification are employed, namely, **subsidence, mechanical filtration,** and **chemical precipitation.**

1630. Subsidence.—When this process is employed the sewage is collected in tanks and allowed to stand until the solids have settled to the bottom, and then the water is slowly drawn off, discharging into the stream or body of water. The water still remains quite highly charged with impurities, however. This process is also called **sedimentation.**

1631. Mechanical Filtration.—By the process of ordinary filtration, the sewage is simply passed through filters or screens; the filters used are of various kinds. This process removes a larger proportion of the solids than can be removed by subsidence, but leaves the effluent still highly charged with impurities.

1632. Chemical Precipitation.—In the various processes of chemical treatment, the sewage is collected in tanks or reservoirs and certain chemical solutions are mixed with it, which precipitate not only the solid matter but also a portion of the matter held in solution. This method of treatment is a direct outgrowth of unsatisfactory sedimentation. Different chemical processes are employed, a large number of which have been patented.

After the sewage has been clarified by chemical precipitation, the remaining water is still far from pure and is liable to decompose after being discharged into a stream. Moreover, the addition of the chemicals used is more or less deleterious to the purity of the water.

1633. The Chemicals Used.—Various chemicals are used in the different processes; in some processes, more than one chemical is employed. The chemicals most com-

›nly used are lime (oxide of calcium), alum (sulphate of ıminum and potassium), and copperas (sulphate of iron). :cellent results have been obtained by a combination of pperas and lime. The proportions of chemicals used will ry somewhat with, and should be adapted to, the character the sewage. The composition of factory sewage varies eatly. For the three chemicals named above, the proirtion, per million gallons of sewage, will generally be thin the following minimum and maximum limits:

	Pounds per Million Gallons.
Lime	1,500 to 3,000
Alum	500 to 1,000
Copperas	850 to 1,700
Lime	600 to 1,200

The minimum limits here given apply to ordinary house wage liberally diluted with water, while the higher values ›ply generally to factory sewage, according to its com›sition.

1634. Application to the soil is, without doubt, the ost efficient means of purifying sewage. Those natural aters which have undergone prolonged filtration through .e soil are the most free from organic matter. Under vorable conditions, water containing sewage may be ren:red reasonably pure by this method; purer, in fact, than ıe common sources of water supply. It is still an open ıestion, however, whether water which has been con.minated by sewage can be so thoroughly purified as to be ıfe for domestic use. The processes of purification em.oyed are those of **irrigation** and **filtration.**

1635. Irrigation, commonly called **broad irrigaıon,** includes quite a variety of methods, all of whichconst essentially in applying the sewage to the soil in such ıantities as to irrigate and fertilize the soil for the growth : vegetation. This would appear to be the most natural ıethod of sewage purification, but, as the amount of sewge that can be applied to a given area, without being

detrimental to the growing crops, is limited, it requires extensive areas. Hence, the name *broad* irrigation. The application of the sewage must be intermittent, in order that renewed supplies of oxygen may enter the soil to maintain the oxidizing process. The application of the sewage is commonly called the **dose.** By increasing the volume of the dose, without reference to the requirements of vegetation, the irrigation may be converted into intermittent filtration.

1636. Filtration.—This may be considered to be copious irrigation. Both upward and downward filtration have been employed. Quite satisfactory results have been obtained by intermittent downward filtration. In this process, the sewage is made to flow evenly over ground that has been prepared for the purpose and thoroughly underdrained, and is filtered by passing through the soil to the drains, during which process most of the organic matter is destroyed by oxidation and nitrification. Separate sewage beds should be prepared, so as to be used alternately, affording opportunity for the entrance of fresh oxygen into the soil, to maintain the oxidizing process. The soil should be loose and thoroughly underdrained; coarse gravel, such as is used for mortar, is generally the best.

The drain tiles should be not less than three inches in diameter, and the lines of tiles generally not more than about forty feet apart. They should be at least four feet below the surface, and a depth of five, or even six, feet is still better.

The amount of sewage per acre that can be successfully treated will vary with the nature of the soil and the thoroughness of its preparation. A thoroughly prepared and well-managed sewage field may easily be depended upon to absorb and purify the sewage for a population of from 200 to 800 per acre of filtration beds, and, where the conditions are favorable, will purify much greater quantities.

The process of intermittent sewage filtration through soil is not a merely mechanical process, but largely a chemical

process, involving oxidation and nitrification, brought about chiefly by means of the micro-organisms, called **bacteria,** contained in the sewage itself. While the soil, to some extent, acts as a mechanical filter in straining out certain parts of the solid matter, the purification is chiefly of the nature of chemical change. The sewage virtually becomes its own purifier, the process of filtration affording the favorable conditions for the purification to take place. The bacteria contained in the sewage will, under the conditions afforded, effect the purification of the sewage, and, having performed their work, will in turn succumb to the action of oxygen and disappear. If the filter beds are properly prepared and the application of sewage is intermittent and properly regulated, the filter beds will not become fouled and ineffective, but will, within limits, become more and more efficient from use.

WATER SUPPLY AND DISTRIBUTION.

GENERAL CONSIDERATIONS.

1978. Utility of a Public Water Supply.—The conveniences of a public water supply for villages, towns, and cities are too well known to need any argument in their favor.

1979. Requisites of a Public Water Supply.—There are four chief conditions which such supplies should fulfil in order to adequately meet the requirements of the communities in whose behalf they are constructed and operated. *First*, the water supplied should be wholesome, and adapted to all domestic uses; *second*, the supply should be abundant; *third*, it should be delivered under a sufficient pressure, and *fourth*, it should be furnished at a cheap rate to the consumer.

1980. Wholesomeness.—As regards the first-named requisite, that is, the quality, or wholesomeness of the supply, without entering into the very interesting and conclusive statistics which prove the statement, it will suffice here to assert that the introduction of an abundant and wholesome public supply of water has invariably exercised a beneficial effect upon the death rate and general health of the community using it. Particularly is this the case as regards diseases of the typhoid and typho-malarial classes, and it must be observed that in studying the sanitary condition of any community, not only the death rate, but the general health and degree of vitality enjoyed by such community should be taken into consideration. Generally, it is true, the one is an index to the other. The advantage of a pure and uncontaminated water supply becomes strikingly evident during the visitation of an epidemic, such as cholera or yellow fever.

For notice of copyright, see page immediately following the tit

1981. Chemical Analysis.—Unfortunately the means which science affords for determining the wholesomeness of any given water are neither easy nor certain, and the literature of the subject is voluminous and somewhat contradictory. It has long been recognized that a chemical analysis is of itself of small value, because, while all the constituents of a given sample may be accurately determined in this way, it is impossible to tell from the analysis alone whether they are present in the water under innocent or harmful conditions. More particularly is this true as regards the attempt to determine by chemistry whether or no a certain water contains the germs of disease. An eminent English chemist has aptly said that we might as well analyze a man's brain to find the ideas which it contains as to analyze water to find the germs of disease which may lurk in it.

As the study of water analysis advances, however, certain facts become established and generally accepted as such, and to these we will now turn our attention.

1982. General Conditions.—In the first place, a water for domestic use should be clear, colorless, tasteless, odorless, and sufficiently soft.

These are physical properties which can be detected by the senses, or by simple and direct experiment. A water may possess all the above desirable characters and yet be highly unfit for use, owing to its containing some poisonous ingredients which can not be detected by the senses. For instance, a perfectly clear and otherwise wholesome water would be rendered dangerous in the extreme by the addition of a very minute quantity of sewage from an infected district; so minute, indeed, as not to affect in the slightest degree its physical properties, and even to escape detection by chemical processes. In such a case the only assurance of the wholesomeness of the water would be the positive knowledge that its source was so situated as to preclude the possibility of the sewage entering it.

Although turbidity does not necessarily render a water

unwholesome, yet it is frequently found that suspended matter exercises an irritating effect upon unaccustomed stomachs, although those habituated to its use experience no ill effects from it.

1983. Most Dangerous Elements.—Of all kinds of contamination to which water is exposed, by far the most dangerous is animal refuse, and of this the worst is that furnished by human beings. This class of contamination is broadly known as **sewage.** In regard to this dangerous element, Professor Mason, in his treatise upon Water Supply, says: "The really serious item of contamination, the one to which the sanitarian's attention is most quickly drawn, is that of sewage introduction, and a consideration of the questions arising upon this topic dwarfs all others into comparative insignificance."

When we know that a certain water is free from sewage contamination, the ascertaining whether or no it is possessed of other harmful qualities is comparatively simple. When we know that it is so contaminated, we need no further test to prove that it is unfit for human use. The difficulty intervenes when we are not certain either way. We then have recourse to chemistry as an aid to our investigations, the analysis not giving us, by itself, any conclusive evidence, but only assisting us to trace the *history of the water*, which is the knowledge necessary and sufficient for us, in our study.

1984. Chlorine.—In reading the reports of chemical experts upon the quality of a water submitted to them, it will be remarked that great attention is paid to the amount of *chlorine* present. This is not because a considerable quantity of this substance may not harmlessly exist in the water, but because, since it also forms an important constituent of urine, we may infer sewage contamination from its presence. It usually exists in sewage in the form of common salt. "If, then, a given sample of water is found to be devoid of chlorine, or very nearly devoid of chlorine, it can not have been charged with sewage." (Wanklyn.) If chlorine is

found to be present in large quantities, sewage contamination is to be seriously suspected, and unless it can be proved that the chlorine has been derived from some other and harmless source—a mineral spring, for instance—the water should be rejected.

As regards the permissible amount of chlorine in a potable water, Professor Mason gives the following, the figures denoting parts in one million, by weight:

"The Rivers Pollution Commission reports the average amount of chlorine in 589 samples of unpolluted water as follows:

Rain	8.22
Upland surface	11.30
Deep well	51.10
Spring	24.90

"Wanklyn considers 140 as possibly suspicious.

"Frankland considers the permissible limit as 50.

"Leed's standard for American rivers, 3 to 10.

"Ordinary sewage, about 110 to 160.

"Human urine (average of 24 samples), 5,872."

It will be perceived from the above that the question is not left in a very satisfactory or certain shape by these high authorities.

1985. Determination of Organic Constituents.—As all sewage contains a great deal of organic matter, the determination of the amount and kind of such substances in a given water is a matter of great importance. Many processes have been devised for this purpose; that now most generally used is the albuminoid ammonia process of Professor Wanklyn.

1986. Albuminoid Ammonia Process.—By this process a knowledge of the amount of free ammonia and of albuminoid ammonia contained in a water sample is secured. As regards the interpretation of the results obtained by this process, Wanklyn says:

"If a water yield 0.00 part of albuminoid ammonia per million, it may be passed as organically pure, despite of

much free ammonia and chlorides; and if, indeed, the albuminoid ammonia amount to 0.02 or to less than 0.05 part per million, the water belongs to the class of very pure water. When the albuminoid ammonia amounts to 0.05, then the proportion of free ammonia becomes an element in the calculation; and I should be inclined to regard with some suspicion a water yielding a considerable quantity of free ammonia along with more than 0.05 part of albuminoid ammonia per million.

"Free ammonia, however, being absent, or very small, a water should not be condemned unless the albuminoid ammonia reaches something like 0.10 per million. Albuminoid ammonia above 0.10 per million begins to be a very suspicious sign; and over 0.15 it ought to condemn a water absolutely. The absence of chlorine, or the absence of more than one grain of chlorine per gallon (*about 15 parts per million*) is a sign that the organic impurity is of vegetable rather than of mineral origin; but it would be a great mistake to allow water highly contaminated with vegetable matter to be taken for domestic use.

"Drinking water falls into three classes, according to the degree of organic purity, as follows:

"*Class I.* Water of extraordinary purity, yielding from 0.00 up to 0.05 part of albuminoid ammonia per million. This class comprises the most carefully prepared distilled water and highly filtered waters, both *natural* (i. e., deep-spring waters) and *artificial* (i. e., such water as has passed through a 'silicated carbon filter' in good working order). Occasionally a river-water in its unfiltered condition falls into this class. Water of this class can not be objected to organically.

"*Class II* comprehends the general drinking waters in this country (*Great Britain*). It gives from 0.05 to 0.10 part of albuminoid ammonia per million. I believe any water falling fairly into this class is safe organically.

"*Class III* comprehends the dirty waters, and is characterized by yielding more than 0.10 part of albuminoid ammonia per million."

As regards "dirty water," the same author says: "If a water be dirty, a knowledge of its history, and, of course, its mineral constituents, may be of assistance in forming a judgment as to the degree of risk attendant on its employment for domestic use."

1987. Moist-Combustion Process.—Another method of determining the amount of organic matter is the "Moist-Combustion Process." This process consists in ascertaining the amount of oxygen required to completely, or nearly so, oxidize the organic bodies contained in the water. The results are sometimes given as "required oxygen." The greater the amount of oxygen required, the greater the amount of organic matter contained in the water. Wanklyn states, as the result of investigation, that water of first-class purity does not consume more than 0.50 part by weight of oxygen per million, average drinking water between 2 and 3, while "dirty" water considerably exceeds the latter figure.

1988. Tests Required.—Wanklyn appears to consider that the tests for chlorine and for organic matter, either as free or albuminoid ammonia or as required oxygen, are all that are necessary as regards sewage contamination, but other authors attach a great deal of importance to the amount of nitrogen contained in the water under the two forms of *nitrates* and *nitrites*.

1989. Nitrates and Nitrites.—Regarding this subject, Frankland writes: "When fresh sewage is added to water already containing nitrates, the latter are generally reduced to nitrites," and adds: "When nitrites occur in shallow wells or river-waters, it is highly probable that these waters have been very recently contaminated with sewage."

In order to understand the distinction made between nitrates and nitrites, we must remember that Nature's method of purifying "dirty" water is by the oxidation of its impurities. Nitrites, indicating probable sewage contamination, are destroyed, as regards their dangerous prop-

erties, by being raised by oxidation to the condition of nitrates. Nitrites, when found in water, constitute a suspicious feature; when nitrates are found, they may or may not have previously existed as pernicious nitrites; if they did so exist, then they have been neutralized by oxidation.

Mason says: "Nitrites should always be looked upon with suspicion if found in ground or surface waters." He adds: "The absence of nitrites, moreover, proves nothing. I have recently had a most foul cistern-water for analysis which showed but a trace of nitrites and no nitrates, and yet the water was contaminated with the entire house drainage, and produced most serious illness."

Frankland also says: "The presence of these salts (nitrites) in *spring* and *deep-well* water is absolutely without significance; for, although they are in these cases generated by the deoxidation of nitrates, this deoxidation is brought about either by the action of reducing mineral substances, such as ferrous oxide, or by that of organic matter, which has either been embedded for ages, or, if dissolved in the water, has been subjected to exhaustive filtration." In quoting the above, Mason adds: "This is merely another instance of how careful the analyst should be to become familiar with the source of the water before undertaking to pass judgment upon its quality."

"Leed's standard for American rivers, 0.003." That is, the above proportion of 0.003 part by weight, per million, marks the "danger line" for this constituent.

As regards nitrates, although, as we have seen, their presence does not necessarily indicate danger, it points to the probability, or, at least, possibility, of previous sewage contamination. Therefore, when they are found to exist in large quantities, unless satisfactorily accounted for, they lead to the rejection of the water containing them. Mason quotes the "Analyst," xviii, 293, in this connection, as follows: "The proposal to consider a water safe so soon as the nitrogen has assumed the oxidized condition, irrespective of the quantity that may be present, is entirely irrational."

It will be seen from the above that nitrates constitute a

rather confusing element in the interpretation of water analyses. Mason gives Leed's standard for American rivers, or the danger line, as from 1.11 to 3.89 parts by weight per million, which certainly seems very elastic. Not so much so, however, as the standards given by other authorities; for Mason states that the Vienna Commission allows 1.04 parts and the Brandes Commission 7 parts. No doubt these were for certain localities possessing distinctive features.

The presence of nitrates is still less of an indication in the case of "deep wells of good character"; for the "Analyst," xx, 84, gives the parts of nitrogen as nitrates in various wells of this character as varying from 0.00 to 11.43.

Mason says, moreover: "Fresh sewage is often found entirely free of either nitrites or nitrates, simply because the organic nitrogen present has, as yet, not had sufficient opportunity to become oxidized thereto."

1990. Value of Chemical Analysis to Determine Sewage Contamination.—It will be seen, from what precedes, that chemical analysis in the case of water, and as regards its freedom from sewage contamination, has no absolute value by itself, but is valuable only as an aid in establishing the *history of the water*, and to find out whether or no it is, or has been at any previous time, subject to contamination, and, in the latter case, whether it has become subsequently sufficiently purified for domestic use.

This branch of the subject falls within the province of the chemist rather than of the engineer, and the above points have been given only to enable the latter to have an intelligent understanding of the expert reports which may be submitted to him by the former.

1991. Metallic and Mineral Substances.—As regards the presence of metallic substances, such as lead, copper, arsenic, and iron, the results of analysis are positive and satisfactory.

1992. Hardness.—Hardness, also, is readily and accurately determined by means of the soap test. Hardness is

due generally to the presence of lime or magnesia, which impedes the formation of *lather* when soap is dissolved in the water. By using a standard soap solution, and noting the amount which a given volume of water requires to form a lather, the degree of hardness, expressed in parts of lime and magnesia per million, can be ascertained.

Mason says: "The average hardness of good waters, as given by the British Rivers Pollution Commission, stands:

Rain	3
Upland surface	54
Deep well	250
Spring	185"

He adds: "Wanklyn allows 575; Leed's standard for American rivers, 50 for soft, 150 for hard."

1993. Collecting Samples.—As regards collecting samples of water for analysis, Mason gives the following directions:

"Large glass-stoppered bottles are best for sampling, but as they are seldom at hand, a *new* two-gallon demijohn should be employed, fitted with a *new* soft cork. Be careful to notice that no packing straw or other foreign substance yet remains in the demijohn, and thoroughly rinse it with the water to be sampled. Do not attempt to scour the interior of the neck by rubbing with either fingers or cloth. After thorough rinsing, fill the vessel to overflowing, so as to displace the air, and then completely empty it.

"If the water is to be taken from a tap, let enough run to waste to empty the local lateral before sampling; if from a pump, pump enough to empty all the pump connections; if from a stream or lake, take a sample some distance from the shore, and plunge the sampling vessel a foot and a half below the surface during filling, so as to avoid surface scum.

"In every case, fill the demijohn nearly full, leaving but a small space to allow for possible expansion, and cork securely. Under no circumstances place sealing-wax upon the cork, but tie a piece of cloth firmly over the neck to

hold the cork in place. The ends of the string may be afterwards sealed if necessary.

"Bear in mind, throughout, that water analysis deals with materials present in very minute quantities, and that the least carelessness in collecting the sample may vitiate the results. Give the date of taking the sample, as full a description as possible of the soil through which the water flows, together with the immediate source of possible contamination."

It may be added to the above that every possible item of information bearing upon the history of the water, from source to point of sampling, should also be collected for the use of the analyst in making his report.

1994. Difficulty of Procuring Uncontaminated Water.—In thickly settled districts it is generally very rare that water is found in sufficient quantities, meeting the requirements of a perfectly clear, soft, uncontaminated supply. Frequently, particularly in the older countries of Europe, it has been found necessary to have recourse to filtration before the water can be used as a public supply. The subject of the purification of such water will be fully treated under a separate heading.

1995. Quantity of Water Required.—Water is needed for many purposes in towns and cities, both public and private. Public service comprises a fire supply, street sprinkling, the flushing of sewers, public fountains, etc. Private service comprises the water needed for drinking, cooking, washing, bathing, lawn sprinkling, etc.

In the early days of the general introduction of water supplies, it was customary to itemize all these various services, allotting so many gallons for this and so many for that, and taking the sum total. At present, with all the experience of the past to guide us, it is found that they may all be combined in a general per capita allowance, and in making the count of the number of inhabitants to be supplied, it is usual to forecast the probable growth of the

town, providing for the prospective needs of the community for the next ten years at least.

In our American cities it would not be safe to estimate the per capita supply, for all purposes, at less than 100 gallons per head per twenty-four hours, on the estimated population ten years from date. Naturally, a supply sufficient for the community after ten years' growth would be more than is necessary for present needs, and care must be exercised from the start to guard against the waste incident to a redundant supply. This subject will be reverted to more in detail later on.

Very frequently cities are so fortunately situated that their supply admits of an almost unlimited extension as time goes on. In such cases the question of the daily supply will be found to affect principally that of the diameter of the mains and branches laid in the city for the purpose of distribution. This also will be treated of subsequently, when we come to the consideration of the discharge of pipes. The question of quantity will come up from time to time in this section, under different aspects, and will be discussed more in detail in reference to each particular case. For the present it will suffice to say that in laying out a water system for a thriving and growing town, the engineer should not be satisfied with a smaller supply than that already indicated.

1996. Pressure.—This is a very important consideration. Most towns cover a territory more or less varied as to elevation, part being on low ground—usually the older portion of the town—and the rest spreading out, and climbing, street by street, the surrounding heights. A great difference of level in the different parts of a town is a somewhat complicating feature of the problem, because a pressure sufficient to carry water to the most elevated districts will be embarrassingly great in the lower ones. It is frequently necessary to reduce the pressure in the house connections in the latter case, either by means of tanks in the upper stories of the houses, filled from the surface pipes, and from

which the water is distributed through the house under reduced pressure, or else, as a less desirable expedient, by introducing a short length of pipe of very small diameter—perhaps quarter inch—in the service pipe, which will also reduce the pressure to reasonable limits.

1997. High-Pressure Service.—On the other hand, it is frequently necessary to have a separate high service system to reach the more elevated portions of the town. This is generally accomplished by pumping into a standpipe, or, better still, if possible, into a small distributing reservoir containing a few days' supply. This necessitates an independent system of mains. When there are great differences of level, there will usually be a general supply for the larger part of the town, with reducing tanks in the buildings in the lower part of the city, and a high service, with separate mains and connections, for the highest portions.

The great advantage of a high pressure at the hydrants is in connection with the fire service. Frequently, if the pressure is sufficient, streams may be got upon a fire by merely connecting the hose with the hydrant, and even when fire-engines are used, it is a great desideratum to have the water fed in large volumes to the engine, so that no suction is required.

1998. Water Rates.—Naturally, the water rates must be sufficient to at least cover the expenses of the water supply, both as regards interest on cost of works and expenses of operation and maintenance. Therefore, the works should be built and operated with the greatest economy, and from the start all unnecessary loss from waste and leakage should be guarded against.

1999. Meters.—Undoubtedly the most effective means of checking waste is by the use of meters, whereby the quantity of water consumed in each house or industrial establishment is measured and paid for. This not only prevents waste, but forces the ratepayer to give close attention to the condition of the plumbing of his premises.

small leaks which would ordinarily escape notice manifest their existence in the shape of increased water bills, and lead to a careful examination of pipes and connections.

It is frequently found, when meters are introduced to replace a general water rate based upon frontage or number of taps used, that the yearly cost to the consumer is increased, particularly when wasteful habits have already been contracted. This, however, may, and generally will, be only a temporary increase of the cost of water, because by suppressing waste the necessity of a large increase of the supply may be avoided, so that the slightly augmented yearly cost to the consumer may result in warding off a very large outlay for an increased water supply, which would have to be met, in one form or another, by the taxpayer.

There are a number of excellent water meters in use, of small cost, very suitable for use in private houses or small manufactories. When, however, very large quantities are to be measured, the Venturi meter gives the best results.

SOURCES OF WATER SUPPLY.

2000. Rainfall the Primary Source of all Water Supply.—All the water supply of the earth is derived primarily from the rain which falls upon its surface.

2001. Surface Flow.—As the rain falls, some of it runs off directly, following the valleys and depressions of the ground in rivulets and brooks, which make their way to the larger streams, or rivers. These latter finally empty into the sea, whence the water which fell in rain was originally drawn up by evaporation. The remainder sinks into the ground.

Of the above-mentioned valleys and depressions, the smaller ones become dry immediately after the cessation of the rain; other larger ones continue to discharge a certain amount of water for a longer or shorter period of time after the rain has ceased, only becoming dry during the "dry

season," while other still larger ones, rising to the dignity of rivers, continue to discharge water throughout the entire year.

2002. Absorption by the Soil.—In order to account for this permanent flow of the large streams, it is necessary to inquire what has become of the water which has soaked into the ground. Whenever we dig into the earth, except in some exceptional localities, we encounter water at a greater or less depth below the surface. This is the rain-water which, instead of running directly off to the river or nearest stream, was absorbed by the ground. This water, also, is seeking its way towards the rivers, but very slowly, as it must percolate through the soil, and to do this a considerable pressure, exerted during a long period of time, is required. Indeed, all of it can not escape, for some will always be retained by capillarity. A large portion, however, is constantly and gradually working its way to the rivers, mainly underground, although some reaches the surface, when it finds a line of least resistance in that direction, and constitutes the *springs* which are found in greater or less abundance in many localities.

2003. The Earth a Great Storage Reservoir.—Thus, the earth constitutes a great storage reservoir, receiving the rain which falls upon it, and slowly yielding it up in a continuous flow to the streams and rivers, some of it, however, remaining permanently fixed in the earth, either from being retained by capillarity or by having sunk below the level of the valleys, in which latter case it must move with a still more retarded and imperceptible flow under the river-beds towards the sea itself. This permanent supply can only be reached by deep wells.

2004. Two Great Classes of Water.—All the water which is derived from lakes, rivers, and streams of all kinds having exposed surfaces is known as **surface water**; all that which is derived from wells, deep or shallow, or from filtering galleries, or from springs when taken very near their source, is known as **ground water.**

2005. Surface Water.—These two classes, although having originally the same source, always differ somewhat, and often very considerably, from each other as regards their properties. In the first place, surface water is generally much softer than ground water, because the latter is more or less impregnated with the earthy salts through which it has passed. Naturally, surface water is much more exposed to contamination, because it gathers up all the surface wash and general drainage of the area over which it flows. Rivers form the natural outlets for the sewerage of the towns, cities, and villages upon their banks, and are frequently so contaminated as to be unfit for use. In thickly settled districts this is generally the case, and as a remedy for the evil, the communities established in the neighborhood of rivers are very generally compelled to purify their liquid sewage before permitting it to enter the river. Even this precaution very often does not suffice, and many European cities are forced to take a polluted river as the source of their supply, and by means of filtration render it fit for use.

2006. Ground Water; Shallow Wells.—Shallow wells when dug in the vicinity of human habitations are also greatly exposed to contamination, because the depth of soil through which they infiltrate is not sufficient to remove their impurities. The ground water taken from deep wells, however, is generally free from this objection, on account of the vast thickness of the strata through which it percolates. Unfortunately the deeper the well, and, therefore, the greater the degree of filtration, the more liable is the water to become impregnated with mineral salts which render it unfit for many domestic uses.

2007. Practical Example.—Supposing it were desired to supply a certain city or town with water and that several sources of supply were available for the purpose. There might be a large river capable of furnishing an abundant quantity of water flowing near the town, the point at which its water could be most conveniently taken

being probably at a lower level, so that the water would have to be pumped up in order to distribute it for use. There might also be a smaller stream situated at a higher or a lower level, so that, by the construction of a dam and reservoir, the water would either flow by gravity to all or nearly all parts of the town or require pumping. There might also be reason to suppose that deep wells could be sunk in the valley which would furnish a sufficient volume of pure water, either rising naturally to the surface or requiring the use of pumps; or filtering galleries might be established near the large river, and parallel to it, forming what might be called an elongated horizontal well, whence water in large quantities could be pumped. There might, also, be some locality, at either a higher or lower level than the town, where springs in large numbers gushed forth to the surface, forming, as it were, natural artesian or flowing wells, of which the waters might be collected in a general basin and supplied to the town. These cases cover all those which could occur, although there might also be modifications or combinations of these, almost to the extent of forming a separate class.

The engineer called upon to determine which of these supplies is preferable would have a task of much responsibility, which could only be properly performed by a careful examination of each one separately from the hygienic, engineering, and commercial point of view.

2008. Large Rivers.—As to the supply from the large river, the chief recommendation would be the fact of its furnishing a very abundant quantity, rendering storage unnecessary. It would be very doubtful, however, if the quality of the water would be satisfactory, and its temperature would certainly be very high in summer. If the quality should prove unsatisfactory, it would be necessary either to abandon the scheme or provide for some process of purification by filtration. This might be by the construction of filter beds, by so-called mechanical filtration, or by the construction of filtering galleries, as already mentioned.

2009. Small Rivers and Brooks.—As to the smaller stream, the same inquiry would be necessary as regards quality, and the above observations hold good as regards abandonment or purification, with the exception that filtering galleries would not be practicable in the case of a relatively small stream.

2010. Yield of Such Streams.—Supposing the quality of the water to be proved satisfactory, the next most vital question becomes that of quantity.

In entering into this question, the probable growth and future needs of the community must be carefully considered. Many towns and cities in the United States and elsewhere have been seriously inconvenienced by finding that not only has their present supply become inadequate, but the source itself, from which the supply is derived, is insufficient and incapable of being extended. When this occurs, a new source of supply, with, perhaps, new appliances, pumps, reservoirs, etc., must be sought for, entailing a great additional expense. It is very desirable, therefore, that the source selected should be capable of furnishing a large excess over that which is needed, both for the present and in the near future.

When a certain stream is under consideration, it becomes necessary, therefore, to ascertain its total yearly yield, and also its minimum and maximum seasonal yields; that is, the smallest daily amount which it may furnish during the dry season, and the greatest which it may furnish in times of freshets. Moreover, since the flow will vary from year to year, it is important to know these particulars for a cycle of several years.

As regards the supply, the most important element of the problem is the *minimum* yield, both per day and per year.

2011. Gauging Streams.—The most obvious way to collect these statistics is by gauging the stream, by building a weir at some convenient locality, and making daily or at least weekly observations of the flow. It is evident, however, that these observations, to be of any value, should

extend over at least a year, and even then, unless the year happened to be a very dry one, the results would not be safe to count upon.

2012. Watershed and Rainfall.—The best way of determining the probable yield of a stream is to ascertain the area of the territory which drains into it and the amount of yearly rainfall. The area of "watershed," as it is called, is ascertained by a survey which consists in running a line locating the division between the basin of the stream under consideration and all the adjacent ones, so that the entire area over which the rainfall drains into the stream, above the site of the proposed dam, may be calculated. This survey is apt to be a tedious and difficult one, for when working among the headwaters of the different streams draining into one basin and of those which run into different watersheds, mistakes are apt to be made, and it is frequently found that the work of several days has been useless from getting into the wrong watershed. Local information is very valuable to the surveyor in such cases, and should be sought and made use of. Extreme accuracy is not needed in this work, and in most cases a simple compass survey is all that is required, with bearings read to quarter degrees.

As regards rainfall, if there have been no records kept in the valley under consideration, the same difficulty would occur as regards the great length of time necessary for collecting complete data as in the case of weir measurements. But it is very rare that no records can be obtained, if not for the district actually studied, at least for neighboring ones, where the conditions are similar. Some intelligent guess can always be made, even if nothing better, and in all cases it is well to commence observations of rainfall and gaugings of the stream at the same time as the survey. There is never any danger of having too many data, provided they are trustworthy.

2013. "Run Off" and Soakage.—It has already been seen that when rain falls in a catchment basin, or drainage area, some of it runs directly off from the surface,

ınd some soaks into the ground. The proportions in both :ases will depend largely upon the character of the basin, ›oth geologically and topographically. A hard, rocky basin vill be unfavorable to soakage, while a very steep one will ıaturally allow a rapid escape of the water to the stream. t must be borne in mind, however, that all of the water hat is absorbed by the soil is not necessarily lost to the tream. We have already seen that a great deal of it is ;radually recovered by the stream, by underground drainage. t is pretty safe to say in this country, and within rather /ide limits as to basin formation, that from one-quarter o one-half of the annual precipitation may be counted upon.

2014. Average Yields.—A common available yield hroughout the Middle and Eastern States is 8,000,000 J. S. gallons per inch of rainfall, per square mile. Thus, yearly rainfall of 46 inches will generally yield about 368 ıillion gallons per year. So generally is this the case, that n the Croton district, where the average yearly precipitaion is about 46 inches, it is customary to count on an averge yield of one million gallons per 24 hours, per square nile, and it is believed that this average is, if anything, xceeded in this district. This corresponds to nearly half ıf the precipitation. If we allowed only one-quarter of the ainfall as available, then we should have but about 4½ millon gallons per inch per square mile of drainage area, which vould yield, from an assumed rainfall of 46 inches, a supply ›f about 200 million gallons per year.

2015. Deductions to be Made from Area of Watershed.—In estimating the area of a watershed, all :xposed water surfaces, such as ponds and lakes, should be leducted, because the evaporation from these will, with ıverage rainfalls, balance the amount of precipitation upon hem. Even swamps and marshes when extensive in area hould be considered, and a certain deduction made, deıending upon their greater or less degree of saturation, ssimilating them, to a corresponding degree, to exposed 'ater surfaces.

2016. Numerical Example.—In order to illustrate the manner of making an intelligent study of quantity, it will be best to assume a particular case. Suppose the town in question to contain 25,000 inhabitants, which it is desired to supply with water from a stream of satisfactory quality and location, the only uncertain element of which stream being the quantity which it can furnish. The first thing is to decide upon the quantity required. The present population of the town is 25,000. It may reasonably be expected to double in 25 years, and in erecting water-works we should look as far ahead as this, in preference to the minimum limit already given, of 10 years. Assuming a total consumption of 100 gallons per capita per day, the town would need within 25 years a daily supply of 5 million gallons, or 1,825,000,000 per year.

A survey has shown that the stream in question possesses a watershed of 20 square miles above the point at which it is proposed to erect the dam. Observations carried on in the neighborhood for a number of years back give an average yearly rainfall of 46 inches, with a recorded minimum of 38 inches during the year of greatest drought. Unless the records extend over a long period of years, we will assume that a year of still greater scarcity may occur, when the precipitation shall fall to 36 inches. A careful examination of the geology of the territory leads to the conclusion that it is neither very favorable nor the reverse for realizing a large percentage of this precipitation, and that a reasonable, conservative estimate would be one-third. This would amount to 12 inches, or one foot. A depth of one foot over one square mile = 208,530,432 gallons; consequently, the yield of the drainage area of 20 square miles is 4,170,608,640 gallons per year, or say 11.42 millions per day. That is to say, the required quantity is not quite 44% of the total estimated minimum flow of the stream. This is a satisfactory showing, and proves that, as far as the *yearly* supply is concerned, there is a considerable factor of safety as between supply and demand. But while the demand consists of the daily quantity of 5 million gallons, day in and day out, the supply,

though more than sufficient for the total yearly demand, is not furnished with the same daily uniformity, for experience shows that the minimum daily flow of most streams (except the largest rivers) may for short periods fall to less than one-fortieth of the average, while the freshet flow may exceed it by at least the same proportion.

It is clear, therefore, that, except in the rare cases where the minimum daily flow of the stream is at least equal to the maximum daily consumption, recourse must be had to **storage reservoirs,** whereby the surplus water of freshets may be impounded, to be drawn upon in times of drought. The principle governing the calculation of the capacity of these reservoirs will now be considered.

STORAGE.

2017. Amount of Storage Required.—To intelligently estimate the amount of stored water necessary to convert a total yearly supply into a daily average is one of the most responsible and difficult tasks of the hydraulic engineer, from the fact that the data are so fluctuating in the various cases which present themselves that it is impossible to frame a perfectly satisfactory general rule. To accurately determine the question, it would be necessary to know the daily yield of the stream in years of minimum flow, and thus find the deficit of each day when the flow fell below the daily consumption. The sum of these several deficits would be the amount of stored water required. It is very rare, however, that an opportunity is afforded to gather such exhaustive data before establishing a new water-works system, and the engineer is generally obliged to content himself with an intelligently founded approximation. In this, a due consideration of the special case, and the teachings of previous experience elsewhere, are invaluable guides.

The two limits are, first, the case already mentioned where the minimum daily flow of the stream is equal to the maximum daily consumption, when no storage is required, and, secondly, that in which the total yearly flow is just equal to the total yearly demand, when the maximum of storage is

required. In this latter case, not a drop of water may be lost, and the only sure way to accomplish this result would be to have a storage capacity equal to the entire yearly flow, and commence supplying the town only when the reservoir was full. In this way, there would be a year's supply on hand, to start with, and a scarcity of water could never occur unless the consumption were to be increased.

These two cases would be the extremes: one in which no storage was necessary, and the other in which the storage should equal the entire yearly yield of the watershed. Hence, as a first general principle, we see that the nearer the yearly consumption approaches the total yearly flow of the stream, the greater must be the storage, and *vice versa*.

A rough approximation, based upon this principle, would be to make the amount of stored water the same percentage of the total yearly consumption, as the total yearly consumption was of the total yield.* Applying this rule to the present case, it would be necessary to provide 44% of 1,825,000,000 = 803,000,000 gallons. This is equal to about 160 days' supply, and would accord with safe practice.

2018. Number of Reservoirs Required.—The next step in the investigation of this particular source of supply would be to ascertain if the topography of the valley admitted of the economical construction of one or more storage reservoirs of the desired capacity. The considerations governing this feature of the problem will be treated of in their proper place; for the present it will suffice to say that, in the comparison of projects, a careful estimate must be made of the cost of these reservoirs. It may be noted that it would be a very advantageous circumstance if the ground permitted of the building of at least two reservoirs, either on the main stream or on some one or more of its tributaries. In such case, a dam and reservoir

* This relation may be expressed in a formula, thus: Let S = required storage; c = yearly consumption; Y = yearly yield. Then, $S = \frac{c^2}{Y}$.

could be built, preferably upon a principal tributary, which should suffice for perhaps ten years, leaving the construction of another larger one to a later period when required. Although the combined cost of the two might exceed that of a single reservoir of equal capacity, the advantage of raising a smaller sum of money at the start, with the consequent saving of interest for a number of years, would go far to outweigh this consideration.

The above two sources of possible supply belong to the general class of *surface water;* the others belong to that of *ground water*, and involve the consideration of *Wells*.

WELLS.

2019. Deep and Shallow Wells.—Wells may be either deep or shallow. Shallow wells reaching only to the upper surface of the water-table possess many objectionable features disqualifying them as sources of large supply. They are very apt to be polluted by surface impurities, and are highly objectionable when dug in the vicinity of human habitations, where they are nearly certain to be contaminated. Moreover, each well furnishes only a comparatively small quantity of water, and is liable to go dry in seasons of drought, for then the neighboring streams are drawing heavily upon the underground storage. Deep or driven wells only will be considered.

2020. Deep Wells.—These wells have long been employed in the arid regions of the West for irrigation, but it is only comparatively lately that they have been used in the East as a source of water supply. They are destined to play a more important part in the future, for as the country surrounding our larger towns becomes more and more thickly settled, the difficulty of securing large uncontaminated supplies of surface water increases, and more attention is consequently paid to the vast quantities of pure water stored away beneath the surface of the earth. These reserves of water due to absorption of rainfall have already been several times referred to. They constitute a body of

great but unknown extent, and are apparently slowly working their way along to lower levels, somewhat like glaciers, following the lines of least resistance offered to them by the character of the geological strata through which they pass. Each of these underground lakes and rivers—for these waters may possess both characters—has no doubt its own watershed or area of absorption whence it is recruited, but the utmost uncertainty exists as to what the bounds of these areas may be. It is impossible to make gaugings and surveys. It is even difficult to predict, in a section where none of these deep wells have been put down, whether or no water will be found at all, within reasonable depths. The judgment of experts as to the chances of finding water in sufficient quantities in any given locality, based upon surface indications, is of small value, and the only test, and that not a sure one, is to put down trial borings at those points which appear most favorable. Naturally, wells sunk in the valleys of rivers, near the stream itself, would be looked upon as more likely to develop large bodies of water than those on higher ground.

The principle of the deep well is that wherever a permeable stratum, having an exposed elevated outcrop, is found lying between two impermeable strata, the enclosed permeable stratum will be found to be saturated with water. If, then, the upper impermeable stratum is pierced with a water-tight tube, the water will rise to a greater or less height in the tube, according to the hydraulic pressure at that point, and will either rise to the surface or at least enable us to raise the water by pumping. If the tube happens to strike a depression in the water-bearing stratum, from which it rises both ways, an underground basin or reservoir will have been struck, which is an advantageous circumstance. If, however, it strikes a point on the dip or the rise of the stratum, it will be located in an underground river of which the water moves with a very low velocity.

It is very seldom that a sufficient quantity of water can be obtained from a single well; so it is customary to put in a "gang." The several wells composing this gang may

more or less interfere with each other, and it is frequently the case that when a well or gang of wells has been furnishing a large quantity of water, this quantity is materially decreased by other wells, sunk perhaps a mile or more away.

2021. Advantages of a Supply of Deep Ground Water.—The advantages of a supply of water derived from driven wells are, that it is most likely to be pure, that there is no expense for land and construction of reservoirs, and that, generally speaking, it can be increased from time to time as needed, by simply putting down more wells. The disadvantages are, a liability to hardness, a probable high temperature, and considerable uncertainty as to the amount of water obtainable and the permanence of the supply.

2022. Operating Deep Wells.—The facility and economy with which these wells can be operated will depend mainly upon whether the water rises naturally to or near the surface, or only to a considerable depth below it. This is much more particularly the case when a gang of wells is employed. If the water rises either to the surface or within suction distance of it, each well of the gang can be connected with a single collecting pipe, suction or otherwise, and thus be led to the pump plungers. If, however, the water does not rise to this height, a lift pump must be placed in each well, to raise the water to the height of the plungers. This greatly complicates the work, and would go far towards condemning such a supply if there were an alternative. If, in spite of this drawback, it were still decided to use this system, some application of electricity to work the separate pumps from a central station, or an adaptation of the "air lift," might be advantageously employed.

SPRINGS.

2023. As regards the last-named possible source of supply given in Art. **2007**, namely, abundant springs breaking forth at some point where they may be collected into a basin and thence distributed, it will be found that

this supply is the least frequently encountered of all the different ones mentioned—at least in this country, where, probably, sufficient attention has not been directed to the development of this source of supply. Springs constitute a special case of the flowing well, without the expense of boring. Unlike the deep-driven well, the water of such springs is generally of a low temperature. Care must be taken to assure one's self that the spring is not contaminated by surface or subsoil drainage. One of the most nôteworthy instances of such a supply is that of the city of Havana, Cuba, where springs furnish over 5 millions of cubic feet per 24 hours of exceedingly pure and cool water. The domestic supply of the city of Paris, France, is also derived from springs, the water of which is brought from a long distance by a magnificent system of aqueducts. The greatest care is necessary in collecting and handling the springs, or they may be entirely diverted into new and inaccessible channels.

SUMMARY.

2024. Of all the above-mentioned sources of water supply, that one which depends upon the damming of some stream other than a navigable river is probably the most frequently encountered in American practice. It is the simplest case, in that it generally admits of the greatest amount of certainty in determining the quantity and quality of the water and the kind and cost of the work necessary to utilize the supply. The well system is the one about which there will almost always be the most doubt and uncertainty as regards all of these points. The collecting of the water of flowing springs is of such rare occurrence as to scarcely warrant more than a mere mention of the fact that such a supply is occasionally found and utilized. The water drawn from large or navigable rivers, while possessing the great advantages of a practically unlimited supply and of being very soft, must in nearly every case be purified before it is fit for domestic use as a potable water.

PURIFICATION OF WATER.

SELF-PURIFICATION.

2025. It was long thought that all flowing streams possessed within themselves the necessary conditions for self-purification. That is to say, the free exposure to the air, particularly when the water was broken up by dashing over the natural obstructions found in its bed, was supposed to ensure that *complete oxidation of organic substances* which is the essential agent in the purification of a polluted water. This is proved to be a delusion, and while it is true that *some* beneficial effect is thus produced, it is so slight and uncertain as not to be counted upon.

The apparent purification which is sometimes noticed is frequently nothing more than *dilution* by the admixture of a large volume of pure water.

FILTRATION.

2026. The only efficient method by which wholesomeness can be restored to a contaminated water is by filtration, either natural or artificial. In regard to filtration in general, it may be said that here, also, the real effects produced were for a long time, and until comparatively recent years, misunderstood. It was supposed that filtration only removed matters held in suspension in the water, and that it had no chemical effect whatever upon it. It is now known beyond a doubt that its action is not thus restricted, but that it has a distinct and well-defined action in removing noxious elements.

It would be undesirable, within the limits of this Course, to enter fully into the chemistry of filtration. The following extract from Prof. Mason's "Water Supply" gives in a few well-chosen words the main facts of the subject: "Nature disposes in sundry ways of the various elements of impurity added to water, but by far the most efficient of these is the interesting process termed 'nitrification.' This

is a change of state best established by infiltration through soil, a few feet of such passage being capable of doing more to restore a water to its original purity than many miles of flow in an open channel.

"Nitrification is accomplished by a bacillus whose function it is to tear asunder the objectionable nitrogenous organic materials and convert them into harmless inorganic forms, which are at the same time valuable as plant food."

All ground water is subject to more or less of this natural filtration, which is the principal cause of its general purity.

FILTERING GALLERIES.

2027. Filtering galleries constitute a system of filtration partly natural and partly artificial, much practised when the water of large rivers is employed.

Fig. 648 is a cross-sectional sketch, purposely without

FIG. 648.

scale, showing the general arrangement of such a gallery. The action of these galleries was a long time misunderstood, like almost everything connected with filtration. It was supposed that by constructing a gallery as shown in the figure, the water of the river would percolate through the

bank and in a purified condition enter the gallery. Some water no doubt does this, but it is now generally understood that the great bulk of the water which enters the gallery is ground water, intercepted on its way to the river. There will generally be a considerable difference between the two banks of the river as regards quantity of water thus secured. That bank should be preferred which, from the greater abundance of springs, dip of the strata, and other indications, promises the more abundant supply. It will frequently be found that the preferable side, from this point of view, is on the opposite bank of the river from the town to be supplied. When this occurs, either the more unfavorable side must be taken, or the water must be conveyed by tunnel or aqueduct across the river.

The side walls and arch of the gallery, as shown in the figure, are built of hydraulic masonry and are comparatively water-tight, although probably never completely so. The bottom is formed of some permeable material, such as coarse gravel, and the theory of the thing is that the water, filtering through the ground, rises up through the permeable bottom of the gallery and flows through it to the point whence it is withdrawn for distribution. As the pressure is always greater from without to within, the leakage will be in that direction also.

FILTER-BEDS.

2028. The system of purifying water by means of **filter-beds** seems to be of English origin. In principle, it is very simple, the apparatus consisting essentially of an impermeable recipient or reservoir containing successive layers of broken stone, coarse and fine gravel, and sand, the coarser material being at the bottom. Sometimes the broken stone is replaced by tiles or drain-pipe, through which the filtered water is drawn off.

As regards the thickness of the several beds, much diversity exists in practice. A somewhat typical example is given in Fig. 649.

The working of such a filter-bed is readily understood.

The water working its way down through the successive layers of material is gradually separated from its impurities, and passes out in a state fit for domestic use. There is, however, a curious element in the action of the filter, in the presence of a fine film of sediment deposited by the water itself upon the surface of the upper layer of fine sand. This film contains the bacillus already referred to (Art. **2026**) as being instrumental in accomplishing the desired nitrification of the organic matter contained in the substances which it is desired to eliminate from the water. In this way the water itself furnishes the most potent agent for its own purification. It was mainly through ignorance of the fact of the existence of this film with its bacillus which led engineers and chemists to underrate the action of the filter in actually purifying, and not merely straining the water.

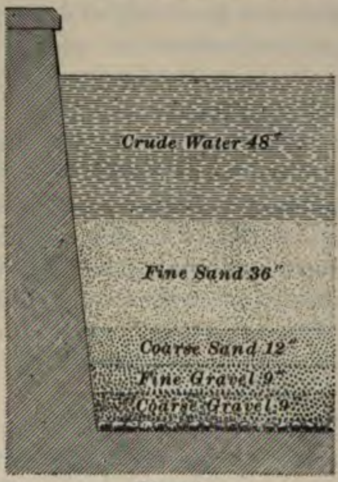


FIG. 649.

Although the principal part of the purification is effected by this film or slimy coating and the fine sand upon which it rests, the remaining coarser beds of material are not without a certain refining effect upon the water which has already passed through the more active upper bed.

2029. Cleaning Filter-Beds.—After the filter has been in use for some time—the length depending upon the degree of impurity of the water—it becomes clogged, and either passes the water very slowly or refuses to pass it at all. This is a warning or a notification that it requires cleaning. In order to clean it, the water is drawn off entirely, and a thin scraping, from half an inch to an inch and a half, is taken off from the top surface of the upper layer

of fine sand. The material thus removed is cleaned by washing, and set aside for future use. When the thickness of the upper bed of fine sand has been reduced by repeated scrapings to about 24 inches—12 inches being the absolute minimum permissible—the washed sand previously removed is replaced until the original thickness of the bed has been restored.

2030. Frequency of Cleaning.—As a general approximation, it may be accepted that a filter-bed will require cleaning as above described every two or three weeks in summer, and every month or six weeks in winter, but these periods may greatly vary in special cases. In the best and most careful practice, after cleaning the filter is saturated with filtered water before putting it again in service.

2031. Accessories of Filter-Beds; Settling Basins.—A well-designed filtering plant embraces a settling basin, in which the water receives a preliminary treatment by sedimentation, before being introduced into the filter. This settling basin is particularly needed in the case of very turbid river-water; otherwise it is commonly dispensed with. Fig. 650 is a sketch illustrating this general

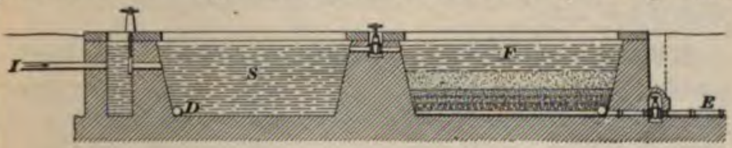


FIG. 650.

arrangement. In this sketch, *I* is the crude water inlet, *S* the settling basin, *F* the filter, *E* the outlet for filtered water, and *D* the drain for emptying the settling basin. The capacity of the settling basin should be sufficient to contain at least one day's supply.

2032. Covering Filter-Beds.—In countries where the winter is very severe, it is found necessary to cover the filter-beds with a roof. In general, wherever the mean January temperature is below the freezing point, good authors

recommend that they be covered. This would include the greater part of the United States.

2033. Rate of Filtration, and Size of Filter-Beds.—As affecting the size of the filter-beds, it is desirable that the water should pass through them at a rapid rate, but this would be at the expense of the efficiency of the filter as a purifying apparatus. It is generally assumed that two and a half million gallons per acre of filter-bed per 24 hours, which corresponds to 4 inches of vertical descent per hour, is the maximum velocity at which the water should run through the filter. A supply of 5 millions of gallons per 24 hours would require, therefore, at least two acres of filter-beds.

It is necessary also to have a reserve area, so that when a filter is laid off for cleaning, the supply of filtered water to the town need not be interrupted. It is not necessary, particularly with large plants, to duplicate the entire system, for by dividing the necessary area into several comparatively small filter-beds, it can be so arranged that only a small percentage of the total area is ever laid off at a given time for cleaning.

The size of each filter-bed will generally vary from one-half to one and one-half acres. A convenient and usual unit is one acre.

2034. Cost of Filters and Filtration.—The cost of building and operating filters varies greatly. An ordinary average for European filter-beds, uncovered, and not including cost of settling basins or land, is from $40,000 to $50,000 per acre. Covering the filters increases the cost about 50 per cent., making the above figures $60,000 to $75,000 per acre. Probably the above estimate would be somewhat exceeded in the United States; still $50,000 for open and $75,000 for covered filters, not including cost of land nor of settling basins (which might not be required), would not in general be exceeded.

As regards operating expenses, the greatest single item is cleaning the filters. This will also vary greatly according to circumstances. The limits of expense of operating the

:ers may be roughly placed at from 1 to 2 cents per 1,000 llons.

2035. Results of Filtration.—In the first place, :ration effectively removes turbidity from water, and to ;reat extent unpleasant tastes and odors. Secondly, it s a distinct effect in preventing the spread of a number water-borne diseases, notably cholera and typhoid fever. may be stated broadly that the water of all large or vigable rivers having towns and villages situated upon eir banks should be filtered before being considered fit · domestic use. It may also be stated that in the case of y town or city using an unfiltered supply, in which the rmal death-rate from typhoid fever exceeds 1 in 4,000, e necessity for filtration is suggested, if not indicated.

As has been already stated, filtration is largely practised European cities, and its use is constantly growing in the iited States. A consideration of the subject is, therefore, :cing itself more and more into the science of water-works gineering.

MECHANICAL FILTRATION.

2036. The slowness of operation of the above-described :ll-tested and approved system of filtration, and the conquent size and cost of the plant, have induced many efforts devise some cheaper and quicker method of accomshing the same result. The result of these efforts has en, mainly in the United States, the development of the ·called **mechanical filtration,** which is described by ason as follows:

"Roughly outlined, this plan consists in adding to the iter to be filtered a minute dose of common alum, avering between one-quarter and one-half of a grain per llon, and then admitting the water to the filter, which is :ylinder of wood or boiler iron, three-quarters full of iformly fine sand. The carbonates present in the water compose the alum, with the formation of a white floccult precipitate of aluminum hydrate, quite jelly-like in pearance. The action of this aluminum hydrate is much

the same as that of the white of egg in clearing coffee. It entangles all suspended matter, disease germs as well as inorganic material, and deposits the same on the surface of the sand, whence it is removed and driven into the waste-pipe by a reverse current of filtered water at the time of cleaning the filter. The cleaning occupies but a short time, not much beyond fifteen minutes, and can be accomplished by a waste of less than ten per cent. (usually four per cent.) of the daily delivery of filtered water. Thus, it is observed the mechanical filter produces an artificial inorganic jelly to replace the 'bacteriæ jelly' of the English filter-bed, already alluded to. In properly managed filters of this type, no alum (or, at most, a trace) reaches the filtrate, for only such a quantity is admitted to the water as will be decomposed by the amount of carbonates present.

"A further action of the precipitated aluminum hydrate is to unite with the soluble coloring matter of the water, thereby rendering the filtrate colorless. The proper 'dose' of alum solution is administered by means of a small automatic measuring apparatus exterior to the filter."

2037. Increasing Use of Mechanical Filtration.—The use of these mechanical filters seems to be rapidly growing; they are still, however, somewhat on trial, so that it is impossible to speak more positively about them, except to say that they give high promise of usefulness. The system has been proposed for filtering the Potomac River water supply of the city of Washington, D. C.

It will be seen from this example that the system is considered adequate for the filtration of very large quantities of water.

2038. Cost of Mechanical Filtration.—An estimate of the cost of filtering the above supply, made by Capt. Symons, U. S. Corps of Engineers, places the cost of a complete mechanical plant for filtering 40,000,000 gallons per day at $600,000, including a considerable amount for connecting with and perfecting the old system. The same authority places the yearly cost of maintenance of the

above plant, and presumably the operation of the same (a comparatively small item), at $18,000. This amounts to about half a cent per 1,000 gallons.

RESERVOIRS.

INTRODUCTORY.

2039. Among the most important of the works of the hydraulic engineer must be classed the designing and building of reservoirs. These structures are divided into two classes: storage and distributing reservoirs. Sometimes a single structure fulfils at once the functions of both classes.

2040. Storage reservoirs are almost universally formed by constructing a dam across the valley of some stream of which it is desired to impound the waters. The water so stored is drawn off, either by pumping or by natural flow, according to elevation, and is usually delivered into a smaller or distributing reservoir, whence it enters directly into the mains. The control of the water is accomplished by comparatively simple appliances.

2041. Distributing reservoirs are frequently, perhaps generally, built in some convenient location apart from the stream whence they are supplied, either by excavating an area furnishing the required capacity, and lining it with some impermeable material if necessary, or by building a structure, commonly rectangular, of earth, or masonry, or both, standing entirely above the general level of the adjacent ground. Sometimes such reservoirs combine both features, being partly above and partly below ground, these points being determined mainly by the elevation it is necessary to maintain.

The appliances for the entrance, withdrawal, and general control of the water will be in most cases more complicated than in the case of the storage reservoir. Distributing reservoirs are also frequently covered. The covering of such reservoirs is a common practice in Europe, and a

growing one in this country, as a means of preventing vegetable growth—notably that of the algæ—and otherwise preserving the purity of the water which they contain.

STORAGE RESERVOIRS.

LOCATION.

2042. The selection of a proper site for a storage reservoir is a matter of the utmost importance. The first desideratum is that the dam should control a sufficiently large area of watershed lying above it to furnish the requisite volume of water. It should also be situated, if possible, at such an elevation that water will flow by gravity to the point of distribution. It should be at a narrow part of the valley, so as to decrease the length of the dam, and it should offer, if possible, a rock foundation. Not only should the site selected be at a narrow part of the valley, but the valley should be flat and wide above it, so that a dam of moderate height and length may impound a large volume of water.

Naturally, it is very rarely that a location embracing all these features can be found, and the skill and judgment of the engineer are exercised in selecting a site which, bearing all the desired features in view, shall most satisfactorily fulfil those which, under the circumstances, may be the most important.

2043. Examination of the Ground.—The first step, in selecting a proper site on any given stream, is to make a thorough reconnaissance of the entire valley, armed with pocket compass, aneroid barometer, thermometer, hand level, and sketch and note book. If possible, an assistant should remain at some fixed point of which the elevation is known, making hourly, or more frequent, observations with another barometer and thermometer, and the explorer should note the time at which he makes his barometrical observations. Without this precaution, his elevations will be much less trustworthy, because the barometer fluctuates with the changes of atmospheric density as well as by changes in altitude.

enerally speaking, in thus exploring a valley, one or e points will be plainly indicated as suitable for the ding of a dam. The next step will be to survey the ershed lying above each of these points, so as to ascer- , as before mentioned, the probable amount of water ch can be furnished at each location. Sometimes it will ossible to estimate, from maps or by simple observation, area with sufficient accuracy for a preliminary study out a survey, particularly when the size of the stream cates that the drainage area of the valley is very large.

hese preliminaries having been accomplished, a careful ey of the site of the proposed dam is made. Cross- ions are taken not only at the point which appears to eye to be the most favorable, but for some distance e and below such point. These cross-sections are then l in making a contour map of the territory covering the of the dam, and on this map a "paper location" of dam is made, more understandingly than would be ible by mere inspection of the ground. This paper tion is then laid out on the ground, and carefully nined to see if any small changes can be made to im- e it.

ext, an elevation is assumed for the level of the overflow, ch will determine the height of the dam. A "flow line" en run around from one end of the proposed dam to the r, which will establish the shape and area of the ground e covered with water when the dam is built and the rvoir filled. Generally several of such flow lines will be out, showing the area and shape of the artificial lake at rent heights of the dam. A sufficient number of cross- ions should be taken in this area to determine the roximate capacity of the proposed reservoir.

PREPARATIONS FOR CONSTRUCTION.

044. Supposing now that the site of the dam has definitely settled, after a comparison of all the locations nined and studied, both on the ground and on paper. next thing is to prepare for construction. For this

purpose the center line of the dam is run out and established by permanent monuments, either of stone, or less advantageously by strong posts driven into the ground with their heads nearly level with the surface, so that an instrument can be conveniently set over them. Two of such monuments should be set at each end of the center line, perhaps 50 or 100 feet apart, making four in all, so that there shall be no danger of "losing the line" by the destruction or displacement of one or more of them. The whole area that is to be in any way worked over in the construction of the dam is then cross-sectioned by being divided up into squares, 25 feet or less on a side, and an elevation taken at each corner, with intermediate points if the topography of the ground requires it. The squares with the elevations above datum of their corners are mapped in duplicate, or even triplicate, one copy being always kept in a place of perfect security, because this map constitutes a record of the ground before any work was commenced, and from it all the excavation and embankment—all the work that changes the face of the ground, in a word—can be and is calculated for the partial and final estimates. The importance of taking and securing this record must not be underrated, as it is the only way to know what work has actually been performed, and to decide disputes regarding such work.

At least two permanent bench marks are also established, from which other temporary ones can be set, in convenient locations, as needed for the work.

EARTHEN DAMS WITH MASONRY CENTER WALLS.

CLASSIFICATION OF DAMS.

2045. Dams may be divided into two classes, according to the material of which they are composed, namely, those built of masonry, either cut stone, rubble, or concrete, or a combination of these materials, and those built of earth, with a masonry center wall. There is still a third class,

composed of earth alone, with or without a "puddle core," but unless the dam is of verv insignificant height, this kind of construction should never be resorted to. Dams so built are eminently dangerous, and they are only mentioned here in order to call attention to this fact and to condemn their general use.

When rock is found on the bottom and at the sides of the valley upon which the proposed dam can be founded, then a masonry dam is always to be preferred. If no rock is to be found, so that the dam must be built upon an earth foundation, then an earthen dam, with masonry center wall, is greatly preferable, as being more secure. If, as is frequently the case, rock exists on one side of the valley and not on the other, then the dam may be of composite character, taking care that the center wall and earth embankment type be preserved on the side where there is no rock foundation.

The two types will now be described, beginning with the earthen dam.

CENTER WALL.

2046. In earthen dams the **center wall** commands our first attention. This wall should be carried down to a water-tight foundation, if possible, and its ends should be deeply embedded in the sides of the valley. Its object is twofold; first, to afford an impermeable and indestructible cut-off to any water which might otherwise percolate through the bank, either because the bank was not itself impermeable, or because it had been perforated by musk-rats or other burrowing vermin; secondly, to afford a means of making water-tight connections for the culverts or pipes used in conveying water from the reservoir, and which run through the embankment from one side to the other. Without such water-tight connection as the cut-off wall offers, there is always danger that the water may follow along *outside* of these pipes, and finally create a channel in the bank, allowing the water to escape thereby. When this occurs, the destruction of the dam, unprovided with a center wall, is prompt and certain.

In building the center wall, the first consideration is the foundation. The wall should be carried down until a stratum is encountered which appears to be impermeable. Generally speaking, the finer the material the better. Fine gravel or sand is probably the best. When these materials are mixed with clay, they also form an excellent material to found the center wall upon. Fine sand with smooth, round grains, mixed with a large percentage of clay, constitutes what is known as *quicksand*, and when found at a certain depth below the surface, so that it can not escape laterally, forms also an excellent and water-tight foundation. Loose, coarse gravel containing large stones or cobbles, is about the worst material that can be encountered. When such material is met with, the center wall must be carried down to a much greater depth than would otherwise be necessary. Increased depth of foundation compensates to a considerable degree for the want of suitable material upon which to build.

The center wall should be carried up as high as the level of the highest water in the reservoir. Its thickness at the surface of the ground may be one-quarter of its height, and this thickness may be reduced by a batter of one inch to the foot on both sides, or, preferably, by building the wall with vertical sides and stepping in two feet (one foot on each side) every ten feet, which amounts to about the same thing. As the foundation ascends the sides of the valley, it is stepped up, care being taken to keep the bottom of the wall well below the surface of the ground and well embedded in good material.

EMBANKMENT.

2047. The center wall, as above described, forms the *core* about which the **earthen embankment** is formed. This embankment rises to a certain height above the top of the wall, depending upon circumstances to be considered hereafter. It is flat on the top, and has a gentle slope on each side, the rate of slope depending greatly upon the material of which the bank is formed. The best material is undoubtedly a fine gravel or coarse sand, such as would

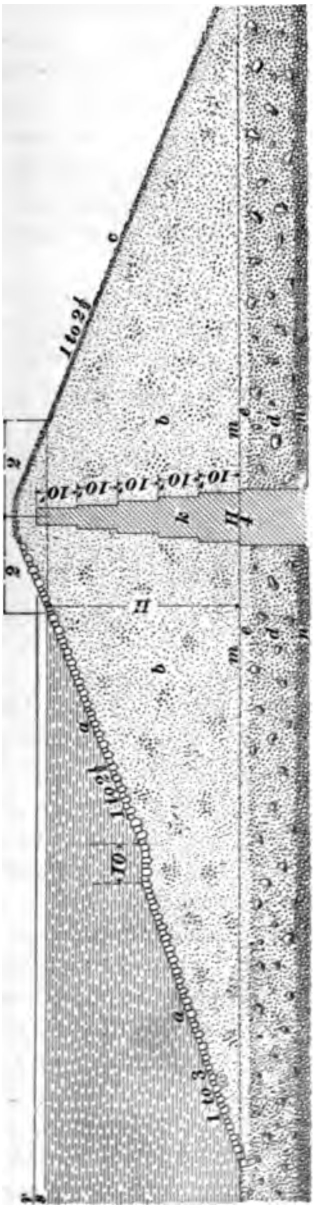


FIG. 651.

be proper for making mortar. Clay, although perfectly water-tight when confined, is a treacherous material in a bank, because it is so fine that it actually dissolves in the water, and is liable to run away in the form of semi-fluid mud.

A fair average for the outside slope, i. e., that on the lower or down-stream side (see *c*, Fig. 651), is 1 vertical to 2½ horizontal. A somewhat flatter slope is advisable on the inside or water side of the bank (marked *a* in the figure), say 1 vertical to from 2½ to 3½ horizontal. In a high embankment, say 50 feet and upwards, it is better to divide the inside slope into two or more steps, as shown in Fig. 651; that is, starting from the top, to carry a slope of say 1 to 2½ down for 25 or 30 feet, and then introduce a level "berme," 8 to 10 feet wide, continuing the slope with a somewhat flatter grade, say 1 to 3, for another 25 or 30 feet, when another level berme is introduced, and the slope then continued either with the same grade as before, or, preferably, somewhat flatter, say 1 to 3½.

The inside slope, next to the water, should be carefully paved, or "riprapped," with stone for a thickness of from one foot to two feet from the bottom to a point well above high-water line. The greatest thickness should be at the level of high water in the reservoir. The stone composing this paving must be placed and packed by hand, not dumped at random. The outside slope should be sodded, or, at least, sown to grass, and carefully tended till a good sod is formed.

The earthen embankment should be carried up in horizontal layers, and kept constantly moist by sprinkling. In many specifications, it is provided that these layers must be consolidated with a roller. This is certainly favorable to good work, but can hardly be considered necessary. If the material be brought to the bank in wagons, and evenly spread by shovels and horse scrapers, being kept constantly moist meanwhile, the travel of the men, horses, wagons, and scrapers is generally all that is needed to secure a good bank.

Before placing the embankment, the natural surface upon which it is to stand must be carefully stripped of all sods, roots, and vegetation, so that the first layer of the earthen embankment may rest upon and incorporate itself with clean earth. This is particularly necessary under the inner slope.

Fig. 651 shows all these features in a general way, and represents a correct type of earthen dam, but requires some further explanation. In this figure, *a* is the inner slope of the embankment, *b*, *b* the earthwork embankment, *c* the outer slope, *d*, *d* loose material under the embankment from which the vegetable soil was stripped, as shown by the lighter shading at *e*, *e*. The original surface of the ground is shown by the dotted line *m m*, and *k* is the center wall, carried well down into the compact sand and gravel *n*, *n*. The line *r* shows the high-water level or level of freshet overflow, and *s* is the low-water level or level at which overflow begins. It will be perceived that several of the dimensions depend upon *H*, *H being the vertical distance*

rom the natural surface of the ground to the level of the verflow, or lip of the spillway. If the dam crosses a deep nd wide stream, then H must be taken *equal to the vertical 'istance from the bottom of the stream to the lip of the spill-vay.*

An excellent, though somewhat more expensive, design or the top of the dam is shown in Fig. 652, where the cen-er wall is extended through the embankment and forms a

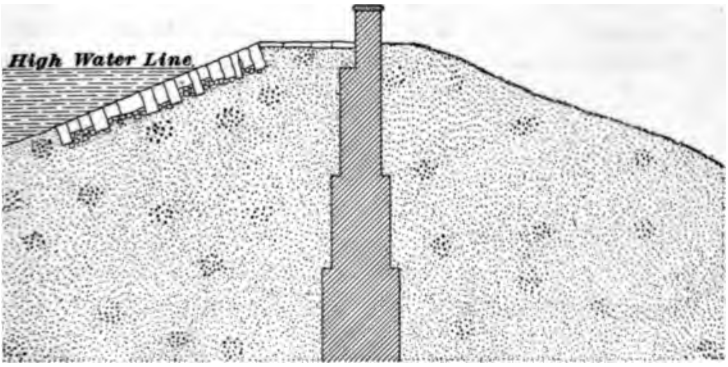


FIG. 652.

arapet wall along the top. If the top of the bank is flagged, s shown in the figure, a very complete and secure piece of ork will be the result.

SPILLWAY, OR OVERFLOW.

2048. The **spillway** is one of the most important eatures of a dam. It is the means by which the surplus ater, when the reservoir is full, is allowed to run to waste, nd want of sufficient discharging capacity in this particular as been probably the most prolific cause of destruction of arthen dams. If such dams are once overtopped by a ood, especially when not provided with a proper center all, they are rapidly cut down and destroyed by the water unning over them.

Frequently a natural overflow can be found in some lateral epression of the ground, by which the surplus water can be

passed into another valley. When such an opportunity occurs, particularly if the depression is in rock, it is generally availed of, as saving a considerable expense. Sometimes it is possible to form such an overflow, by cutting into some rocky ridge leading either into another valley, or into the same one across which the dam is built. In the majority of cases, however, it is found necessary to provide a special piece of masonry construction for the purpose.

The dimensions of this spillway must be proportioned to the amount of water liable to go over in times of freshets. It may be comparatively long and shallow, or short and deep. A convenient length is given by the formula

$$L = 20\sqrt{A}, \qquad \textbf{(173.)}$$

in which L = length of lip of spillway in feet, and A = area of watershed above the dam in square miles.

For the depth, or vertical distance of the lip of spillway below the level of high water in the reservoir, a convenient formula (the length having been determined by the formula just given) is

$$D = \frac{\sqrt[3]{Q^2 \times A}}{16} + C, \qquad \textbf{(174.)}$$

in which D = depth in feet of notch of spillway, Q = cubic feet of water per second, per square mile, A = area of watershed in square miles, as before, and C = a certain additional height above high-water level, depending upon the character of the dam, being less for a rock-founded masonry dam than for an earthen one.

EXAMPLE.—What are (*a*) the proper length, and (*b*) the depth of the spillway of an earthen dam 50 ft. high, built as already described, the area of watershed above the dam being 16 square miles, and the maximum freshet flow being estimated at 70 cubic feet per square mile per second?

SOLUTION.—(*a*) Substituting the value of A in formula **173**,

$$L = 20\sqrt{16} = 80 \text{ ft. Ans.}$$

(*b*) Substituting given values in formula **174**,

$$D = \frac{\sqrt[3]{70^2 \times 16}}{16} + C = \frac{\sqrt[3]{78,400}}{16} + C = 2.68 + C. \text{ Ans.}$$

2049. To determine the value of C, the height and ıaracter of the dam and the area of watershed must be ıken into consideration. Under no circumstances must the ım be overtopped. In the present case, it is probable ıat $C = 4$ to 6 ft. would answer. It must be borne in ind that adding a few feet to the top of the embankment creases the amount of material very slightly, as will be en from Fig. 651, where the embankment could be connued to a considerably greater height with but little adtional volume of material, by somewhat steepening the opes above the water-line. In such dams, therefore, a reat additional security against overtopping can be obıined with but little extra cost.

If we assume $Q = 64$, which corresponds to a little over 1 million gallons per 24 hours, per square mile, and reprents a very powerful freshet flow, although not, perhaps, the ıaximum, then formula **174** reduces in round numbers to

$$D = \sqrt[3]{A} + C. \qquad \textbf{(175.)}$$

2050. We can now examine Fig. 651 more closely. I being the height from lowest point of natural surface at ıe dam to the lip of spillway, the thickness of the earth mbankment at the level of the lip is taken equal to H, venly divided on each side of the center line of the dam.

The thickness of the base of the center wall $= \frac{H}{4}$, and at very 10 feet it is stepped in 1 foot on each side. There is lso a small offset, or footing, given to the foundation, 'hich is carried down to a secure formation of fine sand nd gravel. The top of the center wall will be carried up ɔ the height of the notch of the spillway, or to a height $=$ $) - C$, as determined by formula **174,** above the lip.

The following illustrative example will show how the bove principles are applied:

Referring to Fig. 651, let $H = 48$ feet, and let $D - C =$ .5 ft. The center wall commences with a thickness of 2 ft. At a height of 10 ft. it is drawn in by offsets to 0 ft.; at 20 ft. to 8 ft., at 30 ft. to 6 ft., at 40 ft. to 4 ft.,

which thickness is carried through to the top, the total height of center wall above foundation being $48 + 2.50 = 50.50$ ft. The embankment is carried up 8 ft. above level of spillway. At the level of the spillway, or low-water line, it has a thickness of 48 ft., the top width being 8 ft. At a depth of 27 feet below the spillway, a berme 10 ft. wide is introduced, and the slope continued at the rate of 1 to 3.

2051. Cross-section of Spillway.—When no natural overflow, as already mentioned, is available, an artificial one must be built. This will generally be in the line of the stream across which the dam is built, although it may sometimes be placed at or near one end of the dam, in case rock is found there, or if for any cause the ground seems more favorable. If rock is not found upon which to build the structure, then the foundations must be carried well down, perhaps deeper than those of the adjacent center wall.

2052. Form of Spillway.—The form and dimensions of the cross-section of the spillway vary greatly, excellent examples from actual structures showing considerable difference in the ideas of their designers. This is as much as to say that there are no hard and fast rules governing this point. One of the principal differences that will be noticed in existing works is the form of the face of the spillway, over which the water passes. This is sometimes formed in steps, and sometimes in a concave curve. The steps constitute the most economical form, because the curved face requires a great deal of cut stone of voussoir shape. The object sought—besides economy—in using steps is to break the force of the falling water, so that it shall reach the bottom with very little velocity, either vertical or horizontal. The idea embraced in the curved face is exactly the contrary, the object sought—or at least obtained—being the greatest possible horizontal velocity when the water has reached the bottom of its fall, urging it rapidly forwards away from the foot of the spillway. Of the two, the step system seems preferable, as the water appears to reach the bottom with the least destructive force when its fall is thus

roken up. Of course, lower down in the stream, the veloc-ty of the water is the same, whichever system has been dopted. For low dams, say up to 15 or 20 feet, the face nay be nearly vertical, giving a clear fall to the water upon he apron at the bottom.

For higher dams, say up to 60 or 70 feet, the form shown in Fig. 653 is a very good one. Let $AB = H$, or height to lip of spillway, as already lescribed. Take $AD = \frac{9}{10} H$; also, $AC = \frac{9}{10} H$. Join C and D. Take $BE = \frac{H}{5}$. From E, draw EF with a face batter of $\frac{1}{12}$, intersect-ing CD in F. Give EG a slope of 1 in 4 or 5, and round off the corner at E, taking care not to lose height. This is accom-plished by starting the slope a little back of E. Draw steps of convenient height, following the line FD.

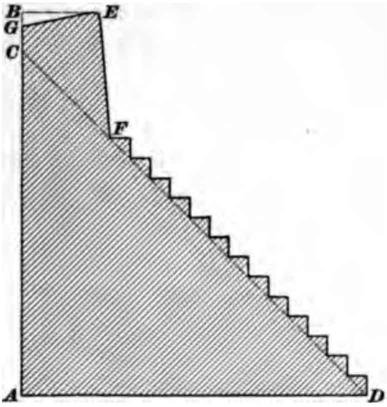


FIG. 653.

EXAMPLE.—Referring to Fig. 653, let $AB = H = 60$ ft., and find the other dimensions.

SOLUTION.— $AD = AC = \frac{9}{10} \times 60 = 54$ ft. $BE = \frac{60}{5} = 12$ ft. Ans.

Spillways for still higher dams require a special study, and can be best considered after a study of high masonry dams.

2053. Accessories of Spillway.—The sides of the spillway, where it cuts through the embankment, must necessarily be protected by wing walls to prevent the earth from falling into it. These wing walls need not have as flat a slope as the exterior embankment, which would make them unnecessarily large and expensive. They may have a slope of say 1 to $1\frac{1}{2}$, and the bank can be graded down to them. The top of the wall may have a coping, well doweled into the stones, or be left with the horizontal courses forming

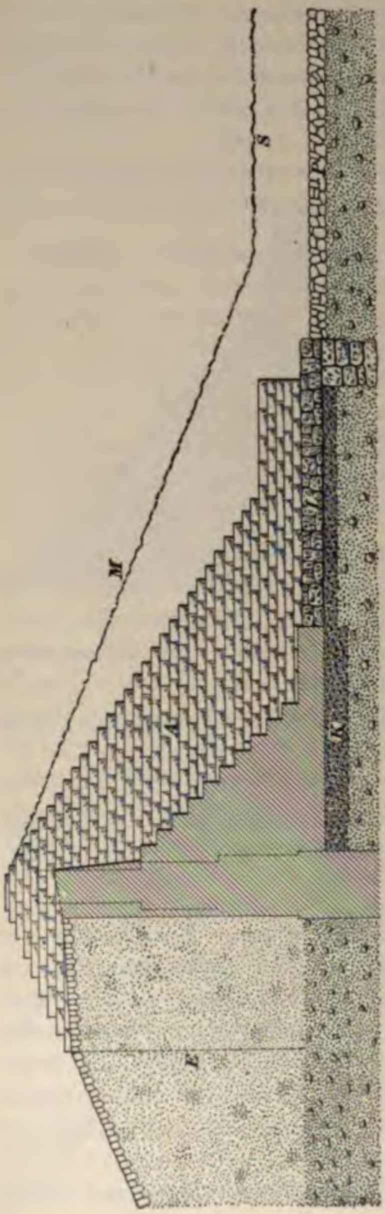


FIG. 654.

steps, a construction which, though not so neat in appearance, is more substantial. When the bed of the stream is not rocky, the foot of the spillway must be protected by an apron composed of very heavy stones, if they can be obtained, laid in cement, upon a bed of concrete. This apron must be extended well beyond the foot of the spillway, the distance depending upon the height of the dam, the volume of water passing over it, and the nature of the bed of the stream. Probably it should never extend less than a distance equal to the height of the dam. Beyond this, it will be well to protect the bed of the stream between the banks for some distance further with heavy dry stone paving. All of these features are shown in the sketch, Fig. 654. No dimensions are given in this sketch, because the conditions of each particular case will greatly modify the details, but the general arrangement

and proportions will always resemble that shown. In this sketch the wing wall of ashlar masonry is shown at *A*; *M* is the exterior slope of the embankment, and *S* the bank of the stream below the dam. Heavy paving-stones laid in cement are shown at *L*, these stones, as well as the masonry composing the apron, being laid on a bed of concrete *K*. The bed of the stream is also protected by the dry paving-stones *P*, to prevent cutting by the current.

APPLIANCES FOR DRAWING OFF THE WATER.

2054. There are many different ways practised successfully for the control of the water. The desiderata are that they shall be simple and effective, not liable to get out of order, and satisfactorily fulfilling their destined purpose. In all cases a communication must be established between the inside and the outside of the dam, and it is of the utmost importance that no water shall follow along the *outside* of the appliance, whether tunnel, gallery, or pipe line, by means of which this communication is established. Herein lies one of the great advantages of a substantial center wall, such as already described, for it affords the means of making a water-tight connection in the center of the embankment, beyond which any trickle of water can not pass. As these features of the dam call for a considerable amount of heavy and expensive masonry, it is also important to so design them that the necessary degree of solidity may be secured with the least possible volume of masonry. This is best accomplished by grouping the different parts together, so that they may be mutually supporting, and so that a portion of one may also form a portion of another.

In Fig. 655, (*a*) shows a plan and (*b*) a vertical section on the line *A B* of a general system of design which experience proves to be strong and satisfactory. In the same figure, (*c*) shows a front elevation and section through gatehouse *n* looking towards the spillway from outside the reservoir, the embankment being supposed to be removed.

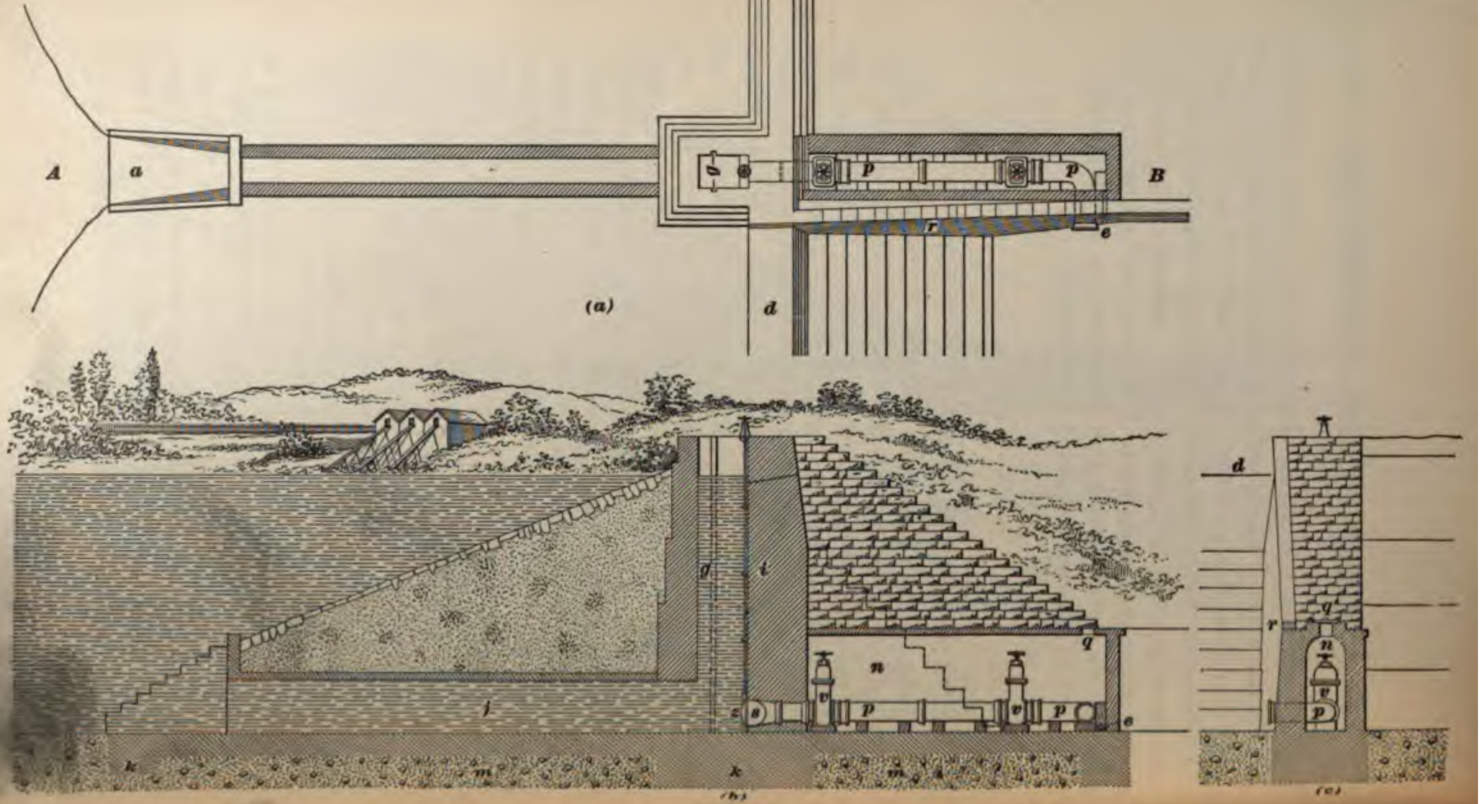


FIG. 655.

These figures are self-explanatory, but a few words concerning them will aid in their proper understanding.

On the inside of the dam, adjacent to the spillway *d*, is built a water tower *i*, one side of which is formed by the prolongation of the spillway itself. This tower communicates with the inside of the reservoir by means of an arched gallery or tunnel *j* passing under the interior embankment and terminating in an open portal *a* with wing walls. The top of the tower is level with the top of the embankment. The reducer *s* leading to the cast-iron pipe *p* is securely built into the opposite wall of the tower, and the pipe itself is placed in an arched gallery *n* under the exterior embankment, one wall of this gallery forming part of the main wing wall *r* of the spillway. The reducer *s* is a special casting, having a rectangular opening at the face, of equal area with the pipe to which the other end is fitted.

The pipe *p* discharges, in some convenient way, into the channel of the stream below the dam. In this way a clear communication is established between the inside and the outside of the reservoir, through which the water can freely pass.

The water passing through this system can be controlled in several ways. The best way is to close the mouth of the reducer on the inside of the tower, with a sliding sluice gate *z*, as shown in the figure. Besides this, there should be a stop-cock, or valve, upon the pipe inside of the exterior gallery *n*, by which the letting on or shutting off of the water will be ordinarily effected, the sluice-gate *s* remaining open, and only closed upon some emergency occurring, such as an accident to the valve. If no such sluice gate is provided, then there should be two stop-cocks on the pipe. In the figures, two valves *v*, *v* are shown, in addition to the sluice-gate, but this would not generally be considered necessary.

Whichever of the two systems is employed and the single valve and sluice-gate is probably the preferable one there should always be a set of grooves *g* cut in the masonry of the tower, in which, in an emergency, stop plank may be placed. These are heavy timbers, of suitable dimensions, which

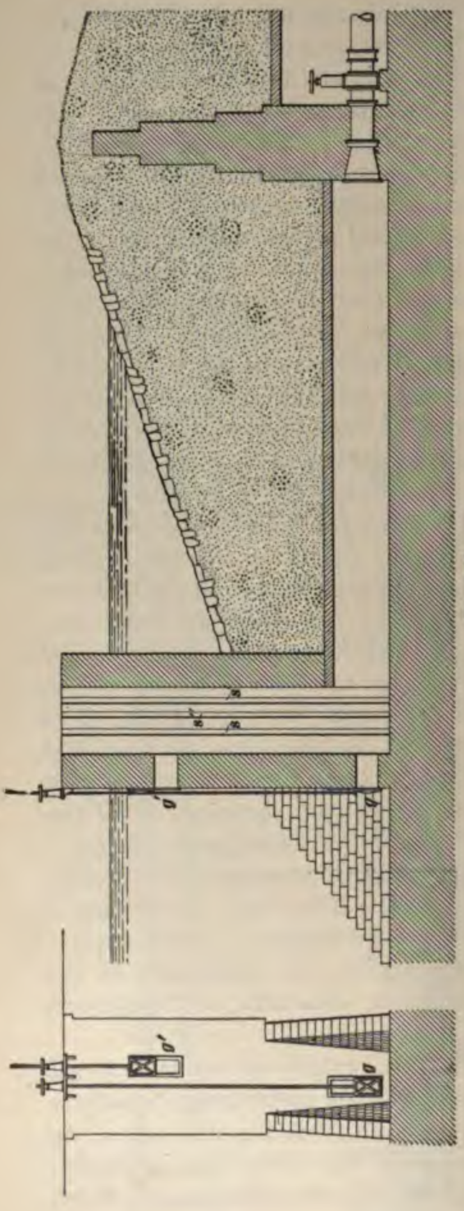


FIG. 656.

are slipped
into the g
ing so reen
iron plates
readily in
These iron
be made
ously in tl
angle irons,
afford mea
draw them
quired wit
hooks.
plank const
valuable
shutting of
in case of
so as to gai
the mouth
without en
reservoir.
access to
n is provi
manhole *g*
gallery is
from wat
small drai
whole stru
on a hydr
foundation
must exten
rock or s
m, *m*.

2055.
Water f
ferent L
Sometimes

reservoir serves both for storage and distribution, it is considered desirable to have the means of drawing water from two different levels, one near the bottom and the other near the surface. There are several different ways in which this may be accomplished; one of the most common arrangements is shown in Fig. 656, which is a vertical section through the gallery, corresponding to the one already shown in Fig. 655. In this figure, the tower is seen to be removed from one end of the gallery and placed at the other, so as to be exposed to the water throughout its entire height. It has two openings *g* and *g'* at different levels, each closed with a sluice-gate and placed sufficiently out of line with each other to prevent interference of the spindles and windlasses of the two gates. The figure is self-explanatory. At the left is an end elevation of the tower, looking at it from the inside of the reservoir. The tower, either in this position or as shown in Fig. 655, is frequently covered with a roof. The tower shown in Fig. 656 is provided with one groove *s'* for stop plank, and, in addition, two grooves *s*, *s'* are provided for the purpose of inserting screens.

2056. Modifications of the Above Designs. For small reservoirs, the appliances and arrangements described are frequently simplified. For instance, the tower, the object of which is to afford means of applying the sluice gates and stop plank, may be done away with, and the pipe run through the center wall to the foot of the interior slope; or the arched gallery may be continued to the center wall and then connected with the pipe. In these cases, the only control of the water will be by means of the stop-cocks in the exterior gate-house.

DISTRIBUTING RESERVOIRS.

2057. Distributing reservoirs are relatively small reservoirs containing usually a few days' supply, sometimes that of a single day only, and sometimes so much as to make them veritable storage reservoirs. Their main object,

however, is to transfer and maintain an undiminished head to a point as near as possible to that of delivery. This feature will be better understood in connection with the subject of the *flow of water through long pipes.* Another very important object is to furnish the means for establishing all the appliances necessary to the perfect control of the water delivered to the city mains. It is also, incidentally, a great advantage to have even a twenty-four hours' supply of water close to the town, as it will often permit small repairs to be executed without interrupting the service. This is particularly advantageous when the supply is pumped.

The general principles of the construction of these reservoirs are the same as those for storage. They usually consist of two contiguous rectangular basins—square, if possible—divided by a separating wall, and are so arranged that either basin can be shut off and emptied for the purpose of cleaning, while the town is supplied from the other. There is generally also a pipe, or culvert, placed in the separating wall, by means of which the supply can be sent directly into the distributing mains without entering either basin. The appliances for controlling the water are therefore more complicated than those already considered. There are many different ways in which the desired end may be attained, each having something to recommend it, according to the circumstances of the case. An example of one method will be given later on.

2058. Proper Site for Distributing Reservoir.—When possible, the summit of a rounded hill or some elevated table-land will be selected, in which the reservoir can be excavated. It is always preferred to have the reservoir in excavation, as all fear of bursting and flooding the town is thereby entirely eliminated. It is very seldom, however, that ground so level and uniform is found in the proper location and at the proper elevation to permit the reservoir being wholly in excavation; generally some portion of the sides must rise above the surrounding ground.

2059. Method of Building When the Reservoir is Wholly or Mostly in Excavation.—Supposing the excavation to be in ground other than rock, the best and most satisfactory method of construction is to level the bottom of the entire excavation and cover it with a bed of concrete. This bed may vary from six inches to one foot; it will rarely be advisable to go below the inferior limit, or necessary to exceed the greater one. If only six inches are used, the entire thickness should be spread in one bed. If twelve inches, then two beds should be used. The beds are best put down in strips from six to ten feet wide, plank being placed on edge, strongly staked in place, to keep the strips of an even width. The top edges of these plank are set by rod and level to the exact elevation of the top of the concrete. In the most perfectly constructed reservoirs, an additional thin bed is applied, having a slight downward slope to one corner of the reservoir where the outlet for emptying it is placed, so that, in cleaning the bottom, it may be readily washed down. The whole surface is finished off with a coat of plaster well rubbed in.

The sides should be heavy retaining walls of hydraulic masonry, the faces showing vertical joints and horizontal beds, so as to admit of careful pointing. A facing of brick makes an admirable finish, and is almost absolutely water-tight, if well done, but this adds greatly to the expense. When the walls rise above the surface of the ground, they are well terraced up with compact earth.

Such reservoirs are sometimes built without the retaining walls just mentioned, by sloping and riprapping the sides of the excavation, and when it is necessary to raise the sides above the surface of the ground, they are then formed with a center wall of masonry and an embankment, as already described for storage reservoirs. This method is never so satisfactory as the one just mentioned; the sides, even when paved with stones laid in cement, sometimes settle, gutter, and wash; the connection with the concrete bottom is never perfect, and when the extra excavation and embankment are considered, the masonry in the center wall, and the

facing of the slopes, the saving of cost will frequently be found less than was anticipated.

2060. Appliances for the Control of the Water.—As has been already mentioned, there are numerous ways in which the water may be admitted to and drawn from the receiving reservoir. Fig. 657 shows in plan the influent and effluent valve chambers of a reservoir consisting of two rectangular basins separated by a dividing wall *W W*. The water is supposed to come to the reservoir through the force main *F*, which enters the influent chamber *I*, and there

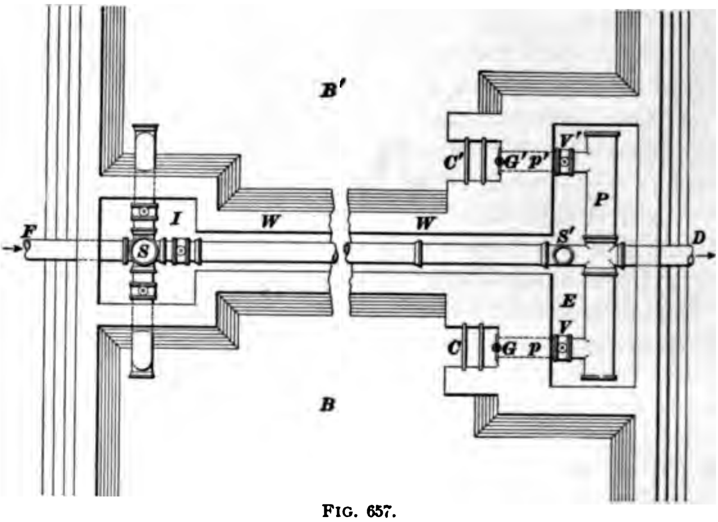


FIG. 657.

separates into three branches, one leading to each basin, *B* and *B'*, and one passing along the dividing wall *W W*. At the junction of these branches, there is an open stand-pipe *S*, of the same or larger diameter, rising a little higher than the top of the reservoir wall. These three branches are controlled by valves, as shown. By means of two elbows, or bends, the branch pipes discharging into the basins are led down to the bottom, so that the efflux of water takes place horizontally without fall or shock.

The central branch, running down the dividing wall,

s the effluent valve house E and connects with the P, by means of a stand-pipe S', similar to that in the influent valve chamber. Neither of these two stand-pipes is absolutely necessary, but they have a very beneficial action as equalizers, and as affording free escape to the air which may collect in the pipes.

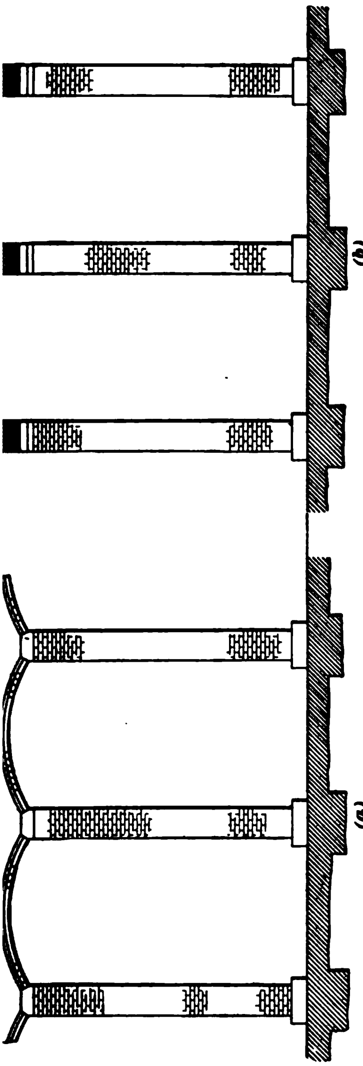


FIG. 638.

The pipes p and p', which connect each basin with the pipe P, are closed by sluice-gates G and G', as well as by the valves V and V', and are each provided with screen chambers C and C'. The collecting pipe P connects with the delivery pipe D by means of a special casting. The figure shows plainly how the system is operated, so as to use one, both, or neither of the basins, as desired.

The basins are each provided with a suitable overflow, and also with an emptying pipe or culvert, opened or shut by means of a :-gate, and discharging into such stream, channel, or as the locality may most conveniently offer.

COVERED RESERVOIRS.

2061. When it is decided to cover a distributing reservoir, the most usual way to do so is by erecting lines of pillars inside the reservoir, the distances between the pillars of each line and between the lines being governed by the height of the pillars and the area to be covered. Twenty feet is generally a convenient distance. The tops of these pillars are then connected by low arches, as shown in elevation in Fig. 658 (*a*), thus forming a series of arcades, with horizontal extrados. These arcades support the cover, which consists of a series of barrel arches turned between the lines of arcades. Fig. 658 (*b*) is a section at right angles to the section shown at (*a*), showing this arrangement. Upon the arched cover a bed of earth is generally spread, which should be sodded, or sown to grass. Ventilation is secured by chimneys, as shown in the figure.

In designing such a cover, care must be taken to give the pillars a sufficient area of cross-section to enable them to support the weight of the roof and earth covering without crushing. For this point, consult the next section, on "Masonry Dams." The material commonly used for pillars and arches is brick.

MASONRY DAMS.

2062. General Considerations.—In the structures hitherto treated of, there has been no attempt made to determine their dimensions by calculation. The character of the structures did not permit of it. In earthen dams all dimensions are fixed by empirical rules; that is, experience has taught us that certain thickness of bank and certain ratios of slope lead to safe results; we know that, by adopting these dimensions, we ensure the stability of our work, but we only know that it is strong enough, and do not know what, if any, is its factor of safety.

In masonry dams the case is different. When we have a wall sustaining a certain head of water, we can calculate almost exactly the character, intensity, point of application, and direction of action of the destructive force or forces

bearing upon it, and we can also calculate with a great degree of precision the resisting force which the wall presents in opposition. The task of designing such structures is, therefore, more satisfactory than in the case of earthen embankments, and admits of a more scientific course of procedure.

2063. Elementary Principles.—Only the simplest principles of hydrostatics are involved in the subject under consideration, but they must be thoroughly understood. The principal property of quiet water which we are concerned with is its *horizontal thrust* upon any surface against which it presses.

Referring to Fig. 659, the horizontal thrust T, expressed in pounds, against any surface $A\ B$, whether vertical as at

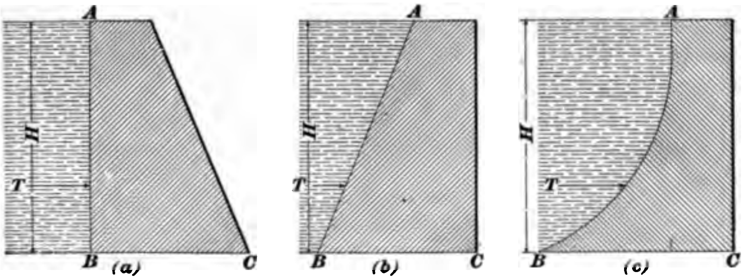


FIG. 659.

(a), inclined as at (b), or curved as at (c), is the same, and is equal to half the square of the height or head of water H, in feet, pressing against $A\ B$, multiplied by 62.50, which is the weight in pounds of a cubic foot of water in round numbers and under ordinary conditions. Thus :

$$T = \frac{62.5\,H^2}{2} = 31.25\,H^2. \qquad \textbf{(176.)}$$

Also, the point of application of this thrust is the same for (a), (b), and (c); namely, at one-third of the height H from the bottom of the wall. Hence, the overturning moment $M\ T$ of the thrust about the point C, expressed in static foot-pounds, is :

$$M\,T = 31.25\,H^2 \times \frac{H}{3} = 10.42\,H^3. \qquad \textbf{(177.)}$$

NOTE.—The value $10.42\,H^3$ will always be slightly greater than the true value, as the decimal is a little too great.

EXAMPLE.—The depth of water H pressing against the curved surface $A\,B$ [Fig. 659 (c)] is 23 ft. 7 in. (a) What is the intensity in pounds of the horizontal thrust? (b) What is the overturning moment, in foot-pounds, about the point c, per foot of length of wall?

SOLUTION.—(a) $T = 31.25\left(\frac{283}{12}\right)^2 = 17{,}380.43$ lb. Ans.

(b) $M\,T = 10.42\left(\frac{283}{12}\right)^3 = 136{,}673.18$ ft.-lb. Ans.

2064. Action of the Thrust Against the Dam.—The first effort of the water against the wall or dam against which it presses is to push it bodily forward, by causing it to slide upon its base. The force tending to produce this effect is the horizontal thrust T alone. If the wall, from its stability in this respect, refuses to move, the next effort of the water is to endeavor to overturn it by causing it to rotate about its outer toe C, Fig. 659. The force tending to produce this effect is the overturning moment $M\,T$, or the combination of the intensity of the stress into the lever arm with which it acts.

2065. Resistances of the Dam.—The resistance which the dam opposes to the tendency of the thrust of the water to move it forwards upon its base is that due to two causes: its weight and its coefficient of friction. The heavier the mass and the greater the friction, the greater the resistance which it offers to any force tending to shove it forwards. Its total resistance is, therefore, its weight multiplied by its coefficient of friction. Much uncertainty attends the determination of this last-named factor. An ordinary estimate places it at about 75%. It must be noted that friction only is now considered, as if the dam were merely standing upon a level base, with no mortar joint intervening, and the resistance only that of the friction of stone against stone. The adherence of the mortar and the bond of work are both neglected. The resistance thus estimated is, therefore, much below the truth. It is, however, at least *safe*.

and, as will be presently shown, the tendency of the wall or dam to slide forwards is not, under ordinary circumstances, the destructive force most to be feared. We will take the coefficient of friction, therefore, in what follows, as 0.75.

In all calculations relating to the resistance of such walls as are now being considered, a length of one foot, or a *slice* of the wall one foot thick, is always taken, because, since the whole of the wall, if of uniform height, is made up of a succession of such slices, what is proved true of one will be true of all. The convenience of this consists in the fact that the area of vertical cross-section of the wall is then equal to its volume in cubic feet. It is unnecessary to state that throughout all the following calculations the units of length and weight are the foot and pound avoirdupois.

The resistance R of the wall to sliding is, therefore, the area A of its vertical cross-section multiplied by its density D, or the weight of a cubic foot of the material of which it is composed multiplied by its coefficient of friction, which we have agreed to call 0.75. Hence,

$$R = 0.75\ A\ D. \qquad \textbf{(178.)}$$

EXAMPLE 1.—A trapezoidal wall, Fig. 660, 12 ft. high, 3 ft. wide on top, and 8 ft. at bottom, has a density of 115 lb. (*a*) What is its resistance to sliding, and (*b*) what is its factor of safety?

SOLUTION.—(*a*) Substituting in formula **178**,

$$R = 0.75 \times \frac{8+3}{2} \times 12 \times 115 = 5{,}692.5 \text{ lb. Ans.}$$

(*b*) To ascertain the factor of safety F, it is necessary to find the amount of thrust to be contended with. From formula **176**,

$$T = 31.25 \times 144 = 4{,}500 \text{ lb.}$$

Hence,
$$F = \frac{5{,}692.5}{4{,}500} = 1.26. \text{ Ans.}$$

EXAMPLE 2.—Let the wall, in example 1, be built of granite, with a density of 170. Determine (*a*) the thrust and (*b*) the factor of safety.

SOLUTION.—(*a*) $R' = 49.5 \times 170 = 8{,}415$ lb. Ans.

(*b*) $F = \dfrac{8{,}415}{4{,}500} = 1.87.$ Ans.

2066. The **resistance to overturning** will be the **moment of resistance** of the wall, and will be the product of its weight, multiplied by the horizontal distance of its center of gravity from the point about which rotation tends to take place. This point, in the case of a dam, is the exterior toe.

If the wall were a "plumb" wall, i. e., one with vertical sides, as Fig. 661, its weight (considering a length of one

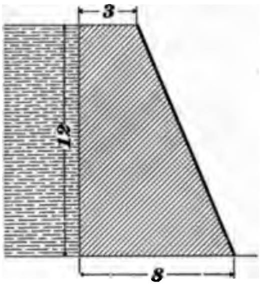


FIG. 660.

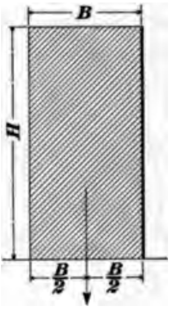


FIG. 661.

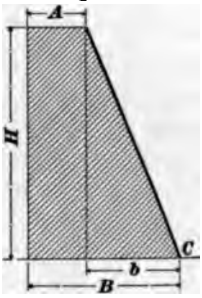


FIG. 662.

foot, as already stated) would be DHB, and as the vertical line passing through the center of gravity would cut the base in the center, its moment of resistance MR would be

$$MR = DHB \times \frac{B}{2} = \frac{DHB^2}{2}. \qquad \textbf{(179.)}$$

But dams are very rarely built plumb on both faces. The almost invariable section, or "profile" as it is generally termed, except for very high dams, is trapezoidal.

To calculate the moment of resistance of such a wall, Fig. 662, it is divided into a rectangle HA and a triangle of height H and base b. The moment of resistance mr of the rectangle about the toe C is

$$mr = DHA\left(b + \frac{A}{2}\right).$$

And of the triangle, since the center of gravity is horizontally distant $\frac{2b}{3}$ from C,

$$mr' = \frac{DHb}{2} \times \tfrac{2}{3}b = \frac{DHb^2}{3}.$$

Adding the two together,

$$MR = DHA\left(b + \frac{A}{2}\right) + \frac{DHb^2}{3} =$$

$$DH\left(Ab + \frac{A^2}{2} + \frac{b^2}{3}\right) = \frac{DH}{6}(6Ab + 3A^2 + 2b^2).$$

Observing that $b = B - A$,

$$MR = \frac{DH}{3}\left(AB - \frac{A^2}{2} + B^2\right). \qquad \textbf{(180.)}$$

EXAMPLE 1.—Referring again to Fig. 660, (*a*) what is the moment of resistance of the wall when $D = 115$? (*b*) What is its factor of safety?

SOLUTION.—Substituting in formula **180**,

$$MR = \frac{115 \times 12}{3}\left(3 \times 8 - \frac{9}{2} + 64\right) = 38{,}410 \text{ ft.-lb. Ans.}$$

(*b*) To ascertain factor of safety, we must ascertain the overturning moment of the water thrust. From formula **177**, we have

$$MT = 10.42 \times 1{,}728 = 18{,}005.76.$$

Hence, $F = \dfrac{38{,}410}{18{,}006} = 2.133$. Ans.

EXAMPLE 2.—Suppose the same wall to have a density of 170. Determine (*a*) the moment of resistance, and (*b*) the factor of safety.

SOLUTION.—(*a*) $MR = \dfrac{170 \times 12}{3}\left(3 \times 8 - \dfrac{9}{2} + 64\right) = 56{,}780$ ft.-lb. Ans.

(*b*) $F = \dfrac{56{,}780}{18{,}006} = 3.15$. Ans.

2067. Designing Profiles.—The above formulas and examples show how to ascertain the resistances of walls of which the dimensions, etc., are given. But they give no help in designing a wall to fulfil certain requirements, except by "trial and error."

Supposing now it were desired to design a wall of height H, of density D, of top width A, and it were desired that it should have a factor of safety C as against sliding and overturning. What should be the bottom width B?

Considering first the case of sliding, since the factor of safety is C, the thrust must be taken as

$$31.25\,CH^2.$$

The resistance, from formula **178**, must be

$$R = 0.75\,H\left(\frac{A+B}{2}\right)D.$$

These two values must be equal. Hence,

$$31.25\,C\,H^2 = 0.75\,H\left(\frac{A+B}{2}\right)D;$$

$$31.25\,C\,H = 0.75\left(\frac{A+B}{2}\right)D;$$

$$83.33\,C\,H = (A+B)\,D;$$

$$B = \frac{83.33\,C\,H}{D} - A. \qquad \textbf{(181.)}$$

Having now established the general formula, we can work any example where A, C, D, and H are given.

EXAMPLE.— $H = 30$ ft.; $A = 6$ ft.; $D = 140$ lb., and $C = 2.5$; what must be the bottom width of the wall?

SOLUTION.— $B = \dfrac{83.33 \times 2.50 \times 30}{140} - A = 38.64$ ft. Ans.

2068. To determine the breadth of base to resist overturning, consider formula **177** (using its more exact form of $\frac{31.25}{3}H^2$), and we have

$$\frac{31.25}{3}\,C\,H^2 = \frac{DH}{3}\left(A\,B - \frac{A^2}{2} + B^2\right);$$

$$62.50\,C\,H^2 = 2\,D\,A\,B - D\,A^2 + 2\,D\,B^2;$$

$$B^2 + A\,B = \frac{62.50\,C\,H^2 + D\,A^2}{2\,D};$$

which, solved for B, gives us

$$B = \sqrt{\frac{62.50\,C\,H^2 + D\,A^2}{2\,D} + \frac{A^2}{4}} - \frac{A}{2},$$

or $$B = \tfrac{1}{2}\sqrt{\frac{125\,C\,H^2}{D} + 3\,A^2} - \frac{A}{2}. \qquad \textbf{(182.)}$$

EXAMPLE.—What must be the width of base of the wall in the last example to resist overturning?

SOLUTION.—Substituting the given data in formula **182**, we have

$$B = \sqrt{\frac{62.50 \times 2.50 \times 900 + 140 \times 36}{280} + \frac{36}{4}} - 3 = 20 \text{ ft.} \quad \text{Ans.}$$

2069. In examining the results given by formulas **181** and **182**, one is struck with the fact that a much greater width of base is required to ensure security against sliding than is required to guard against overturning. It must be borne in mind, however, that, as has already been mentioned, only the mere friction of stone upon stone is taken into the account when calculating the resistance to sliding, such as might occur if two level surfaces of stone were brought in contact. When it is remembered that a well-bonded piece of masonry is by no means in this condition, but is knit together in a more or less homogeneous mass, it will be seen that the tendency to move forwards is counteracted, not by mere friction alone, but also by the resistance to shearing of the stonework. This is a very strong combination, and makes a total resistance so great that experience proves that, when a dam is safe against overturning, it is safe against being moved forwards bodily upon itself. If, however, the whole dam were placed upon a smooth surface, such as a timber grillage, particularly if the planks were laid in the same direction as the pressure, or if it rested upon an unctuous clay, with only a small depth of foundation, very serious doubts might exist as to whether it would remain immovable, and careful examinations and calculations would be called for. In such cases, the coefficient of friction may fall considerably below 75%. *As, however, all masonry dams should stand on a rock foundation, into which the footing course is well embedded, no danger of their moving bodily forwards need be apprehended, if the stability is satisfactory as regards overturning.*

2070. Average Dimensions.—Calculations made with various practical values for density D and top width A show that a bottom width equal to from $\frac{2}{3}H$ to $\frac{3}{4}H$ will always give a satisfactory factor of safety, and in nearly all cases the smaller of these two values, i. e., $B = \frac{2H}{3}$, will give a perfectly secure profile.

HIGH MASONRY DAMS.

2071. General Considerations.—The trapezoidal profile hitherto considered is the one almost universally adopted for masonry dams up to say 50 or 60 ft. in height. Beyond this limit, it would no longer be economical nor, in very high dams, possible. So far, only resistance to sliding and overturning has been considered, but in very high dams another element of destruction comes in; namely, the crushing of the material under its own weight.

As the resistance to crushing of all materials has a limit, it is quite evident that a structure composed of masonry might be piled up so high that the bottom courses would finally be crushed by the superimposed weight.

In the case of symmetrical figures like Figs. 663 and 664, the amount of pressure per square unit of base is obtained by dividing the whole weight resting upon the base by the number of square units (always square feet in the present calculations) in the base. Thus, in Figs. 663 and 664, if W

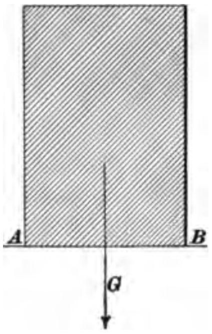


FIG. 663.

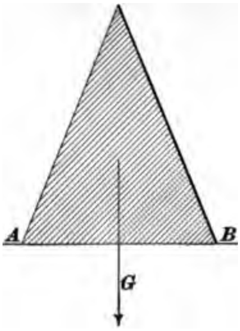


FIG. 664.

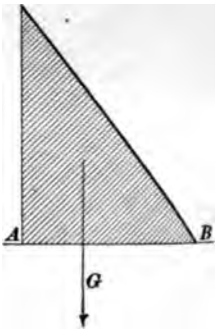


FIG. 665.

represents the total weight of the mass above the base $A B$, then the uniform weight borne per square foot of the base is $\frac{W}{A B}$. If, however, the profile were that of a triangle, as in Fig. 665, then, the figure not being symmetrical about the line G passing through the center of gravity of the profile, the weight is not uniformly distributed over each square

unit of the base, and it is impossible, merely at sight, to say what the *maximum* unit stress may be. All that is clearly evident is that the unit stress must be more intense upon the shorter segment into which the resultant G of the weight cuts the base.

Even when the profile is symmetrical, the influence of an exterior force may cause the resultant of the weight to move from its central position towards one or the other extremity of the base. Thus, in Fig. 666, let $A\ B\ C\ D$ represent the profile of a rectangular wall sustaining water pressure, as shown. The resultant of the weight W passing through the center of gravity of the profile cuts the base in the center. But the horizontal thrust T of the water acts upon it to cause it to move nearer to the toe B. The position of the point P, where the new position of the resultant cuts the base $A\ B$, is readily determined, graphically or by calculation. An example will fully illustrate this.

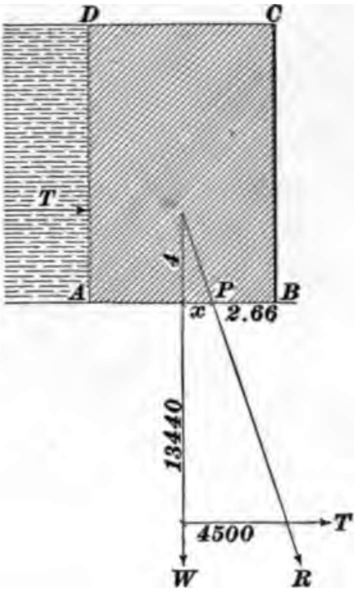


FIG. 666.

Thus, $A\ B\ C\ D$, Fig. 666, represents the profile of a wall sustaining water pressure. The density of the wall is 140 lb., its height is 12 feet, and its thickness 8 feet. The weight is then 13,440 lb. The thrust of the water is 4,500 lb., and it acts at the height of 4 feet above the base.

We, therefore, have the triangle of forces, as shown in the figure, which, by drawing the sides to scale, will give the distance $P\ B$ graphically. It may be also determined very readily, and, of course, more accurately, by calculation.

Thus, let x represent the interval between the two

positions of the resultant. Then, by similar triangles, we have the proportion

$$x : 4 = 4,500 : 13,440,$$

from which we have

$$x = \frac{18,000}{13,440} = 1.34.$$

The desired distance $P\,B$ is $PB = 4 - 1.34 = 2.66$ ft. At the point P, the oblique resultant R can be resolved back into its two components. The weight, 13,440 lb., is, therefore, merely transferred from the center of the base to a point within 2.66 ft. of the nearer toe B, without change in its value.

2072. Maximum Unit Stress from Unequally Distributed Load.—Suppose $A\,B$, Fig. 667, to be the base or any given horizontal course of a mass of homogeneous masonry. Let W be the resultant or the vertical component of the resultant of its weight. It divides the length L into two unequal segments, one d, and the other $L - d$. The maximum unit stress is now in the shorter segment d. What is its amount ?

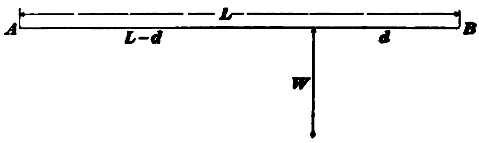


FIG. 667.

This question can not be answered with mathematical exactness, because it is unknown in what way the weight is now transmitted to the base. There are two empirical, or quasi-empirical formulas, however, which experience shows give satisfactory results. They are:

$$P = \frac{4\,W}{L^2}(L - 1.5\,d). \qquad \textbf{(183.)}$$

$$P = \frac{2\,W}{3\,d}. \qquad \textbf{(184.)}$$

Of these, formula **183** is to be used when d is equal to or greater than $\frac{L}{3}$, and formula **184** when d is equal to or

less than $\frac{L}{3}$. When $d = \frac{L}{3}$ the two formulas give the same results. Good authorities warn us that formula **184** should be used with great judgment and only when d is not much less than $\frac{L}{3}$. Others consider the results obtained by formula **184** to be too small when the obliquity of the resultant of the weight of the mass and the thrust of the water exceeds a certain amount, the error increasing with the increase of obliquity; in other words, with the decrease in the value of d. A preferable formula, though one which has not yet been generally accepted, perhaps because it is not generally known, is

$$P = \frac{W(L - d)}{L d}. \qquad (185)$$

which applies to all values of d, and will be used throughout this Course. This formula gives results very nearly the same as formula **183** for cases in which the latter applies, and all three agree for $d = \frac{L}{3}$. For values of d less than $\frac{L}{3}$, formula **185** gives pressures increasingly greater than formula **184**, which, to conform to the ideas of some good authorities, it should do. It will be seen, however, that, in designing the profile of a masonry dam, whether high or low, the point where the resultant pressure cuts the base should always be kept within the "middle third" of the base and as near the center as possible. That is, d should always lie between $\frac{L}{3}$ and $\frac{L}{2}$, and the nearer the latter, the better.

2073. The next point to be considered in this connection is the permissible degree of pressure per square unit on the masonry. It is safe to say that good hydraulic masonry will stand from 15,000 lb. to 30,000 lb. to the square foot, according to circumstances to be treated of later on. The character of the stone, no doubt, has a great influence upon the amount of stress which it will bear, but the use of

mortar, of the same quality in all cases, tends to equalize the quality of the work as regards endurance of stress.

2074. When studying the stresses sustained by a high masonry dam, two conditions are to be considered: the stress when the dam is supporting its own weight solely, and the stress when it is also sustaining water pressure, the two corresponding, respectively, to an empty and a full reservoir. In the former case, the resultant pressure is between the center of the base and the inside toe of the dam, and in the latter between the center and the outside toe.

In designing the proper profile for such structures, therefore, it is necessary to give it such a form that if a line is drawn through it at any point, parallel to the base, and the resultant of the weight of the mass above such line is determined both with and without water pressure, the maximum unit stress in the shorter of the two segments into which the resultant divides the line shall not exceed the limit fixed upon.

It is possible to design a profile which shall not only fulfil the above requirement, but which shall also be exactly, or very nearly, a profile of "equal resistance"; that is to say, one in which the maximum stresses shall be equal at all elevations. The result, however, would be a profile which could not be adopted for an actual structure, because it would conflict with practical constructive features which are of still greater importance than a profile of equal resistance.

Several methods have been devised for determining by means of formulas a proper practical profile satisfying the condition of nowhere exceeding the adopted limit of unit stress, but the formulas deduced, even when simplified by many preliminary assumptions, are still very complicated and of tedious application. Moreover, the outcome of all the study which has been bestowed upon the subject is that a certain type of profile has been evolved to which all designs must very nearly conform, so that at the present day it is needless to go through a series of elaborate calcu-

lations, the only result of which must be to reproduce the general type already established. It suffices, therefore, in every particular case, to lay down this general outline and test it at certain elevations according to the rules already given. The best presentation of the subject will be by a general example in which the method of procedure will be discussed step by step.

2075. General Illustrative Example.—It is required to design the profile of a masonry dam 250 ft. high above the surface of the ground, the foundations extending to rock lying 100 ft. below. The dam to be built of hydraulic masonry, the average weight being 140 lb. to the cubic foot. The top width of the dam to be 20 ft.

The conditions of the design are that the crushing stress at the base shall not exceed 20,000 lb. per sq. ft., nor 30,000 lb. at the bottom of the foundation. It is required, moreover, that at the distance of 100 ft. from the top, the unit stress at the back of the dam, when the reservoir is empty, shall not exceed, or but slightly exceed, 16,000 lb. per sq. ft., and shall increase progressively down to the base, when it may reach 20,000 lb., as aforesaid. The pressure at the face, when the reservoir is full, shall at any elevation be less than at the back when the reservoir is empty. The slope or batter of either face shall nowhere form an angle of less than 45° with the horizon. N. B.—This condition is to prevent the weak edge that would result from a flatter slope.

In calculating the water pressure, the surface of the water shall be considered as level with the top of the dam.

This example will be solved by commencing at the top and working downwards, considering first a height of 100 ft., and adding 50 ft. successively until the total of 250 ft. shall have been reached. As already mentioned, a length of dam of 1 foot is taken, so that areas in square feet represent volumes in cubic feet.

Referring to Fig. 668, the first step will be to lay down the right-angled triangle *A B C*, whose altitude = 100 ft., and base = $\frac{2}{3} \times 100 = 66.67$ ft. This is surmounted by the

small similar triangle CDE, of which the base $CD = 20$ ft., as required. In the actual dam the face DE would have a slight batter, about one inch to the foot, but, for convenience of calculation, in the figure it is drawn vertical.

It is now necessary to find the position of the vertical line passing through the center of gravity of the area $ABEDC$. This is best done by finding, separately, the moments of the two triangles in reference to a common axis. This axis may be assumed anywhere, and in order to be sure of having it sufficiently distant to lie outside of the whole figure as we add to its height, and consequent width of

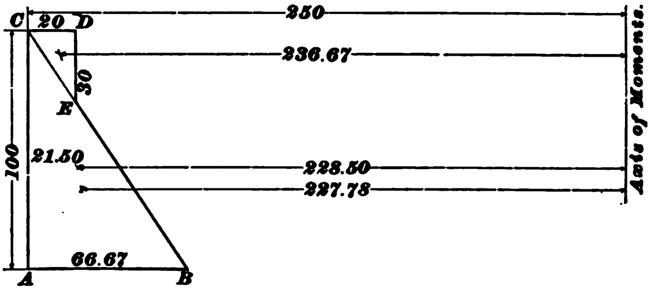


FIG. 668.

base, we will assume the axis to be a vertical line 250 ft. to the right of the side AC. It will be observed that the moments of the *areas*, and not of the weights of the triangles, are those taken. This is to avoid unnecessary multiplications, and the weights are found afterwards, when wanted, by multiplying the areas by 140.

The area of the triangle CDE is 300 sq. ft., and its center of gravity is distant, horizontally, from the line AC $\frac{2}{3} \times 20 = 13.33$ ft., and, consequently, $250 - 13.33 = 236.67$ ft. from the axis of moments. The area of the triangle ABC is $\frac{50 \times 2 \times 100}{3} = 3{,}333.33$ sq. ft., and its center of gravity is distant, horizontally, from AC, $\frac{1}{3} \times \frac{2}{3} \times 100 = 22.22$ ft., and, consequently, $250 - 22.22 = 227.78$ ft. from the axis of moments. We must now multiply the area of each triangle by the horizontal distance of its center of gravity from the axis of moments, add the two together,

and divide by the combined areas of both triangles. The calculation is worked out as follows, taking the areas to the nearest square foot, and the distances to the nearest tenth of a foot:

$$\begin{array}{rr} 300 \times 236.7 = & 71,010 \\ 3,333 \times 227.8 = & 759,257 \\ \hline 3,633 & 830,267 \end{array}$$

$$\frac{830,267}{3,633} = 228.50 \text{ ft.}$$

This is the horizontal distance of the center of gravity of $A\ B\ E\ D\ C$ from the axis of moments.

The vertical line drawn through the center of gravity of the mass $A\ B\ E\ D\ C$ at this distance from the axis of moments cuts the base $A\ B$ at the distance of $250 - 228.5 = 21.50$ ft. from the point A, and thus determines the shorter segment of the base $= 21.50$ ft., upon which the maximum unit stress comes when the reservoir is empty. To obtain the intensity of the stress, the area 3,633 sq. ft. must be multiplied by 140 lb., which gives the weight of the mass above the line $A\ B$ as 508,620 lb.

Referring to Fig. 669, we now construct the triangle of forces, composed of the vertical line representing the weight of the mass, and the horizontal one representing the thrust of the water $= 31.25 \times 10,000 = 312,500$ lb. This thrust intersects the vertical at the distance $\frac{100}{3}$ from the base $A\ B$, and thus determines the apex of the triangle of forces from which the weight is laid off by scale if the graphical method is pursued. The hypotenuse of the triangle of forces determines by its intersection with the base the shorter segment $= 24.67$ feet upon which the maximum unit stress comes when the reservoir is full. This intersection can be

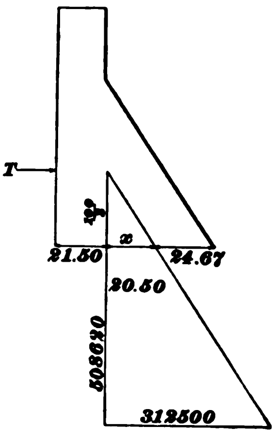


FIG. 669.

determined graphically, as above stated, or by calculation, based upon the principle of the proportionality of similar triangles, thus:

$$\frac{3x}{100} = \frac{312,500}{508,620}$$
$$x = 20.50.$$

Then, the shorter segment d is given by the equation: $d = 66.67 - (21.50 + 20.50) = 24.67$, all of which is shown by Fig. 669.

We are now in a position to test the conformity of our profile, so far, with the imposed conditions. Let us consider first the stress in the neighborhood of A, when the reservoir is empty.

Referring to formula **185**, we have
$W = 508,620$; $L = 66.70$; $d = 21.50$; $L - d = 45.20$, taking distances to nearest tenth.

Then, $$P = \frac{508,620 \times 45.20}{66.70 \times 21.50} = 16,031 \text{ lb.}$$

Deferring any comment upon this result till presently, we next ascertain the maximum unit stress in the neighborhood of B when the reservoir is full. Here $d = 24.70$ and $L - d = 42$, the remaining factors W and L being the same as before, and we have

$$P = \frac{508,620 \times 42}{66.70 \times 24.70} = 12,966 \text{ lb.}$$

It is to be noted that it was not necessary to calculate this last stress, because, since the shorter segment was *longer* than that corresponding to an empty reservoir, it was certain that the stress would be *less*, which was all the conditions of the problem required. It is more satisfactory, however, to fully work out both stresses.

2076. In discussing the profile so far, two facts become evident: As regards the limiting stress when the reservoir is empty, we have practically reached our limit, and the resultant of the weight passes just outside of the middle third of the base AB. A study of Fig. 668 will show that this last fact is owing to the influence of the small upper

triangle. Had the problem been to construct a profile for a dam 100 ft. high only, the base A B would have been widened a little to the left of A, by giving the back of the dam a slight flare, of about 1 horizontal to 4 vertical, commencing at a point about 30 ft. above A. This would give an increased width to the left of A of about 7.5 ft., making total width of base between 74 and 75 ft., instead of 66.67. This would reduce the unit stress on the water side considerably, and on the lower side somewhat. Or, if it were important, owing to great depth of foundation, or other cause, to keep the base as narrow as possible, and, therefore, to keep the back of the dam vertical, then, in building the dam, an embankment with a berme followed by

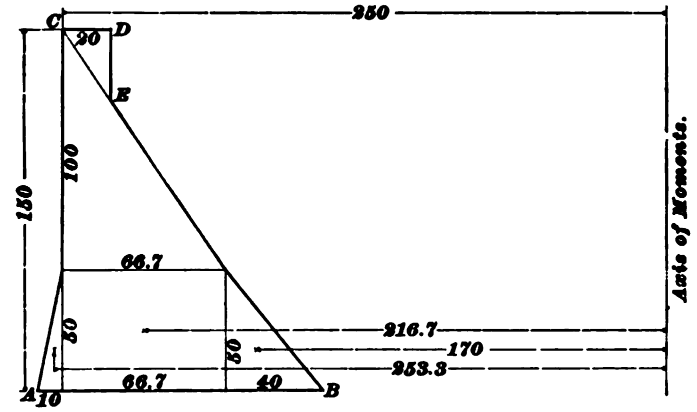


FIG. 670.

a flat slope, well riprapped, would be placed against the back of the dam, rising to a height of about 30 ft., so as to maintain a constant counter pressure against the back, and thus reduce the stress.

The reason why no change is made when the profile is merely that of the upper part of a much higher dam is that, as will presently be seen, the unit stresses rapidly increase with the height, necessitating a corresponding widening of the base. It is important, therefore, not to begin widening any sooner than is absolutely necessary.

2077. We now add 50 feet to the profile. As already mentioned, it is evident that the stresses will increase in a more rapid proportion as we add to the height, so a batter of 20% will be given to the back and one of 80% to the face. That is, a triangle 50 ft. vertical and 10 ft. base is added to the back, and one of 50 ft. vertical and 40 ft. base to the face, as well as the included rectangle, 50 × 66.70. (See Fig. 670.)

The products of these three areas, multiplied by the distances of their respective centers of gravity from the axis of moments, are now added to the sum of the moments already calculated, and their areas to those already obtained. Thus,

$$\begin{array}{lcr}
3,633 & & 830,267 \\
250 \times 253.30 & = & 63,325 \\
3,335 \times 216.70 & = & 722,695 \\
\underline{1,000 \times 170} & = & \underline{170,000} \\
8,218 & & 1,786,287
\end{array}$$

and $$\frac{1,786,287}{8,218} = 217.4 \text{ ft.}$$

This is the distance of center of gravity of entire area $A\ B\ E\ D\ C$ from axis of moments, and the distance from the toe A is, therefore,

$$250 + 10 - 217.4 = 42.6 \text{ ft.}$$

Referring now to Fig. 671, which shows the base $A\ B$ of the above figure, the triangle of forces is constructed, the weight of the mass being $8,218 \times 140 = 1,150,520$ lb., and the thrust of water $31.25 \times \overline{150}^2 = 703,125$ lb., applied at a height $= \frac{150}{3} = 50$ ft. above the base.

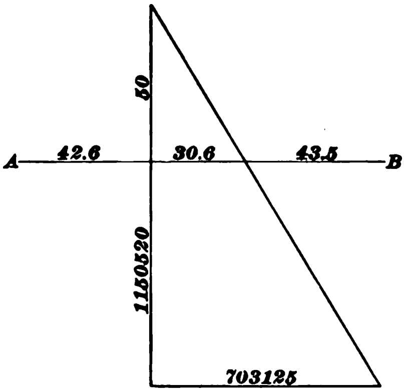


FIG. 671.

To ascertain the maximum unit stress in the segment adjacent to A when the reservoir is empty, the data to be used in formula **185** are:

$L = 116.7$; $d = 42.6$; $L - d = 74.1$, and $W = 1,150,520$.

Then, $$P = \frac{1,150,520 \times 74.1}{116.7 \times 42.6} = 17,149 \text{ lb.}$$

When water pressure is added, the maximum unit stress on segment adjacent to B is

$$P = \frac{1,150,520 \times 73.2}{116.7 \times 43.5} = 16,590 \text{ lb.}$$

This is a reasonable progression in the stresses, and shows a satisfactory profile, so far.

2078. For the next 50 ft., which will bring the profile up to the height of 200 ft., the base must widen out more

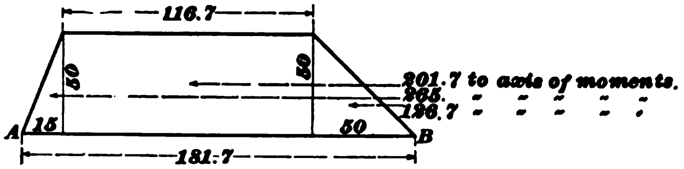


FIG. 672.

rapidly. A batter of 30% will be given to the back, and one of 100% to the face. (See Fig. 672.) This adds a triangle of 50 ft. altitude and 15 ft. base to the back; a rectangle of 50×116.7, and a triangle of 50 ft. altitude and 50 ft. base, to the face, as is shown, with distances to axis of moments, in Fig. 672, which gives the additions only. Utilizing previous work, and adding the new data:

8,218	1,786,287.0
375×265 =	99,375.0
$5,835 \times 201.7$ =	1,176,919.5
$1,250 \times 126.7$ =	158,375.0
15,678	3,220,956.5

and $$\frac{3,220,956.5}{15,678} = 205.4 \text{ ft.}$$

This is the distance of the center of gravity of the whole profile from the axis of moments, and the distance from the toe A is

$$275 - 205.4 = 69.6 \text{ ft.}$$

Referring to Fig. 673, which shows the base $A\ B$ of the above figure, the triangle of forces is constructed, the weight of the mass being $15{,}678 \times 140 = 2{,}194{,}920$ lb., and the thrust of the water $31.25 \times \overline{200}^2 = 1{,}250{,}000$ lb. applied at a height $\frac{200}{3}$ ft. above the base.

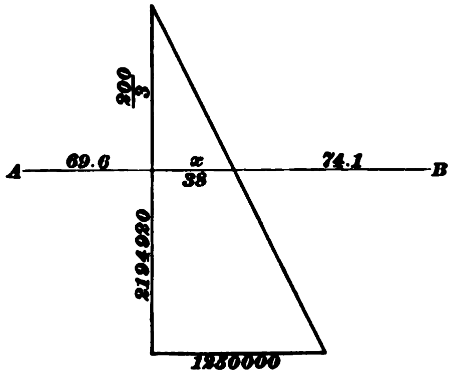


FIG. 673.

To ascertain the maximum unit stress in the segment adjacent to A, with an empty reservoir, the data are: $L = 181.7$; $d = 69.6$; $L - d = 112.1$, and $W = 2{,}194{,}920$. Then, in formula **185**,

$$P = \frac{2{,}194{,}920 \times 112.1}{181.7 \times 69.6} = 19{,}456 \text{ lb.}$$

And with a full reservoir, on the segment adjacent to B,

$$P = \frac{2{,}194{,}920 \times 107.6}{181.7 \times 74.1} = 17{,}541 \text{ lb.}$$

This is also perfectly satisfactory under the given conditions.

2079. The next 50 ft. will complete the total height of 250 ft. The base is widened at the back, by adopting a

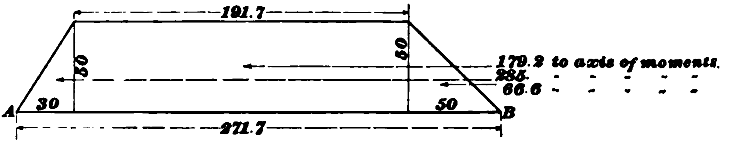


FIG. 674.

batter of 60%. On the face, the limiting slope has been reached, so the only means of widening out is by an offset. Accordingly, a step of 10 ft. is made, from which the 45° slope is continued to the base. The additions are shown in Fig. 674.

Then,

$$\begin{array}{rcr} 15,678 & & 3,220,956.5 \\ 750 \times 285 & = & 213,750.0 \\ 9,585 \times 179.2 & = & 1,717,632.0 \\ 1,250 \times 66.6 & = & 83,250.0 \\ \hline 27,263 & & 5,235,588.5 \end{array}$$

$$\frac{5,235,588.5}{27,263} = 192 \text{ ft.}$$

Fig. 675 shows all the necessary data for using formula **185**; namely, for segment adjacent to A, $L = 271.7$; $d = 113$; $L - d = 158.7$; $W = 3,816,820$.

Then, from formula **185**,

$$P = \frac{3,816,820 \times 158.7}{271.7 \times 113} = 19,729 \text{ lb.}$$

For the segment adjacent to B,

$L = 271.7$; $d = 116.1$; $L - d = 155.6$; $W = 3,816,820$.

Then, from formula **185**,

$$P = \frac{3,816,820 \times 155.6}{271.7 \times 116.1} = 18,827 \text{ lb.}$$

2080. All the conditions of the problem have been complied with, as far as the superstructure goes. If, however, it had been desired to go another 50 feet higher, making the total height 300 ft., the base would assume extravagant dimensions. The batter on the back would have to be increased to 80% or 100%, and another considerable offset taken on the face. It is evident, therefore, that with the given conditions, 250 ft. represents nearly the practical limiting height of a masonry dam. If it were desired to go higher, choice material should be used, and the best workmanship exercised in the lower courses, so as to admit of increasing the limiting unit crushing stress, and at the same time the necessity for a base, much exceeding the height of the dam, would have to be confronted. So far, engineers have not been called upon to design dams of such stupendous magnitude, and the profile already worked out will cover anything likely to be called for in practice.

It now remains to proportion the foundation block. It will be rectangular in section, and the limiting stress is raised to 30,000 lb. This is because the vertical sides are firmly and squarely compressed by the earth pressure against them, and the masonry can, therefore, safely withstand a greater stress. The two cases of a full and empty reservoir are to be considered. The additional width must be given by front and back offsets.

In determining these offsets, it will be best, in the foundation, that they should be such as to make the maximum unit stresses equal for a full and an empty reservoir. Since

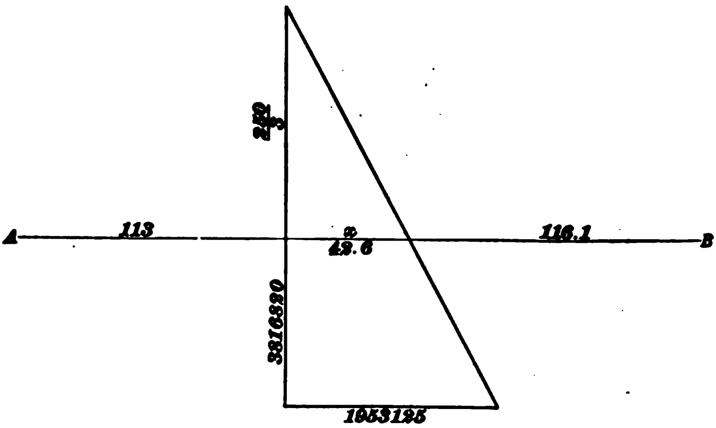


FIG. 675.

there is now no thrust of the water to be taken into consideration, and the addition consists of a rectangle, this may easily be done. Fig. 675 shows that the short segment for an empty reservoir is 113 ft. from the toe A, and 116.10 ft.—which we will call 116 ft.—from the toe B for a full one. If we give an offset at the back of 14 ft., and at the front of 11 ft., then (Fig. 676) the two resultants will pass 127 ft. from either end of the base AB of the foundation block. This base would $= 271.7 + 14 + 11 = 296.7$; or, for round numbers, which are now permissible, 297 ft. If now we determine the maximum unit stress for either assumption, of a full or an empty reservoir, it holds good for the other.

To determine this stress, we will take moments about the point B, the weight of the block being $297 \times 100 \times 140 =$ 4,158,000 lb., which weight passes through its center of gravity at a distance $= \frac{297}{2} = 148.5$ ft. from either end of the

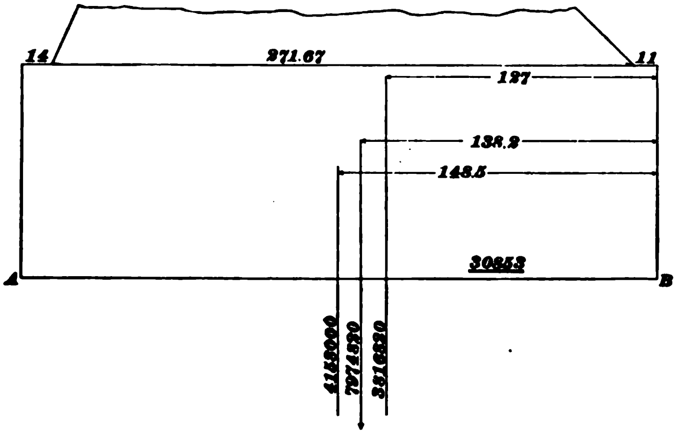


FIG. 676.

base. When the reservoir is full, the resultant of the weight of the superstructure passes at 127 ft. horizontally from B. We have, therefore,

$$\begin{array}{lr} 3,816,820 \times 127.0 = & 484,736,140 \\ \underline{4,158,000} \times 148.5 = & \underline{617,463,000} \\ 7,974,820 & 1,102,199,140 \end{array}$$

$$\frac{1,102,199,140}{7,974,820} = 138.2 \text{ ft.}$$

The data are, therefore,

$$L = 297;\ d = 138.2;\ L - d = 158.8;\ W = 7,974,820.$$

Hence, $$P = \frac{7,974,820 \times 158.8}{297 \times 138.2} = 30,853 \text{ lb.}$$

This slightly exceeds the limit, but in a practical design would be considered as fulfilling the conditions. Fig. 677 shows the entire profile with maximum stresses in the corresponding segments, and tne "curves of pressure" for an empty and full reservoir, in dotted lines. These curves are

constructed by drawing a line through the points where the resultants cut the bases of the various partial profiles, as already determined, except for the upper 30 ft. of the profile.

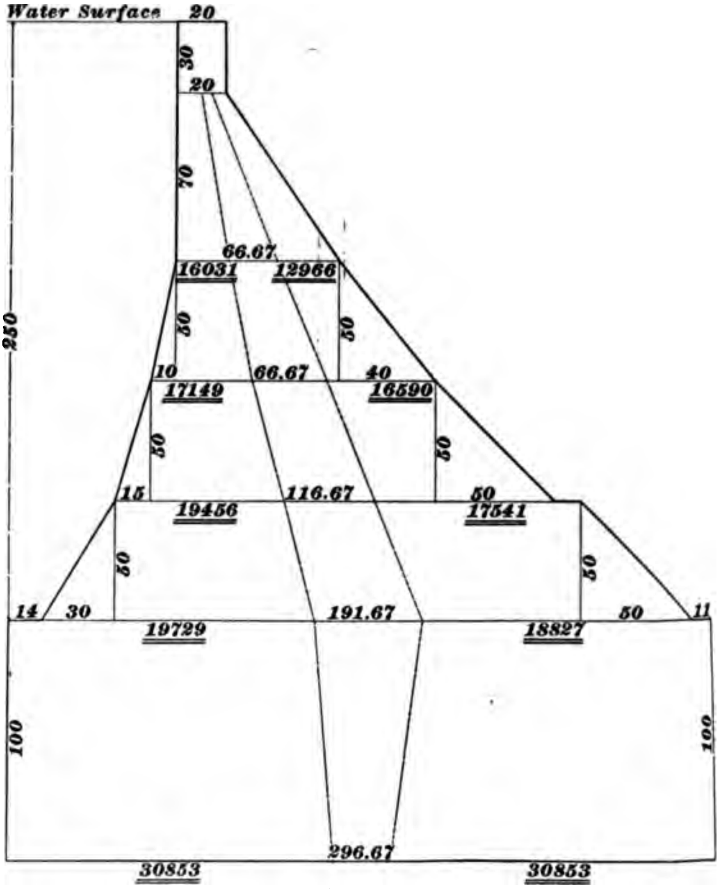


FIG. 677.

2081. It will be observed that no account has been taken of the *downward* water pressure bearing upon the sloping surfaces at the back of the dam. This is generally neglected, because the transference of its action throughout the mass to the base is somewhat uncertain. Its effect is to diminish in some degree the stresses in the lower side

of the dam. Should it be desired to determine the theoretical effect of this pressure, it may readily be done as follows, taking the upper 150 ft. of the profile already established as an example.

Referring to Fig. 678, to find the amount and moment of the downward pressure of the trapezoidal prism of water

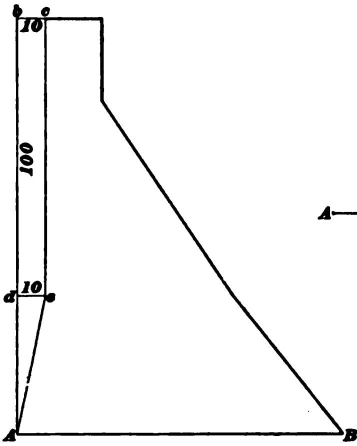


FIG. 678.

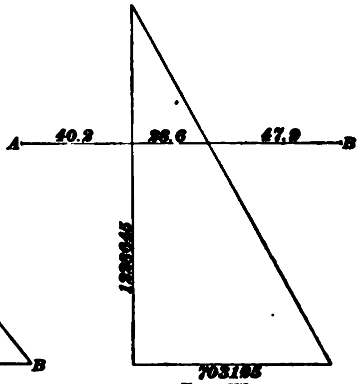


FIG. 679.

A b c e upon the sloping surface *A e*, it is first necessary to ascertain the position of the vertical line passing through its center of gravity. To do this, the area is divided into a rectangle *b c e d* and a triangle *A d e*, and their moments ascertained about any convenient axis, in this case the one already established 250 ft. from the vertical back of the dam. The area of the rectangle *b c d e* is 1,000 sq. ft., and its center of gravity is in the center of the figure, distant, therefore, 255 ft. from the axis of moments. The area of the triangle *A d e* is 250 sq. ft., and its center of gravity is distant $250 + \frac{2}{3} \times 10 = 256.7$ ft. from the axis of moments. Hence,

$$\begin{array}{rcl} 1{,}000 \times 255.0 & = & 255{,}000 \\ \underline{250} \times 256.7 & = & \underline{64{,}175} \\ 1{,}250 & & 319{,}175 \end{array}$$

$$\frac{319{,}175}{1{,}250} = 255.30 \text{ ft.}$$

This is the horizontal distance of the center of gravity of the prism of water from the axis of moments, but this is not needed for our calculation. All that is needed is to multiply the areas and the sum of the moments by 62.5, the weight of a cubic foot of water, and add the weight and moment of the corresponding mass of masonry, which is the same as is shown more in detail in Fig. 670. We have, therefore,

$$1{,}250 \times 62.50 = 78{,}125$$
$$319{,}175 \times 62.50 = 19{,}948{,}438$$

and

$$8{,}218 \times 140 = 1{,}150{,}520$$
$$1{,}786{,}287 \times 140 = 250{,}080{,}180$$

Finally,

78,125	19,948,438
1,150,520	250,080,180
1,228,645	270,028,618

$$\frac{270{,}028{,}618}{1{,}228{,}645} = 219.8 \text{ ft.}$$

This is the distance of the center of gravity of the mass made up of the prism of water and the masonry of the dam from the axis of moments. The distance from the toe A (Fig. 678) is

$$260 - 219.8 = 40.2 \text{ ft.}$$

Fig. 679 shows the new triangle of forces. The new data for the maximum unit stress in segment B are: $L = 116.7$; $d = 47.9$; $L - d = 68.8$; $W = 1{,}228{,}645$.

Hence,

$$P = \frac{1{,}228{,}645 \times 68.8}{116.7 \times 47.9} = 15{,}122 \text{ lb.},$$

as against 16,590 lb., which is the stress when the downward water pressure on the back of the profile is not considered, a gain of about 10%. As already stated, the precise action of this pressure is somewhat uncertain, but it is well to remember that it is always operating, more or less, as a factor of safety, when the reservoir is full.

2082. Summary of Results.—The profile just established is typical, and applies rigorously to a structure of 140 lb. to the cubic foot, with a top width of 20 ft. Neither

the assumed density—140 lb.—nor the top width can vary greatly in practice; so for densities and widths differing but slightly from the above, the profile shown in Fig. 676 holds good for any height from 100 to 250 ft. In any case, and with any different density or top width, this profile will always furnish a safe basis to start from, and each height can be tested with the new data precisely as has been done in establishing the typical profile.

2083. Accessories of High Masonry Dams.—It is still more desirable for dams reaching and exceeding 100 ft. in height that a natural overflow or escape for surplus water should be provided, and generally it is easier to find such an outlet for very high dams than for comparatively low ones, because the water-line approaches so nearly the surrounding summits. Indeed, in the case of such dams it is frequently necessary to construct one or more subsidiary dams to prevent the water, when raised to its full height, from escaping over the "divides" into other valleys. If it should become necessary, however, to allow the waste water to pass over the face of the dam or a portion of it, the profile already established will in general suffice for all heights where the base is equal to 90% of the height. When it falls below, it should generally be brought up to this percentage by extending the outer toe B in the figures. No further general rule will be given, because all such exceptional structures should be considered as special cases, and studied accordingly.

The means of controlling the water will be the same in principle in masonry dams of all heights as for earthen dams. They will always be simpler, however, because there is no earthen embankment to penetrate.

BUILDING MASONRY DAMS.

2084. (*a*) **Execution of the Work.**—It is not sufficient that hydraulic work should be correctly designed; it is equally important that it should be carefully and properly executed. The closest attention is necessary, down to the most minute details. As indicating the manner in which

the work should be carried on, the following extracts are given from "Gould's Specifications for Dams and Reservoirs," which embody those followed in the construction of the storage reservoirs of the Scranton Gas and Water Co., of Scranton, Pa.:

(*b*) "**Clearing and Grubbing.**—The whole area of the reservoir will be cleared by cutting all trees, stumps, and bushes even with the ground and removing or burning the same. The area covered by the dam embankment shall be grubbed, so as to remove all stumps, roots, and sods.

(*c*) "**Rock Excavation.**— * * * As it is important that, in the rock, the trenches shall be shattered as little as possible, hand drilling and light charges of explosives must alone be used.

(*d*) "**Embankment.**—The material for the embankment shall be such as will produce a solid, water-tight bank, and will be selected subject to the approval of the engineer. It will be taken as far as practicable from the land inside the reservoir. No stones larger than two inches in any direction will be allowed in the bank on the upper side of the center wall, or in any slopes exposed to the action of the water, and none larger than four inches on the lower side of said center wall. The material shall be laid down in horizontal courses, carried on the work in wagons, or carts, or in barrows, so as to ensure its being well traveled over with a view to its consolidation. If, in the judgment of the engineer, these means do not produce a sufficiently compact bank, he may, in addition, order the use of rollers or edge-runners of a design approved by him, or such other means as may be necessary to secure the desired compactness. In all places which can not otherwise be reached, the bank shall be tamped with heavy rammers. The bank shall be kept thoroughly moistened while in process of construction, by means of pumps, hose, or sprinkling carts.

(*e*) "**Rubble Masonry.**—The masonry for the center wall shall be composed of quarry stones containing not more than one-third of one cubic yard each, unless otherwise

ermitted by the engineer. They shall all have clean quarry aces, beds, and joints, and each stone shall be thoroughly ret just previous to being laid; every stone shall be laid in full and swimming bed of mortar, and the interior vacancies shall be filled with mortar before any spalls or small tones are put in, the object being to make the wall perfectly rater-tight, and have no spaces, however small, that are iot filled with compact mortar. The bed and end-joints of oth faces of the wall shall be raked and struck as the work rogresses. No stones will be allowed to be deposited nor lressed upon the wall, but all stone shall be deposited and lressed on planks furnished by the contractor at his own xpense, to prevent the stone from coming in contact with he dirt of the embankment. The center wall shall always e kept at least two feet above the adjacent embankment.

"The above specifications apply generally to all rubble nasonry to be built in this work, with the exception that for vork other than the center wall, larger stones will be pernitted, and in some cases required. But all rubble masonry, specially where exposed to the action of the water, shall be trictly hydraulic, as described for the center wall.

"The face of all rubble masonry, except the center wall, hall be hammer-dressed, and pitched to true and fair lines, vith horizontal and vertical beds and joints, extending back t least one foot from the face. No spalls will be allowed n the face, nor any stones less than three inches thick. All work will be thoroughly bonded, with a proper proporion of headers, and breaking well the joints."

WATER SUPPLY AND DISTRIBUTION.

(CONTINUED.)

FLOW THROUGH PIPES.

THE HYDRAULIC GRADE LINE.

2085. In the section on Hydraulics, the **hydraulic grade line,** or **hydraulic gradient,** was defined as the line drawn through a series of points to which water would rise in piezometer tubes attached to a pipe through which water flows. It was also stated that with a smooth pipe of uniform cross-section and without bends or other obstructions to flow, the hydraulic grade line is a straight line extending from the reservoir to the end of the pipe.

In Fig. 680 is shown a long horizontal pipe leading from

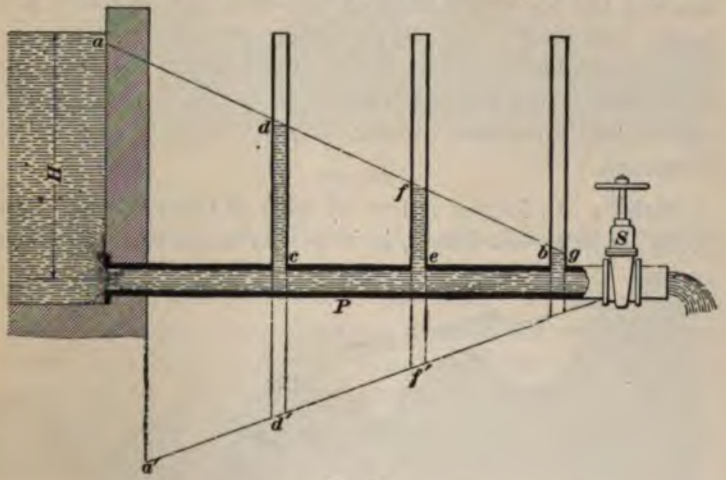


FIG. 680.

a reservoir to a stop-valve S. When the valve is open so that water from the pipe discharges freely into the

For notice of copyright, see page immediately following the title page.

atmosphere, the hydraulic grade line is the line $a\,d\,f\,g$. The distance of the point a below the surface of the water in the reservoir represents the head absorbed in overcoming the resistances of entrance to the pipe, and in producing the velocity with which the water flows. In the same way the difference in the height to which the water rises in any two piezometer tubes represents the head absorbed in overcoming the resistance to flow in the pipe between the points at which the tubes are joined.

2086. The flow of water through the pipe P would be the same whether it were horizontal, as shown in the figure, or if it were laid along the grade line $a\,d\,f\,g$. The flow would also be the same if the reservoir were deepened and the pipe laid along the line $a'\,d'\,f'$. The pressures in the pipe, however, would vary greatly with the different positions. If it were laid along the line $a\,d\,f\,g$, there would be little or no pressure in any part of it, and if it were perforated at the top, little or no water would flow from the perforations. In the horizontal position, however, and still more in the position $a'\,d'\,f'$, there would be pressure at all points, the pressure for any point in the pipe being equivalent to the head represented by the vertical distance from that point to the hydraulic grade line, and if the pipe were perforated anywhere, water would issue from the perforations.

2087. In laying a line of pipe to connect two points lying at different levels, it is of the utmost importance to

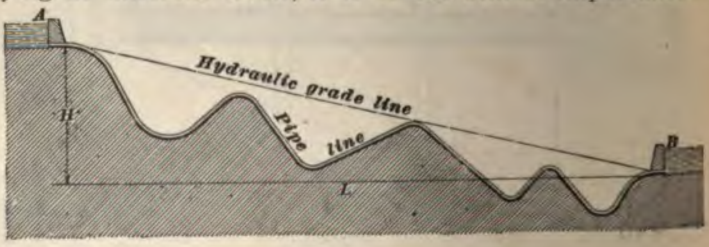


FIG. 681.

ascertain the position of the hydraulic grade line. Let A

and B, Fig. 681, represent two reservoirs, connected by a pipe line of uniform diameter, through which the water flows by gravity from the upper to the lower level. The hydraulic grade line will be the straight line connecting the two reservoirs; in order to cover the most unfavorable conditions, it is usually drawn between the two ends of the pipe line, and not from surface to surface of the water in the two reservoirs, as the level of these surfaces may vary. The slope of the grade line will be represented by $\frac{H}{L}$. *In order that the discharge may take place under the full head, the pipe line must never rise above the grade line at any point.*

Should the pipe rise above this grade line, as is shown at b, Fig. 682, the rate of slope is no longer $\frac{H}{L}$ through the

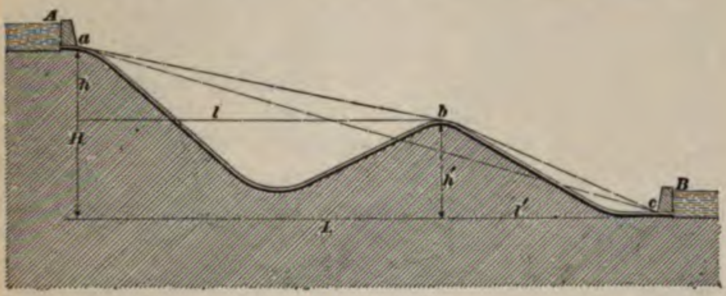


FIG. 682.

entire pipe line, but it is broken into two others at the point b, one, $\frac{h}{l}$, flatter, and the other, $\frac{h'}{l'}$, steeper than $\frac{H}{L}$. If the pipe were of the same diameter throughout, it would not discharge as much water as if it were kept entirely under the hydraulic grade line $a\,c$, because its flow would be governed by the flatter hydraulic grade line $a\,b$. From b to c the water would flow without completely filling the pipe. Sometimes, when a rocky ridge has to be crossed, where it would be very difficult and expensive to keep the pipe low enough, two diameters are used; one, the larger,

being laid between a and b, and the other, the smaller, between b and c. By properly proportioning the diameters to the grades, according to the rules for the flow of water through pipes, the desired discharge can be economically secured in this way.

FLOW OF WATER THROUGH LONG PIPES.

2088. It has been shown in the section on Hydraulics that the mathematical basis upon which the velocity of water issuing from a pipe depends is the well-known expression for the velocity of a body falling freely in a vacuum; namely, $v = \sqrt{2gh}$. We have seen, however, and it will become still more evident as we go on, that, practically, the velocity so indicated is subject to many modifications. The relation of length of pipe to head of pressure, the diameter of the pipe, and the nature of its interior surface so entirely overshadow the mere height of fall, which is the only factor considered in the formula $v = \sqrt{2gh}$, that, finally, the formula itself fades completely out of the problem. Our only trustworthy knowledge of the velocity of flow, and consequent volume of discharge through pipes of different diameters and under different circumstances, rests wholly upon direct experiment.

2089. An all-important factor of all formulas for computing the flow of water through pipes is a *coefficient* whose value has been determined by more or less careful experiments with pipes under the conditions within which the formulas are applicable. In using any of the formulas, it is highly important that the coefficient be so chosen as to correspond as nearly as possible with the conditions under which it was originally determined. If, for any reason, the conditions are doubtful, an allowance should be made that will cover the worst conditions that are liable to occur.

The formulas for the flow of water through pipes given in the section on Hydraulics are based on the general formula for falling bodies. It was shown, however, that the

neral formula is greatly modified by the conditions under ich the pipes are used, and the practical value of the mulas depends on a proper choice of the coefficient to rrespond with the given conditions. The table of efficients f for Smooth Iron Pipes, given in connection th the formulas in the section on Hydraulics, gives iable values of this coefficient that were determined by ans of a set of careful experiments made on pipes under conditions given in the table.

DARCY'S FORMULAS.

2090. The French engineer, Darcy, many years ago ide a series of elaborate experiments with pipes of ferent diameters, from which he formulated certain gebraic expressions which have remained standard to this y. Attempts have been made to improve these simple d reliable formulas, but practical experience more and ore establishes their accuracy and sufficiency. The merit Darcy's formulas consists not only in the skilful and orough manner in which the experiments themselves ere made, but also in the great judgment and practical ct with which the results were formulated for general e.

In the course of these experiments, it was found that the aracter of the interior surface of the pipe affected to a markable degree the velocity of the water flowing through The amount of water flowing through a clean, smooth pe of given diameter, length, and fall was surprisingly minished when another pipe, exactly similar, except having rough and dirty interior surface, was substituted. The gree of reduction in this case was surprising, because it d been supposed that the small projections caused by the ughness of the surface would at most only affect the flow diminishing to that extent the inside diameter of the pe. This would be the case if water were a perfect iid, for then some of the particles of water would simply vel up the irregularities of the surface, and the other

particles would flow freely over them. Water, however, is very far removed from a perfect fluid. It possesses the property of *viscosity* to a great degree, and the particles of which it is composed, instead of moving freely over each other, are held together by molecular attraction, and it requires considerable force to tear them apart. For this reason, the term "friction" is misapplied when used to express the resistance experienced by water in flowing over a rough surface. It is really a resistance to *shearing* that takes place.

2091. To give an idea of the extent to which the nature of the interior surface of a pipe affects the quantity of water running through it, it has been found, within the extreme limits of roughness and smoothness which exist in practice, that if a smooth pipe of given diameter discharges a certain quantity of water per second, a rough pipe, otherwise similar, will require a diameter 15 per cent. greater to discharge the same amount in the same time. Thus, if the smooth pipe had a diameter of 36 inches, the rough pipe would require one of 41.40 inches to have an equal delivery. Did not this fact rest upon actual experience, it would seem incredible that irregularities amounting to only a fraction of one per cent. of the diameter of a pipe could affect the flow to such an extent. It is explainable, however, the moment we realize the great *viscosity* of water.

These facts led Darcy to divide cast-iron water-pipes into the two great classes already mentioned, "smooth" and "rough," the formula for the flow through each being modified by an appropriate coefficient. Between these two extremes there are infinite gradations, but experience shows that, under practical conditions, neither limit will be passed. That is to say, the cleanest and best-conditioned pipes will not give a greater discharge than that assigned to them by the coefficient for smooth pipes, nor will the greatest amount of roughness, from the incrustations to which pipes are liable in practice, reduce the flow below that for rough pipes, although it frequently approaches it closely.

2092. Fundamental Formula.—Darcy's fundamental formula for long pipes, by which is understood pipes of 1,000 diameters and over in length, is

$$\frac{DH}{CLV^2} = 1. \qquad \textbf{(186.)}$$

In this formula,

D = diameter of pipe in feet;
H = total head in feet;
L* = total length in feet;
V = velocity of efflux, in feet per second;
C = an experimental coefficient.

From formula **186** we deduce,

$$V = \sqrt{\frac{DH}{CL}}. \qquad \textbf{(187.)}$$

Since the quantity Q, in cubic feet per second, is equal to the area A of the pipe in square feet, multiplied by the velocity in feet per second, we have

$$Q = A\sqrt{\frac{DH}{CL}}. \qquad \textbf{(188.)}$$

Since $A = 0.7854\,D^2$,

$$Q = 0.7854\,D^2\sqrt{\frac{DH}{CL}}, \qquad \textbf{(189.)}$$

which may be written

$$Q = \sqrt{\frac{0.617\,D^5 H}{CL}}. \qquad \textbf{(190.)}$$

2093. Coefficients.—The important matter now is to know the value of C. For this Darcy gives the following table, based on his experiments:

---

* Although L is, properly speaking, the actual length of the pipe, it differs in practice so little from its horizontal projection that the latter is taken as being, in general, a sufficiently close approximation.

TABLE OF COEFFICIENTS.

Diameters in Inches.	Value of *C* for Rough Pipes.	Value of *C* for Smooth Pipes.
3	0.00080	0.00040
4	0.00076	0.00038
6	0.00072	0.00036
8	0.00068	0.00034
10	0.00066	0.00033
12	0.00066	0.00033
14	0.00065	0.00033
16	0.00064	0.00032
24	0.00064	0.00032
30	0.00063	0.00032
36	0.00062	0.00031
48	0.00062	0.00031

It will be observed that the coefficient for smooth pipes is in all cases half that of rough ones. As all pipes, no matter how clean and smooth they may be when first laid, become in process of time more or less tuberculated and foul, it is safer in practice to always use the coefficient for rough pipes when a permanent system is being laid down.

2094. In carefully studying the above table, we see that the coefficients for pipes from 8 to 48 inches in diameter do not greatly vary. We see, moreover, from formula **188, 189,** or **190,** that, all other conditions being equal, the quantity discharged is affected by only the square root of the coefficient, so that slight differences in its value are insignificant in reference to the volume of water discharged. Observing now that formula **190** contains the factor 0.617, we perceive that if we take 0.000617 as an approximate coefficient for pipes within the limits of 8 and 48 inches, we shall have

$$Q = \sqrt{\frac{0.617\,D^5\,H}{0.000617\,L}};$$

whence, $$Q = \sqrt{\frac{1{,}000\,D^5\,H}{L}}. \qquad \textbf{(191.)}$$

If, now, we replace $\frac{H}{L}$, or the total head divided by the total length of pipe, by the *head per thousand*, or $\frac{h}{1{,}000}$, we may write the above formula thus:

$$Q = \sqrt{D^5\,h}, \qquad \textbf{(192.)}$$

which may be generalized thus:

$$\frac{Q^2}{h\,D^5} = 1. \qquad \textbf{(193.)}$$

In this formula it must be borne in mind that h is the fall per thousand.

When logarithms are employed—and the hydraulic engineer should be perfectly familiar with their use—formulas **191** and **192** are readily solved. Otherwise, they may be more conveniently written,

$$Q = D^2\sqrt{D\,h}. \qquad \textbf{(194.)}$$

$$\frac{Q}{D^2\sqrt{D\,h}} = 1. \qquad \textbf{(195.)}$$

For pipes of smaller diameter, from 3 to 6 inches, we may assume a coefficient of 0.000785. Then for such pipes we have, from formula **190**,

$$Q = \sqrt{\frac{0.785 \times 0.785\,D^5 H}{0.000785\,L}};$$

whence, $$\frac{Q^2}{hD^5} = 0.785; \qquad \textbf{(196.)}$$

also, $$Q = 0.89\sqrt{D^5 h}. \qquad \textbf{(197.)}$$

That is to say, for these smaller diameters, the delivery

will be, in round numbers, 90 per cent. of that given by formula **192.**

2095. Formulas for Smooth Pipes.—While in practice the formulas for rough pipes should always be used, it is sometimes useful to know the probable discharge through smooth ones. Since the coefficients for the latter are always $\frac{1}{2}$ of those for the former, for smooth pipes formulas **192** and **193** may be written,

$$Q = \sqrt{2 D^5 h}. \qquad \textbf{(198.)}$$

$$\frac{Q^2}{h D^5} = 2. \qquad \textbf{(199.)}$$

Also, from formula **198,**

$$Q = 1.40 \sqrt{D^5 h}. \qquad \textbf{(200.)}$$

That is to say,

In general, the discharge through a smooth pipe is 1.40 times that through a rough pipe of the same diameter; and, reciprocally, the discharge through a rough pipe is 0.70 times that through a smooth one of the same diameter, and these factors represent the practical limits between which the extremes of roughness and smoothness can affect the flow through long pipes.

2096. Formulas for Velocity.—Formulas for velocity are frequently needed. They may be derived from those already established.

Since velocity is equal to quantity divided by area, we have from formula **194,** for rough pipes of from 8 to 48 inches diameter,

$$V = \frac{D^2 \sqrt{D h}}{0.7854 D^2};$$

whence, $$V = 1.27 \sqrt{D h}. \qquad \textbf{(201.)}$$

For rough pipes of smaller diameter,

$$V = 1.13 \sqrt{D h}. \qquad \textbf{(202.)}$$

For smooth pipes of large diameter,

$$V = 1.78 \sqrt{D h}. \qquad \textbf{(203.)}$$

For smooth pipes of small diameter,

$$V = 1.60\sqrt{Dh}. \qquad \textbf{(204.)}$$

The relations of the velocities will be as the quantities; hence, the general rule in Art. **2095** holds good for relative velocities, also.

It is to be understood that the terms "rough" and "smooth," here, as elsewhere, signify the extremes of both cases.

EXAMPLE 1.—A rough pipe, 16" in diameter and 8,700 ft. long, connects two reservoirs, the difference of elevation between the two being 187 ft. With what velocity does the water flow through the pipe?

SOLUTION.—Substituting in formula **187**, we have

$$V = \sqrt{\frac{\frac{4}{3} \times 187}{0.00064 \times 8,700}} = 10.26 \text{ ft. per sec.} \quad \text{Ans.}$$

EXAMPLE 2.—What is the velocity through the pipe in example 1, calculated by formula **201**?

SOLUTION.— $V = 1.27\sqrt{\frac{4}{3} \times 50.5} = 10.42$ ft. per sec. Ans.

NOTE.—In approximate formulas, such as all those which apply to the flow of water through pipes necessarily are, the results obtained in examples 1 and 2 are equivalent to an agreement, and in practice one might happen to be as near right as the other. It is obvious that, when the character of the pipe may vary as to interior surface so widely, a very close result can never be hoped for, and all we can do is to keep within probable limits.

EXAMPLE 3.—A rough pipe, 10 inches diameter, is laid with a fall of 7½ ft. per 1,000. What is the discharge?

SOLUTION.—Applying formula **192**, and using logarithms,

Log 10	1.00000
Log 12	1.07918
	$\bar{1}.92082$
	5
Log of 5th power.....	$\bar{1}.60410$
Log 7.5	.87506
	2)0.47916
Log of square root....	0.23958
Corresponding number	1.736.

Therefore, the discharge is 1.736 cubic ft. per second. Ans.

EXAMPLE 4.—It is desired to discharge 8 cubic feet per second from a pipe line having a fall of 5 ft. per 1,000. What diameter of rough cast-iron pipe will be required?

SOLUTION.—Inserting the data in formula **193**,

$$D = \sqrt[5]{\frac{9}{5}}.$$

Log 9	.95424
Log 5	.69897
	5) .25527
Log 5th root	0.05105

Corresponding number $= 1.125$.

Therefore, the diameter is 1.125 ft. = 13½ inches. Ans.

As cast-iron pipes are made to standard sizes, and there are no half inches, the nearest appropriate size would be a 14-inch pipe.

EXAMPLE 5.—It is desired to discharge half a cubic foot per second from a 4-inch pipe. What head per 1,000 is necessary to accomplish this?

SOLUTION.—Substituting the data in formula **196**,

$$h = \frac{\frac{1}{4}}{0.785 \times \frac{1}{243}} = 77.39 \text{ ft.} \quad \text{Ans.}$$

2097. General Relations Between *D*, *Q*, *L*, *H*, and *C*, and *D'*, *Q'*, *L'*, *H'*, and *C'*.

From formula **186** we have, for a given pipe line,

$$\frac{D H}{C L V^2} = 1.$$

For any other system we should have

$$\frac{D' H'}{C' L' V'^2} = 1.$$

Then,
$$\frac{D H C' L' V'^2}{D' H' C L V^2} = 1.$$

C and C' will generally be sufficiently near each other to be negligible; hence,

$$\frac{D H L' V'^2}{D' H' L V^2} = 1. \qquad \textbf{(205.)}$$

Also, from formula **190**, we have

$$\frac{Q^2L}{D^5H} = \frac{0.617}{C}.$$

Also,
$$\frac{Q'^2L'}{D'^5H'} = \frac{0.617}{C'}.$$

Then, letting $C = C'$,

$$\frac{Q^2L\,D'^5H'}{Q'^2L'D^5H} = 1. \qquad \textbf{(206.)}$$

EXAMPLE.—A pipe (see example 1, Art. **2096**), 16" diameter, 3,700 ft. long, with a total fall of 187 ft., has a velocity of 10.26 ft. per second. Another pipe has exactly the same elements, except that its diameter is 18 inches. What is its velocity?

SOLUTION.—Let the elements of the first pipe be D, H, L, and V, and those of the second, D', H', L', and V'. By the conditions given, $H = H'$ and $L = L'$. Then, in formula **205**,

$$\frac{D\,V'^2}{D'\,V^2} = 1,$$

and
$$V' = V\sqrt{\frac{D'}{D}}.$$

Substituting the data,

$$V' = 10.26\sqrt{\frac{1.50}{1.33}} = 10.83 \text{ ft. per sec.} \quad \text{Ans.}$$

2098. From formula **206**, we have

$$Q' = \sqrt{\frac{Q^2L\,D'^5H'}{L'D^5H}}.$$

If L and H equal, respectively, L' and H',

then,
$$\frac{Q'}{Q} = \sqrt{\frac{D'^5}{D^5}}. \qquad \textbf{(207.)}$$

That is, other elements being equal, the quantities discharged are as the square roots of the fifth powers of the diameters. This is a very important relation.

EXAMPLE 1.—A long pipe of 24" diameter gives a discharge of 2 cubic feet per second. What will be the discharge of a similar pipe, under similar circumstances, of 30" diameter?

Solution.—From formula **207**, we have

$$Q' = Q\sqrt{\frac{D'^5}{D^5}}.$$

Substituting the data,

$$Q' = 2\sqrt{\frac{2.5^5}{2^5}}.$$

$$\begin{array}{lr}
\text{Log } 2.5^5 = 0.39794 \times 5 = & 1.98970 \\
\text{Log } 2^5 = 0.30103 \times 5 = & \underline{1.50515} \\
 & 2)\ \underline{.48455} \\
\text{Log square root} = & 0.24227 \\
\text{Log } 2 \qquad\qquad = & \underline{.30103} \\
 & 0.54330
\end{array}$$

Corresponding number, 3.494.

Therefore, the discharge will be 3.494 cu. ft. per second. Ans.

Example 2.—A 24" pipe, as in example 1, discharges 2 cubic feet per second. What must be the diameter of a pipe to discharge 3 cu. ft. per second, under similar conditions?

Solution.—Here we write formula **206** thus:

$$D' = D\sqrt[5]{\frac{Q'^2}{Q^2}}.$$

Substituting the data,

$$D' = 2\sqrt[5]{\frac{9}{4}}.$$

$$\begin{array}{lr}
\text{Log } 9 \ldots\ldots\ldots\ldots\ldots\ldots & 0.95424 \\
\text{Log } 4 \ldots\ldots\ldots\ldots\ldots\ldots & \underline{0.60206} \\
 & 5)\ \underline{0.35218} \\
\text{Fifth root} \ldots\ldots\ldots\ldots\ldots & 0.07044 \\
\text{Log } 2 \ldots\ldots\ldots\ldots\ldots\ldots & \underline{.30103} \\
 & .37147
\end{array}$$

Corresponding number, 2.352.

Therefore, the pipe must be 2.352 ft. in diameter. This would probably be taken in practice as 28 inches, for the next regular size of cast-iron pipe is 30 inches, which is much larger than is needed. Ans.

Example 3.—A 24-inch pipe, as in example 1, discharges 2 cubic feet per second. How many 8-inch pipes will be required to give the same discharge, the heads and lengths being the same?

SOLUTION.—Let x = the required number of pipes. Then the number required being inversely as the quantity discharged, we invert formula **207** and obtain

$$x = \sqrt{\frac{D^5}{D'^5}}. \qquad \textbf{(208.)}$$

Inserting the data,

$$x = \sqrt{\frac{2^5}{(\frac{2}{3})^5}} = \sqrt{3^5}.$$

$$x = \sqrt{3^3 \times 3^2} = 3\sqrt{27} = 15.588.$$

That is to say, 16 pipes would be required, each 8" in diameter. Ans.

The result obtained in example 3 may be surprising, but is nevertheless correct, and shows the great increase of flow which follows an increase of diameter.

The student is again reminded that all the preceding formulas apply to *long pipes* only; i. e., those of at least 1,000 diameters in length.

BRANCH PIPES.

2099. All the above examples have been very simple, and of direct solution, and they cover the great majority of the practical cases which will come under the notice of the hydraulic engineer. But there are other much more intricate ones which occur, some very rarely and others with comparative frequency, with which the accomplished hydraulician must be prepared to cope. They are all solved—if solvable—by some application of the formulas already established, but a great deal of tact is often necessary in order to adapt these general formulas to special cases.

Among the more common of these special cases are those referring to the discharge through branch pipes. Before entering upon this subject, it will be necessary to establish a few preliminaries.

2100. Elevations Above Datum.—In all the formulas already given, the fall or head has been represented by H or h. In engineering operations, however, heights or elevations are generally stated as heights above some fixed point, or datum line, which is considered as passing below

the entire work to be executed. Near the coast, this datum line is generally the level of mean low water, and if any subaqueous work is done, or any subterranean work below that level, its elevation bears the minus sign. Generally, however, the datum line is taken, as already stated, below the lowest parts of the work.

Thus, supposing that the surface of the water in a certain reservoir stood at the elevation 375 ft. above a certain point—for example, a bench mark, either real or assumed, through which the datum line passed, and of which the elevation would be zero. If this reservoir were connected by a pipe with another reservoir, of which the elevation (above the same datum) was 233, then, in the preceding formulas, we would have $H = 375 - 233 = 142$ ft.

2101. General Illustration.—Suppose, as shown in Fig. 683, we have a reservoir R, the elevation of the surface

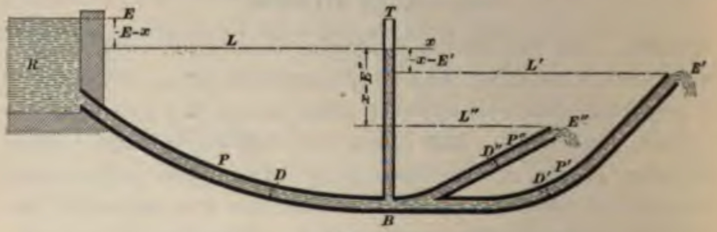


FIG. 683.

of the water in which is E. A pipe P of diameter D and length L runs out of it, and at B is divided into two branches P' and P'', whose diameters and lengths are respectively D' and D'', L' and L''. These two pipes supply points situated respectively at the elevations E' and E''. It is desired to know, when both pipes are discharging freely, how much water is delivered by each of the two branches P' and P''.

Suppose a piezometric tube T to be erected at the point B of embranchment. If we knew the elevation x of the water in this tube when both branches were discharging freely, the problem could be easily solved, for, knowing the

ations of the extremities of the branches, and their ;ths and diameters, we should have all the necessary ı for applying formula **192.**

ow it is evident that the quantity discharged by the pipe ınder the head $E - x$ at the point of embranchment ɪt be equal to the quantity discharged by the pipe P' er the head $x - E'$, *plus* the quantity discharged by the : P'' under the head $x - E''$, because we may consider piezometric tube T as a small reservoir into which the er flows from R through the pipe P, and *from* which ɔws through the pipes P' and P''.

general solution of this problem would be very difficult, ı if practically possible, but in any particular case where the data are given except the piezometric height x, a tion is perfectly practical, although involving a great of tedious calculation. This can be demonstrated by ɛxample.

102. Referring to Fig. 683, let $E = 300$ ft., $E' = 250$ and $E'' = 200$ ft. Let the lengths and diameters and ls of pipes P, P', and P'' be, respectively,

$$L = 3{,}000 \text{ ft.}$$
$$D = 2 \text{ ft.}$$
$$H = 300 - x \text{ ft.}$$
$$L' = 2{,}000 \text{ ft.}$$
$$D' = 1.5 \text{ ft.}$$
$$H' = x - 250 \text{ ft.}$$
$$L'' = 1{,}500 \text{ ft.}$$
$$D'' = 1 \text{ ft.}$$
$$H'' = x - 200 \text{ ft.}$$

hen, using formula **191**, we have:

$$Q = \sqrt{\frac{1{,}000 \times 32\,(300 - x)}{3{,}000}} = \sqrt{\frac{9{,}600 - 32x}{3}}.$$

$$Q' = \sqrt{\frac{1{,}000 \times 7.60\,(x - 250)}{2{,}000}} = \sqrt{3.8x - 950}.$$

$$Q'' = \sqrt{\frac{1{,}000 \times 1\,(x - 200)}{1{,}500}} = \sqrt{\frac{10x - 2{,}000}{15}}.$$

But $Q = Q' + Q''$; therefore, reducing and neglecting insignificant decimals, we have

$$\sqrt{3,200 - 10.7\,x} = \sqrt{3.8\,x - 950} + \sqrt{0.67\,x - 133}.$$

Squaring,

$$3,200 - 10.7\,x = 3.8\,x - 950 + 0.67\,x - 133 + \sqrt{10.20\,x^2 - 4,568\,x + 505,400}.$$

Transposing,

$$\sqrt{10.20\,x^2 - 4,568\,x + 505,400} = 4,283 - 15.17\,x.$$

Squaring,

$$10.20x^2 - 4,568\,x + 505,400 = 18,344,089 + 230.13\,x^2 - 129,946\,x.$$

Transposing and reducing, neglecting small decimals,

$$220\,x^2 - 125,378\,x = -17,838,689$$
$$x^2 - 570\,x = -81,085$$
$$x = 285 \pm \sqrt{81,225 - 81,085}$$
$$x = 285 \pm 11.83$$
$$x = 297 \text{ or } 273.$$

There will frequently be a little doubt as to which is the right value of x, so it will be necessary to try which gives $Q = Q' + Q''$. There can not be two solutions. Trying first the smaller value, as the one most likely to be correct, we have:

$$Q = \sqrt{\frac{1,000\,(300 - 273)\,32}{3,000}} = 16.97 \text{ cubic feet.}$$

$$Q' = \sqrt{\frac{1,000\,(273 - 250)\,7.6}{2,000}} = 9.35 \text{ cubic feet.}$$

$$Q'' = \sqrt{\frac{1,000\,(273 - 200)\,1}{1,500}} = 6.97 \text{ cubic feet.}$$

$Q' + Q'' = 16.32$ cubic feet. A sufficiently close agreement.

2103. Method by Approximation.—The limits within which x can vary in any given case being relatively

narrow, and the process of checking so rapid and simple, it is frequently easier and quicker to operate by "trial and error," particularly as we can closely approximate a true solution from an incorrect one by proportion. An example will make this clear.

Suppose, in the preceding example, it had appeared, from a consideration of the given elevations, that $x = 270$ ft. was a probable value for the piezometric height at the point of embranchment. Trying this value, we would have:

$$Q = \sqrt{\frac{1{,}000\ (300 - 270)\ 32}{3{,}000}} = 17.89 \text{ cubic feet.}$$

$$Q' = \sqrt{\frac{1{,}000\ (270 - 250)\ 7.6}{2{,}000}} = 8.72 \text{ cubic feet.}$$

$$Q'' = \sqrt{\frac{1{,}000\ (270 - 200)\ 1}{1{,}500}} = 6.83 \text{ cubic feet.}$$

Then, $Q' + Q'' = 15.55$ cu. ft., as against 17.89 furnished by the 2-ft. pipe. We could either try again, by guess, making x greater, so as to decrease the head on P, and increase it on P' and P'', or else we could proportion the error, as follows:

Since $Q' + Q'' = 15.55$ cu. ft., let us see what head would be necessary to give that discharge from P. From formula **193**, we deduce

$$h = \frac{Q^2}{D^5}.$$

Substituting the data,

$$h = \frac{241.80}{32} = 7.556.$$

This is the fall per 1,000. Since $L = 3{,}000$, $H = 22.67$. Adding this to the assumed value of x, we have

$$270 + 22.67 = 292.67 \text{ ft.}$$

But, from the data, this should be 300 ft. We have, then, the proportion, neglecting decimals,

$$\frac{x}{270} = \frac{300}{293}.$$

$$x = 276$$

Using this value, we have,

$$Q = \sqrt{\frac{1{,}000\ (300 - 276)\ 32}{3{,}000}} = 16 \text{ cu. ft.}$$

$$Q' = \sqrt{\frac{1{,}000\ (276 - 250)\ 7.6}{2{,}000}} = 9.9 \text{ cu. ft.}$$

$$Q'' = \sqrt{\frac{1{,}000\ (276 - 200)\ 1}{1{,}500}} = 7.1 \text{ cu. ft.}$$

Then, $Q' + Q'' = 17$ cu. ft. This is not quite a close enough agreement, and we must make another approximation, seeing what head would be necessary to discharge 17 cu. ft. through the 2-ft. pipe P. As before, we use formula **193**,

$$h = \frac{289}{32} = 9.$$

The total fall in 3,000 ft. would then be $9 \times 3 = 27$ ft. Adding this to 276 gives 303. But the true elevation we know is 300. Therefore,

$$\frac{x}{276} = \frac{300}{303}.$$

$$x = 273 \text{ ft.}$$

This is the value already found by direct calculation.

2104. Cases of Impossibility.—It is evident that the branches might be of such lengths and diameters or their extremities might be situated at such elevations that water would not flow from both. How can this fact be determined? In general, it may be said that when the piezometric height at the point of embranchment is such that the branch pipe having the lowest point of discharge is capable of taking away all the water that flows to it from the reservoir, then none will be delivered by the other branch having a higher point of delivery. But this will be best illustrated by an example.

Suppose, in the previous examples, illustrated by Fig. 683, that it was desired to have the point of discharge of the 18″ pipe P'' at an elevation $E' = 290$ ft. above datum. Would

it discharge water at that elevation, when the 12″ pipe P'' was discharging at the elevation $E'' = 200$?

The quickest way to solve this question will be to assume a piezometric height $= E' = 290$ ft., and see what will be the discharge through P and P''.

For the discharge through P, we have, from formula **192**, h being $= \frac{10}{3}$,

$$Q = \sqrt{\frac{32 \times 10}{3}} = 10.33.$$

For the discharge through P'' $\left(h = \frac{90}{1.5} = 60\right)$,

$$Q'' = \sqrt{60} = 7.75.$$

Hence, P'' could not discharge all the water flowing from the reservoir through P under a piezometric head of 290. Consequently, the piezometric head is greater than 290, and water will also be delivered through P' at the desired elevation.

Let us take another example, and suppose $E' = 295$ ft. above datum. Then,

$$Q = \sqrt{\frac{32 \times 5}{3}} = 7.30 \text{ cu. ft.},$$

and $$Q'' = \sqrt{63.33} = 7.96 \text{ cu. ft.}$$

Here the discharge from P'', with a piezometric height 295, would be greater than the flow through P; consequently, the piezometric height must be less than 295, and P' would not discharge at that elevation.

2105. Practical Applications. — Many practical applications of the above principles occur. They generally present themselves, however, in inverse order. A numerical example will make this plain.

Referring to Fig. 684, a reservoir R is situated at an elevation of 500 ft. above datum. It is desired to supply three points, situated respectively at elevations 310, 330, and 460 ft. above datum, as follows: The lowest point, whose

elevation is 310 ft., is to receive 2 cu. ft. per second from a branch pipe 3,500 ft. long; the next point, elevation 330 ft., is to receive 1 cu. ft. per second through a branch 2,000 ft. long, and the third point, elevation 460 ft., is to receive 2.5 cu. ft. per second through a branch 4,000 ft. long. The first point of embranchment is 3,000 ft. from the reservoir,

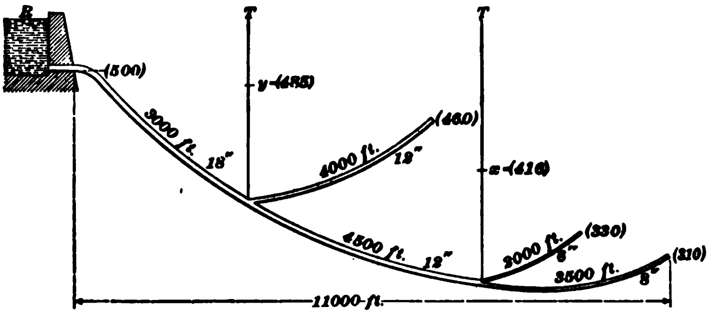


FIG. 684.

and the next occurs 4,500 ft. from the first. All the elevations and lengths of pipe are shown in the figure. It may be noted here that, in this figure, and in all those which follow, elevations are placed in parentheses. This is a very convenient practice, which should be followed in all drawings where elevations are given.

The problem now is to determine the proper diameters of the pipes, and formula **193**, under the two forms,

$$D = \sqrt[5]{\frac{Q^2}{h}}, \text{ and } h = \frac{Q^2}{D^5},$$

will be used.

The diameters will be determined by "trial and error," the only practical way in this instance, and the work will be commenced at the lower end. No diameter less than 6″ will be admitted. (See Art. **2122.**)

Commencing then with the 3,500 ft. of pipe through which 2 cu. ft. of water per second is to be delivered at elevation 310, we will first try the smallest permissible diameter of $\frac{1}{2}$ ft., and see what head x will result at the

imaginary piezometric tube *T*. Here we use the second of the above formulas, and insert the data, thus:

$$h = \frac{4}{\frac{1}{32}} = 128 \text{ ft.}$$

This is the head per 1,000. Since the pipe is 3,500 ft. long, we have total head $= 128 \times 3.5 = 448$. The point to be supplied having the elevation 310, the elevation at the piezometric tube would be $448 + 310 = 758$ ft. But the elevation of the reservoir is only 500 ft.; hence, the six-inch pipe would require a greater head than is available. We will, therefore, try a larger pipe; namely, 8 inches. Substituting this value, we have

$$h = \frac{4}{\frac{32}{243}} = 30.4 \text{ ft.}$$

This gives a total fall of $3.5 \times 30.4 = 106$ ft. (neglecting decimals), which, added to the elevation of the given point, makes $x = 416$ ft.

This result shows the great reduction in *head* brought about by a comparatively small increase of *diameter*. This is because discharges vary with the square root of the head and the square root of fifth power of diameter.

We must next see what diameter is needed for the 2,000 ft. of pipe delivering 1 cu. ft. of water per second at elevation 330, under the head of $416 - 330 = 86$ ft. We are sure, in advance, that 6 inches will answer in this case, but it is always better to try, thus:

$$h = \frac{1}{\frac{1}{32}} = 32 \text{ ft.}$$

The distance being 2,000 ft., the total head necessary is 64 ft. But we have 86 ft. available, so the diameter of 6 inches is more than sufficient.

Before determining the proper diameter of the 4,500 ft. of

pipe lying between the two points of embranchment, we will determine that of the 4,000 ft. of pipe delivering 2.5 cu. ft. of water at elevation 460. We will assume 12 inches for a trial. Then,

$$h = \frac{6.25}{1} = 6.25 \text{ ft.}$$

The distance being 4,000 ft., the total head required is 25 ft., which added to 460 gives 485 ft. of elevation.

We can now ascertain the diameter of the 4,500 ft. of main pipe lying between the two embranchments, which must discharge $1 + 2 = 3$ cu. ft. per sec. As we have the two elevations fixed, namely, 485 and 416, which gives $h = \frac{485 - 416}{4.5} = 15$ ft. (neglecting decimals), we will use the first of the above formulas, thus:

$$D = \sqrt[5]{\frac{9}{15}}.$$

Log 9	0.95424
Log 15	1.17609
	5) $\bar{1}$.77815
	$\bar{1}$.95563

Corresponding number, 0.9029. Hence, a 12-inch pipe is needed.

Finally, we require the diameter of the pipe 3,000 ft. long, leading from the reservoir to the first point of embranchment. This must carry the total quantity of water, $2 + 1 + 2.5 = 5.5$ cu. ft. under a head of $500 - 485 = 15$ ft. As the distance is 3,000 ft., this gives $h = 5$ ft., and we have, inserting the data,

$$D = \sqrt[5]{\frac{30.25}{5}} = \sqrt[5]{6.05}.$$

Log 6.05	5) 0.78175
	0.15635

Corresponding number, 1.434. Therefore, an 18″ pipe would be required.

Very complicated problems of this character may be solved without much difficulty in the same way as the above example, by beginning at the lowest point to be supplied, and working back towards the source.

2106. A Pipe Fed by Two Reservoirs.—An analogous problem is that of a pipe fed by two reservoirs. Let Fig. 685 represent a system composed of a pipe P discharging freely at the elevation E, and fed from the reservoir R, elevation E', by the pipe P', and also from the reservoir R', elevation E'', by the pipe P'', the point of

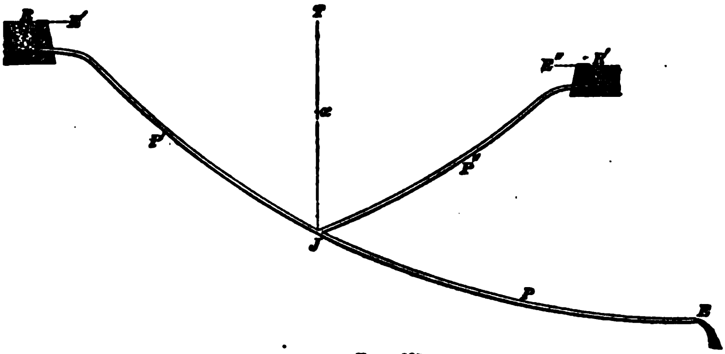


FIG. 685.

junction being at J, where we will suppose the piezometric tube T to be erected.

The question would then be, will water flow from R and R' through the pipe P, or will it flow from R to R' and through P? In each case the answer depends upon the elevation of the water in T, and this depends upon the elevations E, E', and E'', the lengths and diameters of P, P', and P'', and the position of the point of junction J. An example will make this clear.

Referring to Fig. 685, let $E = 50$ ft., $E' = 500$ ft., and $E'' = 400$ ft. Let the length and diameter of P be, respectively, 5,000 feet and 12 inches; those of P', 3,000 feet and 8 inches, and those of P'', 4,000 feet and 10 inches. What is the elevation x of the water in the imaginary piezometric tube T?

NOTE.—In solving such questions, the student should always make a sketch, to scale, on section paper, as this will frequently facilitate a

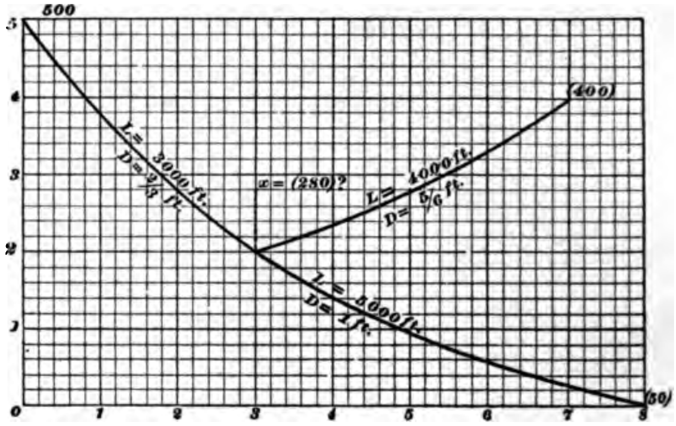


FIG. 686.

solution by mere inspection. Fig. 686 shows such a sketch in which the vertical scale is 10 times as great as the horizontal.

Let us *assume* $x = 280$, and see the result.

We will use formula **193**, written thus:

$$Q = \sqrt{D^5 h}.$$

Then, for the discharge of P, h being $\dfrac{280 - 50}{5} = 46$, we have

$$Q = \sqrt{1^5 \times 46} = 6.78 \text{ cu. ft.}$$

For the discharge of P', h' being $\dfrac{400 - 280}{4} = 30$,

$$Q' = \sqrt{30 \times \left(\frac{5}{6}\right)^5} = \sqrt{30\left(\frac{1}{1.2}\right)^5} = \sqrt{\frac{30}{2.49}} = 3.47 \text{ cu. ft.}$$

For the discharge of P'', h' being $\dfrac{500 - 280}{3} = 73.33$,

$$Q'' = \sqrt{\left(\frac{2}{3}\right)^5 73.33} = \sqrt{\frac{1}{7.6} \times 73.33} = 3.10.$$

$$Q' + Q'' = 6.57 \text{ cu. ft.}$$

Evidently, we have taken x a little too high. Let us try $x = 275$. Then,

$$Q = \sqrt{45.00} = 6.71 \text{ cu. ft.}$$

$$Q' = \sqrt{\frac{31.25}{2.49}} = 3.54 \text{ cu. ft.}$$

$$Q'' = \sqrt{\frac{75.00}{7.6}} = 3.14 \text{ cu. ft.}$$

$Q' + Q'' = 6.68$ cu. ft. per second, as against 6.71 cu. ft. in the same time. This is a close practical agreement.

2107. Influence of Change of Position of Point of Junction and Diameter of Pipe.—Any changes in these elements, the elevations remaining the same, produce great changes in the action of the system.

For example, suppose, with the data in Art. **2106,** the elevations remain the same, but the elements of P are: length $= 6{,}000$ ft., diameter $= 10$ inches, and those of P', length $= 2{,}000$ ft., diameter 12 inches, those of P'' remaining the same. Which way would the water flow?

The question being simply whether water would flow to or from R', we will at once assume a piezometric height at J (Fig. 685) equal to the elevation 400 of R'. If, with this piezometric height, the flow through P is *greater* than that through P', it is evident that the true piezometric height is *less* than 400, and that water will flow from both R and R'. If, on the contrary, the flow through P is *less* than that through P', then the true piezometric height is *greater* than 400, and the flow will be from R to R', as well as through P.

For the flow through P, we have $h = \dfrac{400 - 50}{6} = 58.33$.

Then, $$Q = \sqrt{58.33 \times \left(\frac{1}{1.2}\right)^5} = 4.84 \text{ cu. ft.}$$

For that from R, through P', we have $h = \dfrac{500 - 400}{2} = 50$.

Then, $$Q' = \sqrt{50} = 7.07 \text{ cu. ft.}$$

More water would, consequently, arrive at the point of junction than could be carried away from it, and the surplus would flow from reservoir R into reservoir R', as well as through P.

COMPOUND PIPE LINE.

2108. The term **compound pipe line** is applied to a line composed of pipes of different diameters. Such a combination is sometimes met with in old lines, and it is important to be able to calculate its discharge.

Fig. 687 represents a reservoir R, whose elevation is 500,

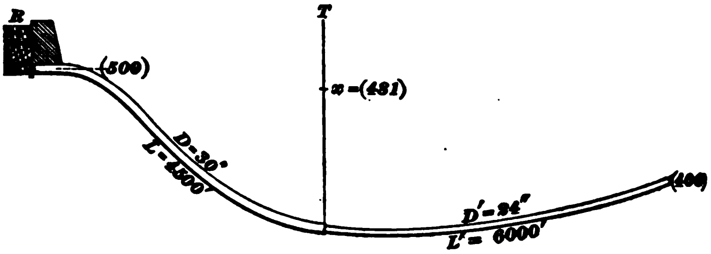


FIG. 687.

tapped by a 30″ pipe, 4,500 ft. long, connected by a reducer with a 24″ pipe, 6,000 ft. long, discharging freely at an elevation of 400 ft. What is the delivery of the system?

It is necessary to find the elevation x of the water in the imaginary piezometric tube T. This is easily done, because the quantity discharged per second by the large pipe must equal that discharged by the smaller one. Hence,

$$\frac{1{,}000\,(500 - x)\,(2.5)^5}{4{,}500} = \frac{1{,}000\,(x - 400)\,2^5}{6{,}000}.$$

$$\frac{(500 - x)\,97.656}{3} = \frac{(x - 400)\,32}{4}.$$

$$(500 - x)\,32.552 = (x - 400)\,8.$$

Neglecting small decimals,

$$40.5\,x = 19{,}476$$

$$x = 481 \text{ ft.}$$

hen, the discharge of the 30″ pipe is

$$Q = \sqrt{4.22 \times 97.66} = 20.30 \text{ cu. ft.}$$

hecking, by the discharge of the 24″ pipe,

$$Q = \sqrt{13.5 \times 32} = 20.78 \text{ cu. ft.}$$

'he slight discrepancy is owing to the neglect of deci-s. In practice, the agreement would be satisfactory, the average quantity taken, $\frac{20.78 + 20.30}{2} = 20.54$ cu. ft.

:**109.** If the number of pipes of different diameters is 'eased, the work becomes somewhat more intricate, as wn by the following:

ı the system shown in Fig. 688, what are the piezometric

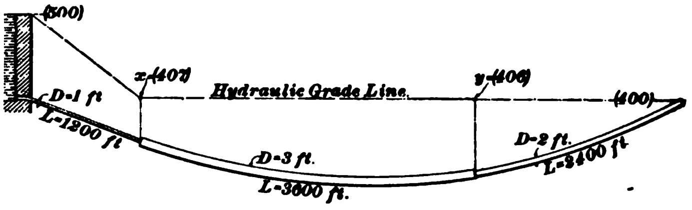


FIG. 688.

ghts x and y, and what the discharge of the system?

$$Q^2 = \frac{10(500 - x)}{12} = \frac{5,000 - 10x}{12}. \qquad (a)$$

$$Q^2 = \frac{10(x - y)\,243}{36} = \frac{(10x - 10y)\,81}{12}. \qquad (b)$$

$$Q^2 = \frac{10(y - 400)\,32}{24} = \frac{(10y - 4,000)\,16}{12}. \qquad (c)$$

'ombining (b) and (c),

$$81x - 81y = 16y - 6,400.$$

$$y = \frac{6,400 + 81x}{97}.$$

Ieglecting small decimals,

$$y = 66 + 0.835x.$$

Inserting this value of y in (c),

$$\frac{(660 + 8.35\,x - 4{,}000)\,16}{12}.$$

Equating this with (a),

$$5{,}000 - 10\,x = 10{,}560 + 133.6\,x - 64{,}000$$
$$143.6\,x = 58{,}440$$
$$x = 407 \text{ ft.}$$

For the discharge of the 12-inch pipe, we have

$$h = \frac{500 - 407}{1.2} = 77.5.$$

Then, $$Q = \sqrt{77.5} = 8.81 \text{ cu. ft.}$$

To find the value of y, insert ascertained data in (c), thus:

$$77.5 = \frac{(10\,y - 4{,}000)\,4}{3}.$$

$$232.5 = 40\,y - 16{,}000.$$

$$y = \frac{16{,}232.5}{40} = 405.8.$$

Then, for discharge through 36-inch pipe,

$$h = \frac{407 - 405.8}{3.6} = 0.33.$$

Then, $$Q = \sqrt{\frac{243}{3}} = 9 \text{ cu. ft.}$$

A good practical check. To see how the discharge from the 24-inch pipe checks, we have $h = 2.417$.

$$Q = \sqrt{32 \times 2.417} = \sqrt{77.34} = 8.80.$$

Practically, we would take 8.81 or 8.80 cu. ft. per second as the correct discharge, the slight inaccuracies which were made by neglecting small decimals affecting the large pipe to a marked degree, but being inappreciable in the smaller ones. The true value of y would be nearer 406, as marked in figure.

Practically this example would generally be solved by

"trial and error," assuming a value for y and working back to the reservoir, as in Art. **2103.**

The calculations will frequently be abridged by making the elevation of the point of discharge = 0.

2110. Replacing a Compound System by a Single Equivalent Pipe.—It is frequently necessary to calculate the diameter of a single pipe to replace an old compound system. This is very readily done as follows:

Calculate the diameter D of a single pipe 7,200 ft. long, to replace the system described in Art. **2109** and shown in Fig. 688.

Calling the elevation of lower end of 2-ft. pipe = 0 as above suggested (see Art. **2109**), we have, for the quantity discharged by that pipe,

$$Q^2 = \frac{1{,}000 y \times 32}{2{,}400} = \frac{320 y}{24}.$$

Reducing, $$y = \frac{3\,Q^2}{40}. \qquad (a)$$

The discharge of the 3-ft. pipe is the same; hence,

$$Q^2 = \frac{1{,}000\,(x - y)\,243}{3{,}600},$$

and $$(x - y) = \frac{4\,Q^2}{270}. \qquad (b)$$

Again, the discharge of 1-ft. pipe is

$$Q^2 = \frac{1{,}000\,(100 - x)}{1{,}200} = \frac{10}{12}\,(100 - x),$$

and $$100 - x = \frac{12\,Q^2}{10}. \qquad (c)$$

Adding (a), (b), and (c) together, and observing that $y + (x - y) + (100 - x) = 100$, we have

$$100 = \frac{3\,Q^2}{40} + \frac{4}{270}\,Q^2 + \frac{12\,Q^2}{10}.$$

$$1{,}000 = Q^2\left(\tfrac{3}{4} + \tfrac{4}{27} + 12\right).$$

Hence, $$Q^2 = 77.52. \qquad (d)$$

NOTE.—We see that our work is correct so far, because this agrees very well with the value of Q^2 in the previous example.

But we have, also,

$$Q^2 = \frac{100}{7.2} D^5, \qquad (e)$$

D being the desired diameter. Substituting the value of Q^2 from (d) in (e), we have

$$77.52 = \frac{100}{7.2} D^5.$$

$$D^5 = \frac{77.52 \times 7.2}{100}.$$

$$D = \sqrt[5]{5.581}.$$

$$D = 1.41 \text{ ft.}$$

This is just about equal to 17 inches. As this size of pipe is not regularly cast, the next regular size of 18 inches would be adopted, with a considerably increased capacity.

2111. Generalization of Above Process.—While it is frequently quite as satisfactory to work out each special case, as has just been done, it will also be well to establish a general formula for such cases as the above by using symbols instead of numbers. Thus, referring again to the previous example, let the total height above the point of discharge (100 ft. in the example) be represented by H. Let L represent the total length of the system (7,200 ft. in the example), and, beginning at the lower end, let l and d represent, respectively, the length and diameter of the pipe farthest from the reservoir, l' and d' those of the next, and l'' and d'' those of the pipe next to the reservoir. The desired diameter will be represented by D. Then,

$$Q^2 = \frac{1{,}000\, d^5 y}{l}.$$

$$y = \frac{l\, Q^2}{1{,}000\, d^5}. \qquad (a')$$

Again, $$Q^2 = \frac{1{,}000\,(x - y)\, d'^5}{l'}.$$

$$x - y = \frac{l' Q^2}{1{,}000\, d'^5}. \qquad (b')$$

Also, $$Q^2 = \frac{1{,}000\,(H - x)\,d''^5}{l''}.$$

$$H - x = \frac{l'' Q^2}{1{,}000\, d''^5}. \qquad (c')$$

Adding, as before,

$$H = \frac{l\, Q^2}{1{,}000\, d^5} + \frac{l' Q^2}{1{,}000\, d'^5} + \frac{l'' Q^2}{1{,}000\, d''^5}. \qquad (d')$$

But we have, also,

$$H = \frac{L\, Q^2}{1{,}000\, D^5}. \qquad (e')$$

Hence, $$\frac{L\, Q^2}{1{,}000\, D^5} = \frac{Q^2}{1{,}000}\left(\frac{l}{d^5} + \frac{l'}{d'^5} + \frac{l''}{d''^5}\right).$$

Canceling common factors,

$$\frac{L}{D^5} = \frac{l}{d^5} + \frac{l'}{d'^5} + \frac{l''}{d''^5}. \qquad \textbf{(209.)}$$

We will verify this by substituting the data of the example in Art. **2110.**

$$\frac{7{,}200}{D^5} = \frac{2{,}400}{32} + \frac{3{,}600}{243} + 1{,}200.$$

$$\frac{6}{D^5} = \frac{1}{16} + \frac{1}{81} + 1.$$

$$\frac{6}{D^5} = \frac{1{,}393}{1{,}296}.$$

$$D = \sqrt[5]{5.582}.$$

$$D = 1.41 \text{ ft., as before.}$$

It is obvious that formula **209** can be extended to any number of lengths and diameters of a compound system.

2112. Importance of Knowing the Position of the Hydraulic Grade Line.—A study of Fig. 688 will show the great importance of knowing the position of the pipe in reference to the hydraulic grade line. It is evident that if any part of the 3-ft. pipe, for instance, rose above elevation 407, the calculation would be vitiated. Hence, after establishing the position of the hydraulic grade line by calculation,

the whole system must be platted to scale to see where the pipe comes. Suppose, for instance, the compound system were laid as shown in Fig. 689. It is clear that the entire

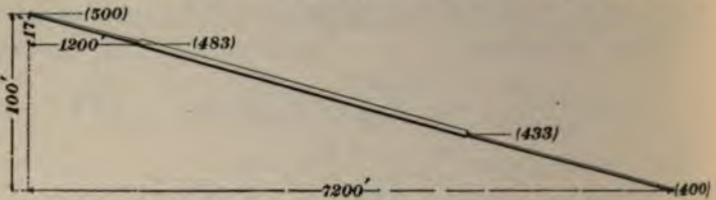


FIG. 689.

discharge would be limited to that of the 12-inch pipe under the total head of 17 feet. This would be

$$Q = \sqrt{\frac{17}{1.2}} = 3.76 \text{ cu. ft.}$$

The water would flow through the larger pipes as through a trough.

FLOW THROUGH SHORT PIPES.

2113. All that precedes refers to the flow of water through long, rough pipes, where only the head necessary to maintain the flow as against the interior resistance of the pipe has been taken into account. In such pipes the additional head necessary to overcome resistance to entry into the pipe, and that necessary to produce the velocity of flow, are so insignificant in comparison with the so-called friction head, that they are neglected, as unnecessarily complicating the formulas.

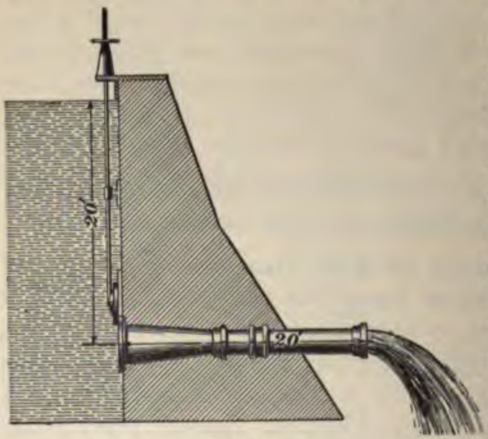


FIG. 690.

In short pipes,

however, the case is quite different, and the velocity and entrance heads must be taken into the account. This is easily done, when we recollect that the resistance head can always be taken as about one-half the velocity head. (See Art. **1020,** Vol. I.)

Suppose, for example, a reservoir R, Fig. 690, is tapped by a 24-inch pipe, 20 ft. long, the center of which is 20 ft. below the surface of the water in the reservoir. What is the discharge, using formulas for rough pipe and ignoring the modifying action of the reducer shown in the figure?

What is wanted here is the velocity of efflux, which can be obtained in the following manner:

The total head, 20 ft., is made up of the velocity head, the entrance head, and the frictional head. We will call the velocity head x, and the entrance head will then be $\frac{x}{2}$. The frictional head we will call y. Then we must have

$$\frac{3x}{2} + y = 20.$$

The velocity head is that required by the law of falling bodies,

$$x = \frac{v^2}{2g}.$$

The velocity and entrance heads together are, therefore,

$$\frac{3v^2}{4g}.$$

From formula **201** we have

$$v = 1.27\sqrt{D \times \frac{1,000y}{L}},$$

where h is replaced by its value $\frac{1,000y}{L}$. Substituting the given data, we have

$$v = 1.27\sqrt{2 \times \frac{1,000y}{20}},$$

from which

$$y = \frac{v^2}{161}.$$

Therefore (neglecting small decimals),

$$v^2\left(\tfrac{3}{129} + \tfrac{1}{161}\right) = 20.$$

$$v^2\left(\tfrac{1}{43} + \tfrac{1}{161}\right) = 20.$$

$$v^2 = \frac{6,923}{204} \times 20 = 678.7.$$

$$v = 26.05.$$

Area of 2-ft. pipe = 3.1416.
Then discharge Q is

$$Q = 26.05 \times 3.1416 = 81.84 \text{ cu. ft. per second.}$$

2114. The formula for finding the diameter of a short pipe to convey a given quantity of water with a given head is derived from the general formula as follows:

Solving the form of formula **201** given in the last article for y, we have $y = \frac{v^2 L}{1612.9\, D}$; this substituted in the expression for the total head, $H = \frac{3\, v^2}{4 g} + y$, gives us

$$H = \frac{3\, v^2}{4 g} + \frac{v^2 L}{1612.9\, D}.$$

Substituting for v its value $v = \frac{Q}{.7854\, D^2}$, and reducing, we have

$$H = \frac{Q^2}{26.45\, D^4} + \frac{Q^2 L}{995 D^5},$$

from which

$$D = .251 \sqrt[5]{\frac{Q^2}{H}(37.6\, D + L)}.$$

To use this formula, we first assume a value for the D under the radical sign and solve, thus finding an approximate value for D. We then substitute this new value for the D under the radical and solve again, and, if the new value of D agrees closely with the first approximation, the next larger commercial size may be taken as the required size of pipe. If, however, the second value differs greatly from the first approximation, it may be substituted for the D

under the radical, and a new value can thus be found. One or two approximations of this kind will usually give a value of D that will enable us to select the commercial size nearest to the theoretical diameter.

Example.—What diameter of pipe must be used in order to draw 17.22 cubic feet of water per second from a reservoir if the total head is 20 feet and the pipe is 20 feet long?

Solution.—Assuming a value of D of 16 inches = 1.33 feet, and substituting it for the D under the radical in the last formula, we have

$$D = .251\sqrt[5]{\frac{17.22^2}{20}(37.6 \times 1.33 + 20)} = 1.0067 \text{ feet, say 1 foot.}$$

Substituting this value under the radical, we have

$$D = .251\sqrt[5]{\frac{17.22^2}{20}(37.6 \times 1 + 20)} = .968 \text{ foot.}$$

Since this value is so near that of the first approximation, it is plain that the required diameter is 1 foot. Ans.

OTHER LOSSES OF HEAD.

2115. Besides the losses of head which have been considered, there are some other, minor ones, such as those occasioned by bends, changes of grade, or by passing from one diameter to another. In general, any change whatever in a pipe line produces some loss of head, but all such as occur in practice are so insignificant in comparison with the loss of head from interior surface resistance, that no account is taken of them. In practice, changes of horizontal direction, when at all pronounced, are effected by special castings called *bends*, which effect the change with very little loss of head, and changes of diameter are made through other special castings, called *reducers*, tapering in form, so as to mold the stream of water into the proper shape for entering into the pipe of different diameter. Moreover, since water-pipes are always cast to even sizes, when calculation calls for a fractional diameter, as it almost always does, the next larger size of even inches is taken, and this is generally more than enough to cover all the small losses which can occur from the above causes.

GENERAL APPLICATIONS.

2116. In the preceding pages we have all the rules and formulas necessary to calculate any practical problem which can present itself. All that is needed is a little ingenuity in their application. A series of examples has been given showing how these applications should be made, but so many different cases may occur, when the data are rather confusing, that it will be best to add a few more problems before leaving this very important branch of the subject.

PUMPING INTO MAINS.

2117. It is desired to raise 15 cu. ft. of water per second by pumping to a reservoir two miles distant from the pumping sump, and at an elevation of 300 feet above it. What theoretical horsepower will be necessary to do this work through a main 24 inches in diameter?

In the first place, we want to know the head per thousand requisite to overcome the resistance of the two-foot pipe. From formula **193**,

$$h = \frac{225}{32} \text{ ft.}$$

The distance being 2 miles, or 10,560 ft., the total head is

$$H = \frac{225 \times 10.56}{32} = 74.25 \text{ ft.}$$

But, besides this, there is the height of the reservoir above the pumping sump to be overcome; consequently, the lift of the pump is

$$74.25 + 300 = 374.25 \text{ ft.}$$

The pump must, therefore, raise 15 cu. ft. of water per second 374.25 ft. high. The formula for the theoretical horsepower required to do this work is

$$\text{H. P.} = \frac{QH}{8.8}, \qquad \textbf{(210.)}$$

in which H. P. = theoretical, or net horsepower; Q = cu. ft. of water per second; H = lift in feet.

Substituting the data of the example, we have

$$\text{H. P.} = \frac{15 \times 374.25}{8.8} = 637.9.$$

COMPUTING A SYSTEM OF MAINS.

2118. A certain town is to be supplied with water from a reservoir R, Fig. 691, situated at an elevation of 600 feet.

The maximum supply is to be 18 cu. ft. per second, to be distributed as shown in the figure, which, however, does not show a complete street plan of the town, but only a skeleton of the principal mains. The large main A conveys the entire volume of 18 cu. ft. per sec. to the first point of embranchment, from which the mains B and C extend to the right and left. B extends 6,000 ft., and will deliver at its extremity and along its course 4 cu. ft. per second. The water delivered at the extremity must rise to the elevation of 550. C extends 4,000 feet, with a delivery of 3 cu. ft. per second, and a maximum elevation at its extremity of 530. Although only a portion of these deliveries of 4 and 3 cu. ft. reach the extremities of the mains A and B, it is always best in such calculations to consider the whole volume as being delivered at the total distance and at the maximum elevation.

From the above point of embranchment, the main J extends 2,000 feet to the next two branches D and E. It must deliver $18 - 7 = 11$ cu. ft. per second. D delivers 3 cu. ft. per second at the maximum elevation 400, through a length of 6,000 ft., supplying all the district lying between the river and that portion supplied by B. E has a length of 4,000 ft., an extreme elevation of 375, and a total delivery of 2 cu. ft. per second, supplying the district lying between the river and the lower limit of the delivery of C. K conveys $18 - 12 = 6$ cu. ft. across the river, having a length of 3,000 ft. F has a length of 5,000 ft., a delivery of 2 cu. ft., and an extreme elevation of 375. G delivers 2 cu. ft. through 5,000 ft. of pipe to a maximum elevation of 350. L conveys the

remaining 2 cu. ft. through 2,000 ft. of pipe to the last two branches *H* and *I*. Both of these have a length of 4,000 ft., a total elevation of 400, and a delivery of 1 cu. ft. All these data are shown in the figure, the elevations in feet being shown by the numbers enclosed in parentheses. (See Art. **2105.**)

The problem now is to fix the diameters of this skeleton system of mains. It is evident that this problem admits of

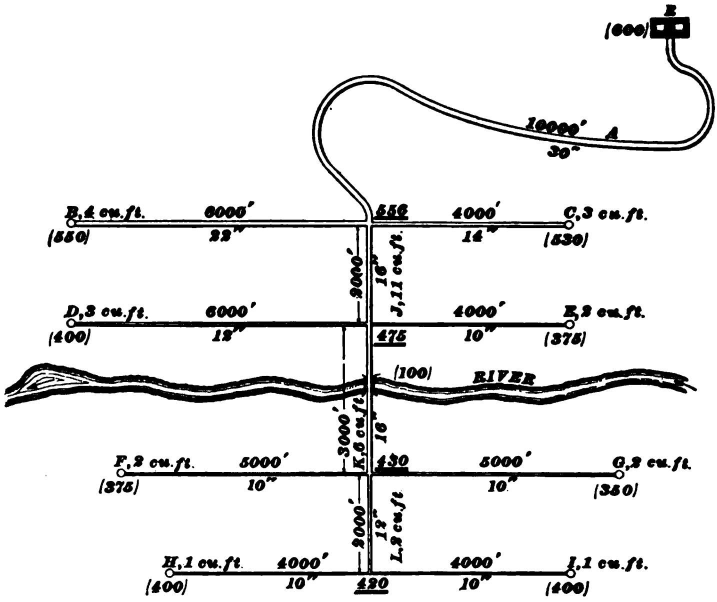


FIG. 691.

an almost endless number of solutions, but practical considerations greatly limit the number. In the first place, we want, if possible, to keep the frictional heads between 5 and 20 per 1,000, the lower number being preferable. We shall find, however, that this will rarely be possible in a territory exhibiting any considerable differences of elevation.

A brief consideration of the problem, as graphically represented by Fig. 691, shows that we had better commence the study at *H* and *I*, and, in general, such problems are best attacked by beginning thus at the most remote point

and working back towards the source of supply. We will assume a value of $h = 5$, which will give a piezometric height at the point of embranchment of $400 + 5 \times 4 = 420$. This height is marked on the figure with double lines, thus: $\underline{\underline{420}}$. All the piezometric heights will be placed on the figure in a similar way to distinguish them from the given elevations enclosed in parentheses.

To calculate the diameters of H and I, we use formula **193**, written thus:

$$D = \sqrt[5]{\frac{Q^2}{h}}.$$

Substituting data, we have

$$D = \sqrt[5]{\tfrac{1}{5}} = \sqrt[5]{0.20}.$$

By logarithms,

$$\frac{5)\bar{1}.30103}{\bar{1}.86020}.$$

Corresponding number, 0.725 ft. Reducing this to inches, we have 8.70, or nearly 9 inches. To conform to current sizes, we must make this either 8″ or 10″. Eight inches will probably suffice, because, since we have calculated upon the whole volume being delivered at the extremity, we have a considerable margin to our credit. On the figure, however, a diameter of 10″ has been marked.

Retaining the same value of $h = 5$, the piezometric height at the other extremity of L is 430, as marked with double lines on the figure. The volume to be delivered being 2 cu. ft. per second, we have, for the diameter of L,

$$D = \sqrt[5]{\tfrac{4}{5}} = \sqrt[5]{0.8}.$$

By logarithms,

$$\frac{5)\bar{1}.90309}{\bar{1}.98062}.$$

Corresponding number, 0.9564 ft. Reducing, we find the proper diameter of L to be 12 inches.

To calculate the diameter of F, we have given the two elevations, 430 and 375, and the length, 5,000 ft. Hence,

$$h = \frac{430 - 375}{5} = 11.$$

Then, $$D = \sqrt[5]{\tfrac{4}{11}} = \sqrt[5]{0.3636}.$$

$$\begin{array}{r} 5)\overline{1}.56062 \\ \hline \overline{1}.91212 \end{array}$$

Corresponding number, 0.8168 ft. = 10 inches, nearly, as marked on figure.

For G, we have

$$h = \frac{430 - 350}{5} = 16.$$

$$D = \sqrt[5]{\tfrac{4}{16}} = \sqrt[5]{0.25}.$$

$$\begin{array}{r} 5)\overline{1}.39794 \\ \hline \overline{1}.87958 \end{array}$$

Corresponding number, 0.7578 ft. Nearest equivalent in current diameters, 10 inches.

2119. The practised calculator would now see that there was trouble ahead in regard to the pipe J, because there will be a great difference in the levels of the piezometric heights at its two extremities, as will presently become quite evident. Besides, there is the river to cross, and if this crossing is effected by means of a reversed siphon (so-called) there will be some loss of head incurred in overcoming extra resistances. The value of h for K will, therefore, be raised to 15, and for the diameter of K we will have

$$D = \sqrt[5]{\frac{36}{15}} = \sqrt[5]{2.4}.$$

$$\begin{array}{r} 5).38021 \\ \hline .07604 \end{array}$$

Corresponding number = 1.19 ft., the equivalent being 14 or 16 inches. Sixteen has been marked on the figure.

Turning to D, we have

$$h = \frac{475 - 400}{6} = 12.5.$$

Then, $$D = \sqrt[5]{\frac{9}{12.5}} = \sqrt[5]{0.72}.$$

$$\begin{array}{r} 5\,)\,\overline{1}.85733 \\ \hline \overline{1}.97146 \end{array}$$

Corresponding number, 0.9364 ft. Evidently we require a 12-inch pipe here.

For E, we have

$$h = \frac{475 - 375}{4} = 25.$$

Here we have exceeded our limit, but by reducing the piezometric height of 475, which is the cause of the trouble, we should only make matters worse for J later on. So, for the diameter of E, we have

$$d = \sqrt[5]{\frac{4}{25}} = \sqrt[5]{0.16}.$$

$$\begin{array}{r} 5\,)\,\overline{1}.20412 \\ \hline \overline{1}.84082 \end{array}$$

Corresponding number, .6932 ft.

Here we are somewhat in doubt again; 8 inches would probably suffice, but 10 inches have been marked in the figure.

Before determining the diameter of J, we must consider the requirements of B and C. The greatest elevation to be reached by B is 550. The piezometric height at the point of embranchment of B and C must, therefore, be greater than 550. We will take it as small as possible by making $h = 1$ for the pipe B, which will give a piezometric height at the point of embranchment of $550 + 6 = 556$, as marked on the figure.

Therefore, for J, we have

$$h = \frac{556 - 475}{2} = 40.5.$$

This is a great deal more than we desire, but is the best that can be done under the circumstances, although we might, perhaps, have advantageously raised the hydraulic grade of K a little, so as to divide the too great difference of level more equally between J and E.

For the diameter of J, we have

$$D = \sqrt[5]{\frac{121}{40.5}} = \sqrt[5]{2.987}.$$

$$\begin{array}{r} 5\,)\,0.47524 \\ \hline 0.09499 \end{array}$$

Corresponding number, 1.245 ft. Equivalent, 16 inches.

For B, we have already fixed $h = 1$. Therefore,

$$D = \sqrt[5]{16}.$$

$$\begin{array}{r} 5\,)\,1.20412 \\ \hline 0.24082 \end{array}$$

Corresponding number $= 1.741$ ft. Here we have to choose between 20 and 22 inches. Twenty-two has been marked on the figure.

For C, we have

$$h = \frac{556 - 530}{4} = 6.5.$$

$$D = \sqrt[5]{\frac{9}{6.5}} = \sqrt[5]{1.385}.$$

$$\begin{array}{r} 5\,)\,0.14145 \\ \hline 0.02829 \end{array}$$

Corresponding number $= 1.07$ ft. We will call this 14 inches, as marked on the figure.

For A, we have, the length being 10,000 ft.,

$$h = \frac{600 - 556}{10} = 4.4.$$

$$D = \sqrt[5]{\frac{324}{4.4}} = \sqrt[5]{73.63}.$$

$$\begin{array}{r} 5\,)\,1.86705 \\ \hline .37341 \end{array}$$

Corresponding number, 2.363 ft., which reduces to 28 or 30 inches. Thirty inches has been marked on the figure.

2120. Although in the figure the different branches of this system have been shown as entirely independent of each other, this would not actually be the case, because the submains, extending through the different streets of the town, and which are not shown in the figure, would connect the entire system, on each side of the river, making each of these systems a "closed circuit," through which the water would circulate in a manner entirely baffling all calculation. Indeed, the only result which can be achieved by calculation is to design a rationally proportioned system based upon maximum values of all the factors. The actual draft upon any part or all parts of the system can not be determined with any considerable degree of accuracy, because it is varying from moment to moment, and does not obey any fixed law. By following out the method just illustrated, however, we can be sure of an effective and satisfactory distribution and pressure of water. Such calculations should always be carefully gone through with in any important project for piping a city, bearing in mind and providing for prospective growth in probable directions. Unfortunately, this is seldom done, at least in this country, and the result is that the water service is frequently less satisfactory than it should be. A few hours spent in an intelligent study of the conditions, based upon measurements and elevations which are at least approximately correct, will suffice to design a pipe plan giving a much more satisfactory result than one laid out in a haphazard manner.

It will be a most instructive exercise for the student to work out the preceding problem with different values of h and note the results, also to estimate the weight of iron pipe required for each case, according to the rules given in Arts. **2125** to **2127**, and thus determine which system gives the best and most economical results.

HYDRANTS.

. . . are set according to agree- . . . authorities. It is a great ad- . . . each block, although they are . . . 1,000 ft. apart. Their prin- . . . and for that purpose it is im- . . . many as possible for any given . . . employing great lengths of

. . . AND THICKNESS OF CAST-IRON PIPES.

. . . foundries furnish tables of the . . . of their pipe, but it is convenient . . . calculating these elements.

. . . n water-pipes are almost invariably . . . Spigot" type, cast with the bell down. . . . weight of such pipes, they are consid- . . . from end to end, and eight inches . . . for all diameters. Thus, if a bell and . . . 12 feet over all, it would be considered, . . . plain cylindrical tube 152 inches long.

. . . FOR WEIGHT OF PIPES.

. . . weight is

$$\ldots + T)\, T \times L, \qquad (211.)$$

. . . in pounds;

. . . meter in inches;

. . . inches;

. . . es.

. . . inches diameter and 12 ft. . . . inch. What is its weight

. . . 1,457.04 lb. Ans.

2 . . . pipe . . . is n . . . supp . . . 4 gal . . . rate . . . daily . . . genera . . . the mi . . . still m . . . hand h . . . ordinaril . . . draft wi . . . above cas . . . 100 gallon . . . pipes must

2122. the diamete . . . single effici . . . per second. . . . suction as w . . . can be draw . . . on up to a ce . . . eter of the pi . . . supply one st . . . would require . . . Art. **2096**), a . . . sidered as the a . . . nish one good f . . . streets of a sma . . . considered poor p

2126. A convenient approximate formula for weight per foot is

$$w = 10\,(D + T)\,T. \qquad \textbf{(212.)}$$

EXAMPLE.—Apply formula **212** to the previous example.

SOLUTION.— $w = 10(16 + 0.7)\,0.7 = 116.90$ lb. per foot.

Then, $116.90 \times 12.66 = 1,479.95$ lb. Ans.

2127. To ascertain by a rapid approximation the weight in long tons (2,240 lb.) per mile of a pipe-line of given diameter and thickness of metal, we have the formula

$$P = 25\,M\,(D + T)\,T, \qquad \textbf{(213.)}$$

in which P = weight in long tons, and M = length in miles.

EXAMPLE.—What is the weight of 17 miles of the pipe in previous example?

SOLUTION.— $P = 25 \times 17\,(16 + 0.70)\,0.70 = 4,968.25$ long tons. Ans.

In estimating, about 5% might be added to cover breakage, specials, and contingencies.

FORMULA FOR THICKNESS OF PIPE.

2128. For calculating the proper thickness in inches of a pipe of given diameter, to resist a given pressure in feet, we have the formula

$$T = 0.00006\,HD + 0.0133\,D + 0.296, \qquad \textbf{(214.)}$$

in which H = head in feet. The rest of the notation is as before.

EXAMPLE.—What should be the thickness of the above pipe, if subjected to a pressure of 300 feet?

SOLUTION.—

$T = 0.00006 \times 300 \times 16 + 0.0133 \times 16 + 0.296 \doteq 0.797$ inch. Ans.

PIPE-LAYING.

2129. A very important part of the practical work of water-supply engineering is the laying of the pipe. As already stated, the kind of cast-iron pipe used in this country is almost invariably of the bell and spigot type.

2130. Joints.—In making the joints, it is important that the spigot should be entered well home in the bell, and the annular space between the outside of the spigot and the inside of the bell evenly divided all around. A hemp or oakum packing, called the "gasket," is then driven in, leaving a sufficient depth—from 2 to 4 inches, generally, according to diameter of pipe—for the lead to be poured in. The lead is run so as to leave a projecting "bead," which is driven in by calking, any superfluous lead remaining outside of the joint being cut off evenly. Sometimes, particularly in wet localities, where it would be impossible to get the joint perfectly dry before running the lead (a *sine qua non*), the lead is put in cold and driven home with the calking tools. In this case a piece of lead pipe may be conveniently used, by being bent around the pipe and calked in, the oakum packing being dispensed with.

2131. Alinement and Grade.—The pipe should be laid to a true alinement, or nearly so. It is generally sufficient to dig the trench to true lines, and lay the pipe in it, when the latter will be found to be sufficiently near a straight line for all practical purposes. A still more important requirement is that it should be laid to a true

FIG. 692.

grade with no sags. This is best accomplished by driving in grade plugs every 50 feet, or oftener, in the trench when the bottom is nearly down to grade. These plugs are driven in until the heads are exactly at the right grade. The pipe can be then sighted in from these plugs.

All pipe above 20 inches in diameter should be laid on blocks, and wedged up. This is good practice, even for smaller diameters, but should never be omitted for the sizes mentioned. The blocks and wedges should be sawed out to regular dimensions, and the blocks laid to grade, by means of a string stretched between the grade plugs already mentioned. The blocks should be laid a trifle below grade, and the pipe raised to its grade by means of the wedges. Figs. 692 and 693 show a 48-inch pipe, properly blocked and wedged. Fig. 694 shows a block and wedges for a 48″ pipe to an enlarged scale. For smaller diameters, the blocks and wedges are lighter.

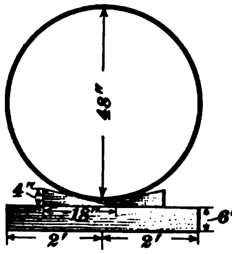


FIG. 693.

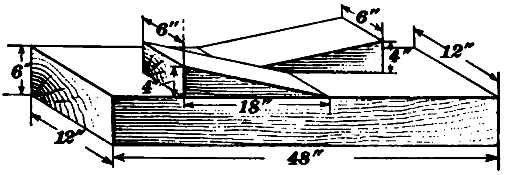


FIG. 694.

2132. Back Filling.—It is very essential that the back filling of the trenches with earth should be properly executed. No stone should be allowed to come in contact with the pipe, and, in particular, great care should be taken that no stone should get under the pipe with the pipe resting on it. The earth filling should be driven well in under the pipe and packed closely around it on all sides, so that it may be firmly held all around, by compact earth, in order to prevent all movement or vibration in case of the occurrence of shocks from "water-hammer" or any other cause.

2133. Air Vents.—In laying a long line of main, it is essential that a vent or air-cock be provided at all summits or high points from which the grade descends both ways. Otherwise, air may accumulate at these points and interrupt the flow by causing an "air-lock." A projection

from the top of the pipe, inclining slightly upwards, and connected with a hydrant, as shown in Fig. 695, is a good arrangement.

FIG. 695.

2134. Blow-Offs.—Similarly, all low points from which the grade rises both ways should be provided with "blow-offs," by means of which the pipe can be "blown off"

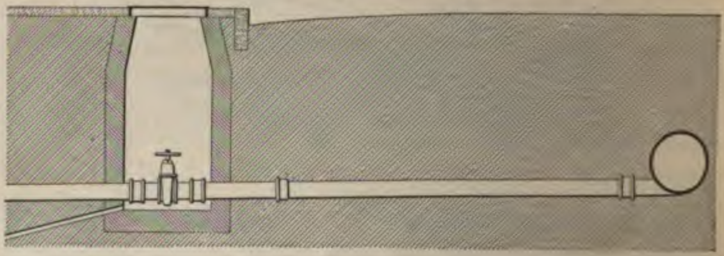


FIG. 696.

under pressure, so as to discharge any accumulation of silt which may have gathered in the depression, a suitable exit being provided for it.

The proper arrangement is shown in its general features in Fig. 696.

2135. Bends.—When it becomes necessary to put a pretty sharp turn in a pipe-line (which should be avoided, if possible), special pipe cast to the proper radius should be employed. Ordinary curves can be overcome by slightly deviating a number of lengths of pipe to one side or the other, the bell affording enough play to accomplish this. Short lengths of pipe, such as generally accumulate from

making closures, may be advantageously employed for changes of direction, by being connected with "sleeves," or short cylindrical special pieces for closing plain or spigot joints.

2136. Stop-Cocks.—When setting stop-cocks, at least one sleeve should be employed in connecting the stop-cock, because, in case of wishing to remove the piece at any time, a sleeve can be readily broken, and then the two spigots which it joins come readily apart.

FILLING A SYSTEM OF PIPES.

2137. When a system of piping, either a new system or one which for any cause has been emptied, is to be filled with water, the greatest care should be exercised in letting it fill very gradually. All the blow-offs and air-cocks should be wide open, and the water admitted through a very small opening of the admission valves. It is hardly possible to go too slow, and in an extensive and intricate system, several days may be required to entirely fill it. The reason for this is that the admission of large volumes of water into empty pipes causes a great agitation of the air which they contain, compressing it here, and expelling it there, with the result that the water, responding to this elastic and fluctuating pressure, is driven forwards and back, producing violent shocks which may prove very destructive to the pipes. In other words, in the conflict between the water which is trying to get in and the air which is trying to get out, the pipe has to bear the brunt. By opening the admission valve only a few turns, so that the water rather dribbles than flows into the pipes, while all the outlet valves are well open, violent perturbations are avoided.

When the water reaches and runs out of the first blow-off, it is closed, and so on with the others. The air-cocks are closed successively, beginning generally with the lower ones, as solid water begins to flow from them, without *spitting*. In a new system, it is well to let the water stand in the pipes, when completely full, for at least 24 hours, as a test for leakage.

DIFFERENT SYSTEMS OF DISTRIBUTION.

2138. Unless the "Direct" or "Holly" system is employed—a system which is falling rapidly into merited disuse, but which will be briefly described presently—the distribution of water to a town is effected by gravity from a suitably located reservoir, into which water is either pumped or allowed to flow by gravity. This reservoir may be of greater or less dimensions; it may be a combined storage and distributing reservoir, as already described, or it may be a simple stand-pipe, to which water is raised by pumping.

2139. Stand-Pipes.—The stand-pipe is nothing more nor less than a very small, but very deep reservoir, with an insignificant storage capacity. It serves as an equalizer of the pressure, which it transfers unimpaired from a more distant to a nearer point. It is in every respect inferior to a larger, shallower reservoir of greater storage capacity, but is frequently useful and economical when a sufficient area, at a sufficiently high elevation, can not be secured for a more efficient reservoir. In contemplating the erection of a stand-pipe, the fact must not be lost sight of that a large number of disastrous failures from collapse and other causes are recorded against this class of structure, which, from its very nature, must be of a comparatively frail and top-heavy character. If the position, topographical and financial, of the town to be supplied in any way, admits, a regular distributing reservoir, designed and executed in accordance with the principles already laid down in the section on *reservoirs*, and containing a week's supply, should be by all means preferred.

2140. The **Holly system**, already mentioned, is a device by which, when a town is supplied by pumping, the necessity of a reservoir or stand-pipe is avoided. By this system, the water is pumped directly into the mains, and if the pumps were suddenly arrested, the whole water supply would be immediately arrested also. The endeavor is made to run the pumps so as to exactly keep time with the con-

sumption, varying their action from minute to minute, and even from second to second, to correspond with the variations of the draft. To this end, relief valves are provided, but it seems impossible to avoid a considerable degree of shock and water-hammer in the pipes, which is, of course, very trying to both pipes and plumbing. In fact, it may be doubted, in view of the increased weight of pipe necessary to resist this more or less intermittent action, as well as the damage to pipes and plumbing, and the uneconomical manner in which the pumps must be operated, whether there is any real economy in this system after all. Its sole advantages are the saving of the first cost of reservoir or stand-pipe, and the fact that, in case of fire, the pressure can be instantly increased, so as in many cases to render the use of fire-engines unnecessary, the streams being taken direct from the hydrants.

WROUGHT-IRON OR STEEL RIVETED PIPE.

2141. At the present day iron and steel riveted pipe are extensively used; in many cases, to great advantage. They are very much lighter than cast-iron, and are consequently easier to handle, and cost less to transport. They are sometimes shipped in sheets, and riveted on the ground, but as good a job can never be secured in this manner as when the riveting is done in the shop. They are made in longer lengths, and there are, therefore, fewer joints to make. Although much lighter than cast-iron, the cost per pound is naturally much greater, so, foot for foot, the price of the two classes of pipe f. o. b. does not probably differ very much. The economy comes in in transportation and laying. They have the advantage of being made to any required diameter, in fractions of an inch, if desired, so they can be more easily made to conform to calculation. When danger from corrosion is apprehended, they should be used, if at all, with great judgment.

As regards the formulas for the flow through these pipes, it may be said that there is considerable uncertainty as to how far those already established for cast-iron pipes are

applicable. Trustworthy data, except in some special cases, are lacking, but so far experience seems to show that the discharge of these pipes is less than that of cast-iron pipes of equal diameter. This is probably due to the obstructions caused by the lap-joints and the riveting. It appears safe to say that diameters should be increased by from 5% to 10%, in order to give equal discharges. Some mortifying failures have resulted from laying this class of pipe with insufficient diameters.

FLOW OF WATER THROUGH OPEN CHANNELS.

DARCY'S FORMULAS.

2142. Kutter's formula, together with the table of coefficients for various kinds of channels (Art. **1033**, Vol. I), furnishes a general method of computing the velocity of flow in any channel the engineer is likely to be called on to consider. It is somewhat long and complicated, however, and, for the special cases of the conduits most often used in connection with water-works, Darcy's formulas as given below will often be found more simple.

Hitherto the application of Darcy's formulas only to the flow of water through closed conduits or pipes, under more or less pressure, has been considered, and the rules and formulas given for such cases do not apply to water running through open channels. The data required in the latter case are :

V = mean velocity of flow in feet per second;

S = water section in square feet (= the area a, b, d, c in (a), Fig. 697);

WP = wet perimeter in feet (= the sum of the lines ac, cd, db in (a), Fig. 697);

R = mean hydraulic radius = $\frac{S}{WP}$;

I = slope of free water surface, ab, per foot of length = total fall of surface divided by total length.

Darcy, in experimenting upon this class of conduit, found that the nature of the lining of the channel exercised the same influence as that of the interior surface of pipes, in modifying the velocity of flow. He, therefore, adopted a series of coefficients, suited to different degrees of smoothness or roughness of lining.

2143. Brick-Lined Channels.—For the ordinary case of a tunnel or channel lined with well-laid brick, the formula is

$$U = R\sqrt{\frac{100,000\ I}{6.6\ R + 0.46}}. \qquad \textbf{(215.)}$$

EXAMPLE.—Referring to Fig. 697 (*a*), what is the velocity of flow where $a\,b = 26$ ft.; $c\,d = 16$ ft.; $a\,c$ and $b\,d = 10$ ft. each, and $e = 9$? Let the longitudinal slope of the surface $a\,b = \frac{1}{1,000}$.

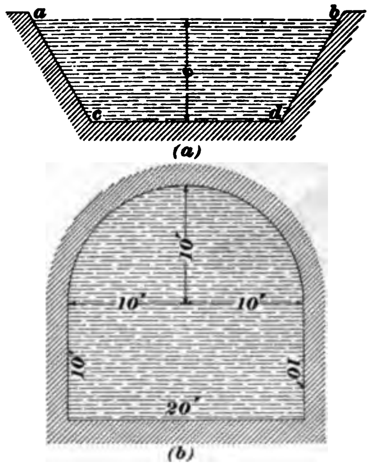


FIG. 697.

SOLUTION.—

U = mean velocity in feet per second.

S = 189 sq. ft.

WP = 36 ft.

$R = \frac{189}{36} = 5.25.$

$I = 0.001.$

Then,

$U = 5.25\sqrt{\frac{100}{35.11}} = 8.87$ feet per second. Ans.

NOTE.—The "mean velocity" is used in these calculations, because there is frequently a considerable difference between the surface velocity and the velocity at various depths and at various distances from the sides. The mean velocity is that which, when multiplied by S, gives the quantity discharged by the channel, in cubic feet per second.

2144. In the case of a **circular, brick-lined conduit, running full,** R is always $= \frac{D}{4}$, D being interior diameter of conduit, in feet. The formula for such is

$$U = \frac{D}{4}\sqrt{\frac{100,000\ I}{1.65\ D + 0.46}}. \qquad \textbf{(216.)}$$

It is to be understood that the conduit, though running full, is not under pressure.

EXAMPLE.—Let $D = 4$ ft. and $I = \frac{1}{1,000}$, and find the velocity.

SOLUTION.— $U = \sqrt{\frac{100}{6.6 + 0.46}} = 3.76$ ft. per sec. Ans.

This result is somewhat in excess of the velocity through a smooth cast-iron pipe of same diameter, with the same fall per 1,000. Formula **216** agrees well with observed velocities, however, and the greater velocity in circular brick-lined conduits is probably due to the fact that they are laid always to a true descending grade, with few and slight lateral deflections, and are quite uniform throughout, as to interior surface.

2145. Maximum Flow through Brick-Lined Conduits of Circular, or "Horseshoe" Section.—This occurs when the conduit is not running quite full (unless it is under pressure), but when running to within about $\frac{1}{10}$ of the radius at the crown. This singular result is due to the fact that, although the cross-section of the body of water moving through the conduit is somewhat reduced, the velocity is more than proportionately increased, owing to the favorable value of the hydraulic mean radius.

EXAMPLE.—Given a cross-section as shown in Fig. 697 (*b*), and $I =$ 0.001, what is the discharge (*a*) when the conduit is running just full, but not under pressure, and (*b*) when running to within 1 ft. of the crown?

SOLUTION.—(*a*) When running full, we have

$$S = 357.08 \text{ sq. ft.}$$
$$WP = 71.41 \text{ ft.}$$
$$R = \frac{357.08}{71.41} = 5.00.$$

Then, $U = 5.00\sqrt{\frac{100}{33.46}} = 8.63$ ft. per sec.

The discharge is $357.08 \times 8.63 = 3,081.60$ cu. ft. per sec. Ans.

(*b*) When running to within 1 ft. of the crown,

$$S = 351.18 \text{ sq. ft.}$$
$$WP = 62.38 \text{ ft.}$$
$$R = \frac{351.18}{62.38} = 5.63.$$

Then, $U = 5.63\sqrt{\frac{100}{6.6 \times 5.63 + 0.46}} = 9.18$ ft. per sec.

And the discharge is $351.18 \times 9.18 = 3{,}223.83$ cu. ft. per sec. Ans.

This is greater than when the conduit is running full.

NOTE.—The section shown in the figure is only given as furnishing an easy numerical example, and not as one to be used in practice. Such sections—"horseshoe," so called—should have battering sides and a curved invert.

2146. Conduits with Rough Sides.—All tunnels or open channels for the conveyance of water are ordinarily lined with brick. When of other material—rubble masonry, for instance—the flow will be proportionately diminished. It is difficult to calculate the flow in such cases, because, for one thing, it is impossible to accurately determine the area and wet perimeter, owing to the projecting points of the masonry. The best way is to make all the calculations as if for brick lining, and then apply a coefficient of from 85% to 90%, according to the degree of roughness of the sides.

IRRIGATION.

INTRODUCTION.

2147. In spite of the large areas which, in certain sections, are already irrigated for the purpose of cultivation, it may be affirmed that irrigation as a science is still in its infancy in the United States. There are other countries, notably India, where it has not only been practised from a remote period, but where its principles are well understood, and its operations carried on in a systematic manner and frequently upon a gigantic scale. In other words, in these countries irrigation has passed beyond the experimental stage, and taken its place as an established branch of agricultural science. In others again, such as Mexico, although it is not practised on the large scale nor in the systematic manner that prevails in India, its advantages are thoroughly appreciated by all, and it is exceedingly interesting, in traveling through the semi-arid agricultural districts of that country, to note the skilful manner in which the small cultivators take advantage of every little streamlet to store and distribute the few scanty drops of water necessary to secure their otherwise very precarious crops.

2148. Necessity of Water in Raising Crops.—Although it is pretty generally understood that water is necessary to the growth of plants, that vegetation thrives with moisture and languishes in drought, it is not, perhaps, so generally understood why this is the case. Briefly, the reasons are the following: Plants require a certain and often very considerable proportion of water as one of their constituent parts. This water they absorb from the soil by means of their roots. The soil must contain a certain

For notice of copyright, see page immediately following the title page.

percentage of moisture—perhaps from 5% to 10%—before it can yield up any to the plant. On the other hand, the plant is constantly giving off moisture, by evaporation from the leaves, and when the amount thus lost exceeds the amount taken up by the roots, the plant droops and dies. Besides, no plant food can be assimilated by crops in the solid state, but it must be presented to them in a fluid condition. It is necessary, therefore, that the substances which they require to build them up should not only be of a soluble nature, but must also be furnished with a sufficient quantity of water to thoroughly dissolve them, so that the plant may drink in its nutriment—for plants do not *eat* their food, but *drink* it.

2149. Natural Irrigation.—In those regions of the United States which enjoy a normal rainfall, comprising, as a rough approximation, all that portion lying east of the 97th meridian, the natural precipitation is depended upon exclusively to supply the necessary solvent. That it should satisfactorily accomplish this result, two conditions are evidently necessary : First, the annual amount must be sufficient, and, secondly, it must be distributed at proper seasons, so as to be timely as regards the needs of vegetation. It may be remarked that in the distribution of the rainfall nature has a double duty to perform, providing both for the needs of vegetation and the water required for animal life. For the former, rains are needed, according to locality, at certain periods of plant growth, while for the latter, in regions where frost prevails, copious rains are needed not only during the heated term, but also in the late fall, in order that the ground may be thoroughly saturated before it becomes sealed up by frost. Naturally this precipitation does not assist plant growth except indirectly.

In the regions enjoying an average yearly rainfall of 30 inches and upwards—which would be much more than sufficient for plant growth if falling opportunely—there may still be serious damage done by drought, even to the extent of losing certain crops, from the fact that rain is lacking

just at the right time. Even in the most favored localities the element of uncertainty introduces a most dangerous factor in all agricultural pursuits. On the other hand, serious loss is often sustained by excess of rain when not wanted.

2150. Artificial Irrigation.—The amount of water actually needed for the growth of crops is relatively very small. As a rough general average, we may say that twelve inches per annum, spread at proper seasons over the duly prepared surface of the area under cultivation, is sufficient. In order that these conditions should obtain, it would be necessary to have under control and in store a volume of water equal to that which would cover the entire cultivated surface to the depth of from one foot to two feet, allowing for loss by evaporation and other causes, and then have facilities for spreading this water over the given area as wanted. It is clear that it would be necessary to provide for a considerable excess over the amount absolutely necessary for irrigation, in order to provide for the various losses of evaporation, leakage, and waste. But given a sufficient quantity, and adequate means of controlling its use, artificial irrigation is superior to natural, in that it gives the proper quantity of water to the plant, at the proper season, and no more, and at no other time. Indeed, the claim seems justified that, instead of artificial irrigation being a substitute for rain, rain is an imperfect substitute for artificial irrigation.

2151. Districts to Which Irrigation Is Applicable.—Although the evident benefits of artificial irrigation, as supplementing and regulating the supply of water to crops, even in districts which have a satisfactory annual average precipitation, will probably in the future lead to its introduction in such sections, yet at present its use in this country will doubtless be confined for some time to come to the arid and semi-arid regions of the West. This includes that portion of the United States and Central America lying west of the 97th meridian, having a rainfall of less than 22 inches per year.

2152. Quantity of Water Required and Irrigation Periods.—"In Colorado, alfalfa and clover are irrigated twice in a season, once in May and once in June, to a depth of 6 inches for each period; wheat and oats are irrigated twice, once in June and once in July, to a depth of 9 and 6 inches, respectively. Meadow or native hay requires considerably more water; there are usually two service periods, each of which lasts several days, the water being allowed to run in a small quantity during that time. The first is usually in May, and is about 2 inches in depth for a week; the second in July or August, of about the same amount; in all from 24 to 30 inches in depth of water are applied. Since the application of water is generally followed by a temporary checking of the growth of the plant, the method preferred in the arid region seems to be to give thorough rather than many irrigations; in other words, to have two ample rather than four to six small services. In general, it may be stated that two or three service periods, varying in depth from 3 to 6 inches, are employed in Colorado, and that the irrigation period extends from May to September—123 days. In Utah the practice seems to be to employ a much larger number of service periods—from three to five on grain crops of 2 to 3 inches in depth each—the water running 12 to 15 hours per service period, and the irrigation period extending from June to August, inclusive. On vegetables as many as six or ten service periods are employed, each lasting from 3 to 6 hours, during June to August, inclusive. The irrigating period in the majority of Western States averages from April 15 to August 15, or about 120 days; while the service period varies from 3 to 15 hours in length, according to soil and crop, and there are from two to eight such service periods in an irrigating period. In India there are from three to five service periods, making up an irrigating period of from 100 to 130 days' duration." (Wilson.) In Mexico, or in certain parts of it, they allow three annual irrigations of about 4 inches each, the rainy season furnishing a fourth irrigation, of uncertain depth.

As regards quantity needed, the above statements seem

to show that it varies from 1 to 2½ feet per annum over the cultivated surface.

2153. Quality of Water.—Evidently it is not necessary to exercise so close a scrutiny into the quality of the water used for irrigation as for a domestic supply. Indeed, some waters totally unfit for domestic use from the presence of a large amount of organic matter are thereby rendered peculiarly favorable for irrigation, owing to the fertilizing properties of the substances held in suspension. While this material is frequently beneficial to the land, its presence is sometimes very troublesome, by obstructing channels and waterways, and filling up reservoirs, particularly when the entrained silt is composed of mineral substances.

A very important factor in the value of water for irrigation is its temperature. The warmth imparted by water of a relatively high temperature is of itself frequently sufficient to greatly stimulate plant growth.

2154. Area of Territory Under Irrigation.—"The extent to which irrigation can be practised is enormous. The total area irrigated in India is about 25,000,000 acres, in Egypt about 6,000,000 acres, and in Italy about 3,700,000 acres. In Spain there are 500,000 acres, in France 400,000 acres, and in the United States 4,000,000 acres of irrigated land. This means that crops are grown on 40,000,000 acres of land which but for irrigation would be barren and unproductive. In addition to this there are some millions more of acres cultivated by the aid of irrigation in China, Japan, Australia, Algeria, South America, and elsewhere.

"The works which provide water for the irrigation of the 40,000,000 acres above specified represent an investment of about $450,000,000, and the area thus rendered cultivable yields annually products valued at about $500,000,000. This represents an interest on the original investment which seems absurd, but, in fact, it means only that the yield of irrigated crops averages about $12.50 per acre controlled." (Wilson.)

In reference to the above computations, it may be said that what makes the interest upon the outlay seem exorbitant is the fact that the value of the land itself is not taken into account, nor the value of the crops which it might produce without irrigation. Upon any basis of calculation there is still, from the figures, a large percentage of profit divided between the investors and the farmers.

2155. Drainage Connected with Irrigation.—In order that the territory operated upon may derive full benefit from irrigation, it is necessary that there should be facilities afforded for the removal of the surplus water after the soil has been thoroughly saturated. No benefit is derived if the soil is allowed to become water-logged. It is necessary that the water applied should slowly pass through the ground, and not remain upon it until removed by evaporation.

Drainage, like irrigation, may be either natural or artificial. Frequently the character of the soil is such that the drainage takes care of itself; this occurs when the ground is underlaid by a porous substratum ; but at other times artificial drainage should be resorted to.

2156. Circumstances Which Render Drainage of the Soil Peculiarly Necessary.—In many parts of the West the presence of "alkali" is a serious impediment to the growth of crops. The presence of alkali is manifested by a white efflorescence upon the surface of the ground, consisting chiefly of chloride of sodium, or common salt, sodium carbonate, or sal soda, and sulphate of sodium, or Glauber's salt. The effect of these salts upon vegetation is most pernicious, particularly the sodium carbonate, known as "black alkali." The deposit of alkali upon the surface of the ground is due to the evaporation of considerable quantities of water which has become impregnated with the above salts.

The best preventive of the formation of alkali is found in underdraining the soil so affected. In regions where alkali prevails, soils not naturally underdrained should, if possible,

be avoided, and only those which have natural advantages in this respect should be selected for irrigation. If the difficulty can not be avoided in this way, it must be combated by the further process of artificial drainage.

2157. Other Remedies for Alkali.—Although underdraining is the most radical and effective means of combating alkali, there are other palliatives which may be employed, either alone or in connection with drainage. Mulching the soil, or giving it a top dressing of any kind suitable to shelter it and impede evaporation, is sometimes a valuable aid. The evil effects of black alkali are greatly diminished by the use of gypsum as a top dressing, but it appears to be thoroughly effective only when the soil is also underdrained. Sometimes, when there is an abundance of irrigating water, the deposit may be washed off the surface by flooding it, and rapidly drawing off the water before it can soak into the ground.

Some crops are less injured than others by alkali. Alfalfa, or lucerne, seems to be the least affected by it, and can be grown to advantage when other crops would fail.

In general, in order successfully to cultivate ground afflicted with alkali, recourse should be had to underdraining, the use of a minimum amount of water in irrigating, cultivation, and mulching, and the application of plaster of Paris, or gypsum.

2158. General Conclusions.—Mere irrigation must not be exclusively depended upon to render arid soil productive, although in any case it may cause a temporary fertility at the start. Applied alone, and injudiciously, it may even increase the impoverishment. It is only one of several factors in the reclaiming of otherwise uncultivable soil. It must be combined with a proper selection of crops suited to the particular soil, and which is a question rather for the agriculturist than the engineer; with proper underdrainage, natural or artificial; by cultivation and mellowing of the soil; by mulching, and in many cases by fertilizing.

In a word, every other resource of the agriculturist should be brought into action, just as would be done in ordinary farming when no artificial irrigation was practised. A neglect of those precautions has, no doubt, often led to disappointment and loss of faith in irrigation.

WATER SUPPLY AND STORAGE.

SOURCES OF SUPPLY.

2159. There are two sources of supply which are commonly looked for in studying an irrigation project, namely, surface and ground waters. Generally speaking, all that has been said upon this subject in the section on Water Supply and Distribution holds good for irrigation also. There are some points of difference, however, which must be noted. In the first place, as has already been mentioned, the question of hygienic quality is virtually eliminated from the problem. The chemical character of the water has, it is true, some bearing upon its fitness for irrigating purposes, as being favorable or the reverse to the formation of alkali; but, broadly speaking, neither biological nor chemical examination plays any prominent part in this branch of hydraulic engineering.

Secondly, since irrigation is mostly practised in districts where the rainfall is abnormally small, general rules are less applicable, as regards the supply derivable per square mile of drainage area, than for districts of average rainfall and evaporation. More attention must be paid and more weight given to gauging, measuring, and observing, at least, until more general knowledge has been obtained of average conditions in the arid and semi-arid districts which form the principal field of irrigating operations; and more pains must be taken to secure accurate results, on account of the small quantities dealt with. Each case will be more or less a special one, requiring special study. We will first consider all that part of the subject which relates to surface water.

SUPPLIES FROM SURFACE WATER.

2160. Preliminary Observations.—The first question to be decided will be the amount of water required. In this estimate it will be well to make very liberal allowances for losses by waste, evaporation, and in transmission. Suppose that in a given project it was thought that a yearly quantity of water, sufficient to cover the whole area to be irrigated to a depth of 48 inches, would be necessary to include all items of use and loss. Then, if the given area contained 10 square miles, or 278,784,000 square feet, the yearly amount of water required in cubic feet would be this area multiplied by 4, or 1,115,136,000 cu. ft. If, therefore, it were desired to secure this amount of water by the aid of a storage reservoir, formed by building a dam across a certain stream, it would be necessary to ascertain if the drainage area situated above the proposed dam, combined with the minimum available rainfall, were sufficient to afford the required quantity. It may be here remarked, that in the study of the quantities necessary for irrigation we are not bound quite so rigidly as in cases of water supply for communities. In the latter case, any failure in the daily quantity of water furnished leads to dangerous, or, at least, very inconvenient results. Obviously, the failure to supply the full quantity which would be desirable for irrigation can not be followed by such serious consequences as a water famine in a populous city.

It has already been shown in the sections on Water Supply and Distribution that the available yield of a given watershed or drainage area is not given by its area multiplied by the depth of yearly precipitation. A large percentage of this amount is lost by evaporation, by absorption, and other sources. The remainder, which finds its way to the stream to which the drainage area is tributary, and which is known as the **run-off**, is all that is available for storage. The difficulty consists in deciding what percentage of the total annual precipitation may be counted upon for run-off. This percentage will differ greatly, according to the soil and the character and degree

of natural moisture, as well as the nature of the vegetation of the district. Probably from one-third to one-half of the annual precipitation is as much as can ordinarily be safely counted upon.

2161. Survey of Watershed.—In any event, the first thing which it is indispensably necessary to do, in the study of an irrigation project derived from surface water, is to collect data. These will consist in a survey of the watershed, gauging the flow of the stream, and measuring the precipitation or rainfall. The survey will be conducted upon the same principles as that for any water supply. As from its nature it must be an approximate one, no time should be wasted in unnecessary refinement of instrumental work. A plain chain and compass survey is all that is needed, and in the case of a very extensive drainage area, it would, perhaps, suffice to merely determine the latitude and longitude of a few leading points, and connect these by a simple reconnaissance sketch.

2162. Gauging Rainfall.—Rainfall is measured by means of an apparatus called a **rain gauge.** This apparatus may be either quite rude or so carefully and accurately made as to be classed as an instrument of precision. As the observations made in reference to irrigation will generally occur in districts of very light rainfall, it is desirable that the most perfect rain gauge should be used. Such gauges are furnished, with full directions, by the dealers in scientific instruments. Frequently, however, it becomes necessary to commence the observations before a proper outfit can be procured. In such cases, a home made contrivance may be used, and this may vary from a simple pail or tub set out of doors to quite an efficient apparatus.

A good rain gauge may be made by any handy tinsmith. It will be understood that the difficulty in measuring rainfall consists largely in the fact that there will be frequent light showers, in which the depth over a given area is s

small as to render its accurate measurement very uncertain. Recourse, therefore, is had to the principle of the "exaggerated scale." This being premised, let Fig. 698 represent the vertical elevation and plan of a tin rain gauge. The dimensions being as given, it is evident that rain falling upon the mouth of the funnel, which is a circle 12 inches in diameter, and being collected in the cylindrical vessel beneath, of which the diameter is 4 inches, will stand in the latter nine times deeper than the same volume spread over the greater area of the funnel, because the respective depths will be inversely as the squares of the diameters, the ratio in the above case being $\frac{12^2}{4^2} = 9$. If, therefore, after a fall of rain, water stood in the cylinder to a depth of $3\frac{5}{16}$ inches, it would indicate a precipitation of 0.368 inch. It would be advisable to have a rule marked with inches divided decimally for the purpose of measuring depths. The apparatus shown in the figure can be supported in any suitable manner. It is generally placed in a cylindrical vessel of a diameter such that its edges support the sloping sides of the funnel. If used in a district subject to heavy rainfalls, the cylindrical receptacle would require to have a greater length than that shown in the figure.

FIG. 698.

Some judgment must be exercised in placing the gauge in a suitable location to secure average results. A wide, level, open space is preferable, and the mouth of the gauge should be about a foot above the general surface of the ground. It is very desirable to have two such gauges, or even more, placed in different localities, and at different elevations, in order to guard against merely local conditions.

Measuring the equivalent precipitation of snow is more difficult. Probably as good a way as any is to select a spot

where the snow has an average depth, and invert the funnel over it; then take up the snow which it covers, melt it, and pour the resulting water into the cylinder, where it can be measured just the same as rain.

A complete record of the rainfall in any locality includes a record of the thermometer and barometer at stated times, with the direction and estimated velocity of the wind, and time of beginning and end of rainfall, and of all other observed meteorological phenomena.

2163. Gauging the Flow of Streams.—Streams are generally gauged by the erection of a dam, in which an overflow weir is placed, as described in the section on Hydraulics. In some cases the construction of such an apparatus is somewhat difficult, because the dam must be quite water-tight, so that the entire flow of the stream shall pass over the weir.

Sometimes a part of the stream to be gauged may be found where it is divided into two branches by an island, as shown in Fig. 699. In such a case one branch may be shut

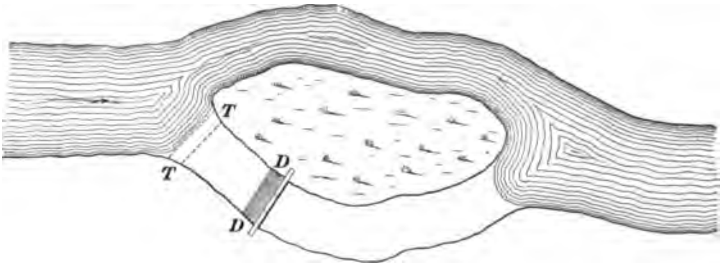


FIG. 699.

off by the temporary dam *T T*, while the dam for the weir *D D* is being built. Sometimes another temporary dam may be needed at the lower end of the island, to prevent back wash. When the weir is completed, the temporary dam is removed, and another one built across the other branch, so as to divert the water to the channel in which the weir is built.

The dam *D D* is built by driving sheet piling across the

stream, the ends entering well into the bank on either side, for which purpose an excavation will be made in the banks, in which the sheet piling will be driven, and the excavations then refilled with closely packed earth. Back of the sheet piling, on the up-stream side, an embankment should be placed, the best material for which is very fine gravel or coarse sand. Clay is of very little use for this purpose, because if the least trickle of water passes through it, it soon becomes washed away, whereas the tendency of the sand and gravel is to clog any aperture which may exist.

The methods of constructing the notch, of making the measurements, and of computing the quantity of discharge were fully described in the section on Hydraulics, Vol. I, Arts. **1000** to **1007.** When great accuracy in the quantity of discharge is not required, or where the conditions under which the observations are made, such as possible leakage around the dam, inaccuracies of measurement, or uncertainty in regard to the velocity of approach, make the result doubtful, the following formula, in which a mean value of the coefficient of discharge has been used, is more convenient than those given in Art. **1006,** Vol. I:

$$Q = 3\tfrac{1}{3}\, l H^{\frac{3}{2}}. \qquad \textbf{(217.)}$$

Should the problem be to ascertain the maximum flow of the stream, instead of only a safe average, it would be necessary to use the more accurate values given in the section on Hydraulics, and to take account of probable leakage, velocity of approach, etc.

Daily observations should be recorded for at least a year, in order to obtain even an approximate knowledge of the normal regimen of the stream. The time required to get an intelligent notion of the run-off and general character of a given watershed, by the above observations and those of the rainfall, constitute an embarrassing delay in districts where such observations have not already been carried on for a considerable period before it is desired to commence work. Unfortunately, it is very seldom that systematic records are kept in advance of the requirements.

2164. Other Methods of Gauging the Flow of Streams.—Although the method of weir measurements is the best adapted to continuous observations, there are many cases in which its use is impracticable, or at least too difficult and expensive, and in such cases one of the less accurate methods of measuring the discharge given in Arts. **1034** to **1038,** Vol. I, may be used.

In order to obtain accurate results by the various methods of measuring the velocity of flow, it is absolutely necessary that the greatest care be used in making the observations. In most cases the measurements should be repeated a number of times, and the different results so obtained carefully compared, and it will often be an advantage to make observations at two or more points along the stream, in order to get a nearer approximation to an average.

2165. Measurement of Evaporation.—In the dry regions, where irrigation is mostly practised, loss by evaporation is often a serious matter, especially when calculating the proper capacity of storage reservoirs. In sections enjoying an abundant rainfall, this item of loss is rarely given much attention, it being admitted that the rainfall upon the surface of a reservoir will compensate for loss by evaporation. Against this view, however, must be set off the fact that during the rainy season the reservoir is apt to be overflowing, when the benefit of any compensating precipitation is consequently lost.

It is very difficult to estimate the probable loss by evaporation in a storage reservoir in advance, because any experiments carried on in the stream which it is proposed to dam will be conducted under conditions different from those which will obtain in the reservoir itself. Evaporation is greatest in warm weather and during the prevalence of high winds. It also varies with the character of the body of water from which it proceeds, as it is less in a deep reservoir than in a shallow one, and less in still than in running water. In the semi-arid regions, it will probably average from 3 to 5 feet in depth from the surface of a reservoir

during the year, by which amount the capacity of the reservoir will be reduced.

To measure the evaporation from a given body of water, a simple apparatus is used by the U. S. Geological Survey. It consists of a pan of galvanized iron, 36 inches square and 10 inches deep, floated in the body of water of which the evaporation is to be measured, and filled with water to within a few inches of the top, care being taken to prevent water from washing in or out of it. An oblique scale is placed in it, as shown in Fig. 700, so that small vertical distances can be rendered appreciable by exaggeration.

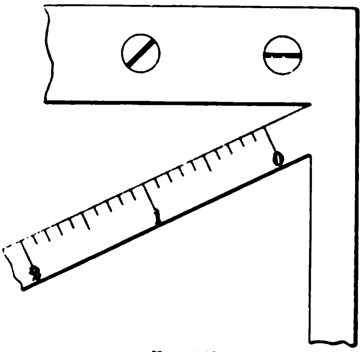


FIG. 700.

STORAGE.

2166. Storage Reservoirs.—Having decided upon the quantity of water required, and satisfied ourselves of the ability of a given stream to furnish this supply, it is next in order to consider how much of the yield of the stream must be impounded in a storage reservoir, for it will very rarely occur that we have occasion to deal with a river so large that no storage is required, and from which a supply of water sufficient for our purpose may be obtained by diverting a portion of the flow.

The general subject of storage reservoirs has been treated exhaustively in the section on Water Supply and Distribution, and need not be gone into here. Owing, however, to the somewhat different services for which they are intended, there will doubtless be some difference between the functions of a storage reservoir for irrigation purposes and one intended for a public water supply. The draft made upon the former will vary from year to year according to the wetness or dryness of the seasons, while the annual supply

that must be furnished by the latter will be nearly constant Irrigation reservoirs will also be called upon to furnish large quantities of water during short periods of time, thus making it necessary to provide them with appliances by means of which this may be accomplished, whereas the demand made on water-supply reservoirs varies but little from day to day, making it unnecessary to provide them with such capacious outlets, unless it be for the purpose of emptying them quickly in a case of emergency.

The appliances for controlling the flow of water from an irrigation reservoir or into an irrigation canal are generally known as **head gates** or **head works.** In spite of the differences between the kind of service required of the two classes of reservoirs, it seems hardly necessary to make a broad distinction between them, the real points of difference being in details rather than in general principles of design and construction.

It has been shown (Art. **2160**) that 1,115,136,000 cu. ft., or about 8,363,000,000 U. S. gallons, per annum would be required to irrigate an area of 10 square miles to a depth of 4 ft. This amount of water per annum would be sufficient to furnish a city of about 230,000 inhabitants with a supply of nearly 100 gals. per day per capita. In round numbers we may say, therefore, that 1 square mile of irrigated area, and 23,000 souls congregated in a town or city, require, on an average, an equal yearly amount of water.

Storage reservoirs, for whatever purpose they may be constructed, are almost invariably formed by building a dam across some valley, and thus forming an artificial lake, as already described in the section on Water Supply and Distribution. In this way, only one side of the reservoir need be built, and the enormous expense incident to building or excavating a tank large enough to contain the volume of water required is avoided.

2167. Small Reservoir Dams for Private Use.—In works for irrigation, there will be more frequent occasion to build small, cheap reservoirs than in the case of water-supply engineering, because such works are often erected by

individuals for private use on their own farms. Such dams are usually constructed without calling in the aid of a hydraulic engineer, but there is a right and a wrong way of executing even these small undertakings, and a few words will not be out of place upon the subject of small dams for private irrigation works.

These dams may be considered as having a limiting height of about ten feet, and impounding not more than one million cubic feet of water. Beyond these limits the danger incurred by imperfect work becomes so great that nothing but the most substantial and scientifically constructed work, such as has been described in the section on Water Supply and Distribution, should be allowed.

The class of work now under consideration may be built upon any one of many different designs, which it would be impossible to consider in detail. Briefly, these designs are all based upon the desideratum of avoiding stone masonry laid up in cement mortar, which would raise the structure to a higher type and require more skilled labor in its construction.

The best type for these home made dams is that which consists of a timber crib work, filled with well-packed stone, and backed on the water side by a well-constructed earthen embankment. Loose stone not confined in cribs should be avoided, as they are liable to be carried away by freshets. The crib work acts as a substitute for mortar, by binding the stones together and compelling them to act as a whole.

Fig. 701 is a section showing the general features and minimum dimensions for first-class work of its kind for such a dam, 10 feet high to the level of spillway or overflow. To build such a dam, a good trench is excavated, deep enough to reach a satisfactory foundation, and a pavement of large stones, well-packed in with spalls, is carefully placed upon the bottom. Upon these stones the crib work is placed, consisting of either round or square timber, notched and treenailed together. The cribs should be carried well into the bank on either side, and the ends very carefully packed, to prevent water from passing around them.

The cribs are filled with stone of various sizes, tightly packed.

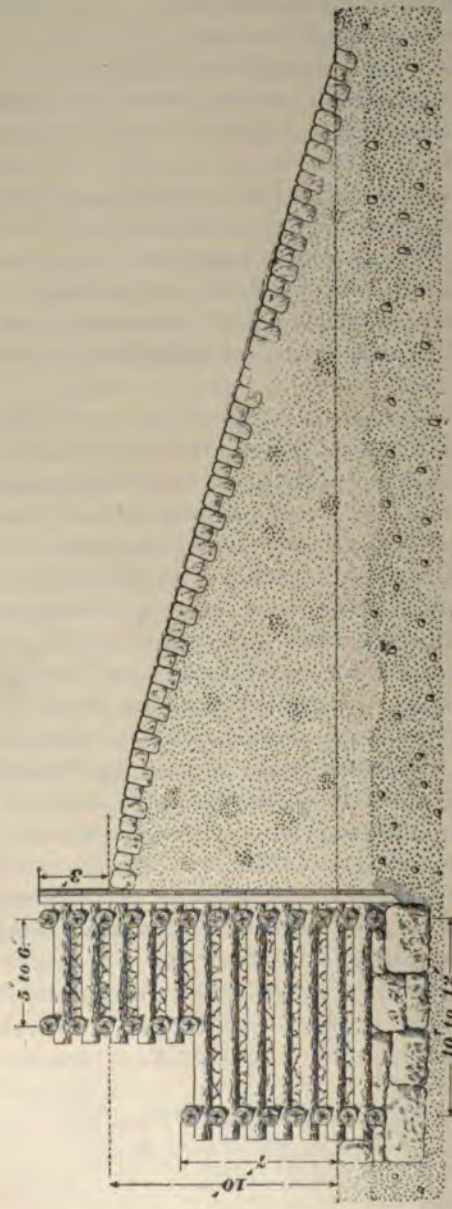


FIG. 701.

The face of the crib on the up-stream side is sheathed with tight planking. Against this sheathing the earthen embankment is placed, the natural surface of the ground beneath it having been roughly excavated, by plowing and scraping, for a foot or so, according to the nature of the soil. This embankment should be riprapped.

The spillway for these dams should be ample; the length may be about 10% greater than for the carefully built dams described in the section on Water Supply and Distribution. At this point the cribs require reinforcing, as shown in Fig. 702 and the face timbers had better be squared, for the convenience of spiking on the planking with which it should be completely sheathed.

It must be repeated that Fig. 701 shows

minimum dimensions for first-class work of its kind. For ordinary work it will be prudent to somewhat exceed them. The thickness of the solid stone and timber work should

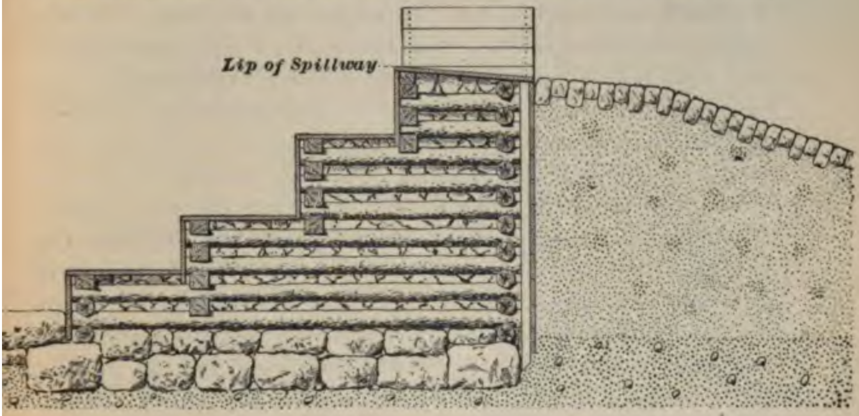


FIG. 702.

never be less than the maximum height to which the water may rise behind the dam.

Such dams, when care and judgment are used in their construction, are safe and substantial as long as the timber forming the cribs remains sound. Under ordinary conditions it will be long before the timber decays. If the embankment has been well made of good material, it will have become so packed and consolidated by time that even after the cribs have lost much of their original strength through decay the dam will continue to be a safe structure. Still, these dams are not to be considered in the light of permanent structures, in the same sense that a stone dam, or even an earthen one with masonry center wall, is.

Probably the simplest and best way to draw off the water from the above described dam is by means of a cast-iron pipe running through it and under the embankment. This pipe should be placed, if possible, upon the natural surface of the ground, to avoid settlement. If this is impossible, a stone foundation resting upon the natural ground should be built up under it. Gates or stop-cocks should be placed

upon this pipe, outside of the dam. Sluice gates, stop plank, etc., will not be needed in this class of work, if the above pipes and appliances are used.

2168. Location of Storage Reservoirs.—If the storage reservoir is to be used as a distributing reservoir, that is, if it is to be connected directly with the canal, flume, or pipe-line by which the water is distributed to the territory to be irrigated, it is advisable, in order to diminish the length of the canal, to place it as near as possible to the district to be supplied. Sometimes, however, the most favorable site for a storage reservoir is at a point far from the locality where the water will be used; in such a case it will generally be advisable to erect a small dam, just large enough to form a sufficient barrier to divert the water into the irrigating conduits below the reservoir, and as near as may be to the territory to be irrigated. In this way a considerable length of conduit can frequently be saved.

CONDUITS.

MAIN CONDUITS.

2169. When a large volume of water is to be conveyed a long distance and distributed in measured quantities over an extensive territory along the route, open canals having a suitable fall are generally employed. Such canals have some objectionable features, one of which is a considerable loss by evaporation and soakage, the latter being an unknown quantity until the canal is built and put in operation. They must also follow a nearly uniform and easy grade, a condition that makes it necessary either to skirt along the valleys which they encounter, thereby greatly increasing their length, or to cross these valleys on aqueducts, which are always more or less expensive structures.

When the volume of water to be conveyed is not too great, canals can be replaced by flumes or by pipe-lines. Flumes, being open channels, generally made of wood, differ from canals chiefly in the material of which they are made. Pipe-

lines, which may be of either cast-iron, wrought-iron, steel, or wooden stave pipe, differ from canals and flumes in the fact that they are not confined to a uniform descending grade, but may go up hill and down, and, if necessary, may be laid in the bed of any streams which they must cross.

These different classes of conduits will be taken up in order, beginning with open canals.

OPEN CANALS.

2170. Preliminary Work.—Surveys of a more or less extensive character are the necessary preliminaries to the design and construction of either a canal or a pipe-line; this is especially true of the former, because, as we have just seen, the course of the canal is confined to narrower limits than that of the pipe-line by the topography of the country through which it passes. Although much of what follows regarding surveys will be common to both, it must be understood throughout that it is a canal line which is specially under consideration.

Such a line naturally begins at some point very near to the stream which furnishes the water, and it will generally follow the same valley for a considerable distance. If time affords, it will be of the highest utility to secure a general survey and profile of the river itself for the entire distance that the canal follows it, noting all tributaries, falls, and rapids. It will be found that it pays well to do a good deal of surveying in all such operations, because more work can frequently be done in a few days with transit, chain, and level than in many weeks with pick and shovel. It must be again impressed upon the engineer that, as far as alinement is concerned, this survey does not call for any great refinement of accuracy. The leveling is of much more importance. It must be remembered that very large errors are liable to occur unperceived in leveling, and that no line of levels can be trusted that has not been checked. Therefore, when the alinement has been completed and leveled, check levels should be run back over the entire line; it will

not be necessary, however, to verify the entire profile, a check on the benches being sufficient. All important tributaries should also be surveyed, carrying the survey up the valley till an elevation approximately equal to that of the starting point has been reached.

A rough estimate of the length of the canal can be made from this survey, in connection with the topographical notes taken at the same time; then the approximate length, together with the total fall to be overcome, will enable us to make a preliminary design of the section and grade.

A trial line for the canal can now be run. There are several ways in which this can be done. One of the best and most expeditious methods is this: Suppose a grade of 4 ft. to the mile is decided upon for the slope of the canal. The tangent of the angle corresponding to this slope is $\frac{4}{5280} = 0.00075$, which corresponds to an angle of nearly 3 minutes. Having a transit provided with a vertical limb, let the telescope be depressed to this angle and clamped. When the transit is set up, let the target of a leveling rod be set at the height of the telescope of the transit from the ground. This can be sufficiently approximated by holding the rod alongside of the transit and sighting across the wyes. Let the rod now be taken as far ahead as possible and moved along the ground, up or down hill, until the center of the target is bisected by the horizontal cross-hair of the transit. The foot of the rod is then on ground falling at the desired rate, and a plug should be driven at this point and the distance measured. The direction will be ascertained by the needle, as this will be quite sufficiently accurate. From time to time measurements will be taken to convenient stations on the line of the river survey, if one has been made, as a check. It will be well to carry this line along, following all the indentations and tributary valleys, for in this way the length and characteristics of a line following the natural surface of the ground for its entire distance will be obtained. It will be very rarely that this line is actually followed by the canal, as it would lead to a too great development. Valleys will be crossed on aque-

ducts, and promontories will be "thorough cut," or tunneled, but only in this way can a full estimate of the comparative advantages of alternative lines be compared.

When an approximate location of the line has thus been determined, it will be accurately re-run and leveled over, so as to establish the final location, and make a more nearly exact estimate of cost.

2171. Grade.—The grade of the canal will be subject to several conditions. In the first place, the character of the bottom and sides will place certain limits upon the velocity of the water, which must be great enough to prevent the deposition of silt, and not so great as to do injury to the canal itself. The grade necessary to maintain the velocity within the desired limits will also depend upon the character of the interior surface of the canal, being very much less for one having a smooth lining—of brick, for instance—than for one merely excavated in the earth. The area of cross-section also affects the question, for the water in a large and deep canal will move with a greater velocity under a given grade than that in a smaller and shallower one having the same slope. The form of the cross-section also exerts a considerable influence upon the velocity of flow, so the question of the determination of the grade becomes a complex one, depending upon the desired discharge of the canal, its character and form, and the dimensions of its cross-section. These points must be taken up in detail, and it will first be necessary to establish some general principles.

2172. General Principles Affecting the Flow of Water Through Open Channels.—Although many of the principles about to be discussed apply equally as well to pipes running full, under pressure, as to open channels, all that follows will be considered in its relation to the latter only.

"Gravity is the sole force that acts upon a mass of water left to itself in a bed of any form; it produces all the motion which takes place—the inclination of the surface of

the water in the channel is the immediate cause of motion, being that which enables gravity to act." (Downing.)

It is a matter of common observation that the steeper the slope the greater the velocity, and as this steepness is determined by the ratio of the vertical height to the distance in which it is overcome, it is evident that the accelerating force producing velocity will be expressed by the ratio $\frac{h}{l}$, in which h = the difference of level between the two extremities of the canal and l = the distance, usually measured horizontally, separating the two.

If there were no resistance to the flow of water running through the canal, the constant accelerating force would cause the velocity to go on increasing indefinitely. But observation shows that water under these circumstances very soon acquires a constant velocity, which it maintains throughout its course, no matter how long the canal may be, provided the ratio $\frac{h}{l}$ remains constant. It is evident, therefore, that there are resistances at work which increase in intensity with the increase of velocity, so that after a certain time the increasing resistance just equals the increasing acceleration, and the velocity then becomes constant. This constant velocity is sometimes known as the **permanent regimen** of the canal.

It is necessary to ascertain what these resistances are and what is the law governing their increase.

2173. Resistance to the Flow of Water Through Conduits.—General principles and definitions relating to the flow of water through conduits and channels, together with a formula for computing the mean velocity of flow under any condition likely to occur in engineering practice, are given in the section on Hydraulics, Art. **1032**, etc. These definitions and principles should now be carefully reviewed before reading the following pages on the construction of conduits.

The laws governing the resistance to the passage of water

over the interior surface of a conduit are almost directly opposite to those governing the resistance of friction when one solid body slides over another.

The laws governing the flow of water that have the most important bearing on the subject now under consideration may be briefly expressed as follows:

I. *The resistance for any given velocity is proportional to the extent of the surface over which the water flows.*

II. *This resistance affects the entire volume of water flowing over the given surface, being greatest for the film in immediate contact with the surface, and becoming less and less for the films and threads more remote from that surface.* See Art. **1034**, Vol. I.

III. *The greater the extent of the surface in contact with a given volume of water, the greater the resistance becomes; conversely, the greater the volume subject to a given resistance, the less will the velocity be affected.*

IV. *The resistance is nearly proportional to the square of the mean velocity of flow.*

V. *The resistance varies with the nature of the surface of the conduit, being greater for a rough surface and less for a smooth one.*

If we let

h = the difference in level between the ends of the canal, or any two cross-sections of the canal;

l = the horizontal length of that portion of the canal included between the sections whose difference of level is h;

s = the slope = the ratio $\frac{h}{l}$;

a = the area of the water cross-section;

p = the wetted perimeter;

r = the hydraulic radius = the ratio $\frac{a}{p}$;

c' = a coefficient depending on the nature of the surface of the conduit; and

v = the mean velocity of flow;

then the laws for the resistance to flow may be expressed by the relation $h a = c' l p v^2$, from which we have the formula

$$v = \sqrt{\frac{h}{c' l} \times \frac{a}{p}} = \sqrt{\frac{1}{c'} \times s \times r}. \qquad (218.)$$

By replacing the factor $\sqrt{\frac{1}{c'}}$ in this formula by an equivalent factor which we may call c, such that $\sqrt{\frac{1}{c'}} = c$, we have $v = c\sqrt{r s}$, which is the same as formula **50**, Art. **1033**, Vol. I.

Kutter's formula for finding the value of the coefficient c, together with the values of the coefficient of roughness given in Art. **1033**, when applied to formula **50**, furnishes the most reliable available method of computing the mean velocity of flow in open channels.

2174. Importance of the Hydraulic Radius.—It is evident from formula **218** that the velocity increases with the hydraulic mean radius $r = \frac{a}{p}$, and that, therefore, the most favorable shape of cross-section will be the one in which a given area is enclosed by the smallest wetted perimeter. In the case of an open canal, this section would be a half circle, since the circle is that geometrical figure which encloses the greatest area within a given perimeter. In the case of the circle, the value of the hydraulic radius is $r = \frac{a}{p} = \frac{\frac{1}{4}\pi d^2}{\pi d} = \frac{d}{4}$, and, since both the area and the wetted perimeter of a half circle are, respectively, equal to one-half of the area and wetted perimeter of a circle when running full, the ratio $\frac{a}{p}$ for the half circle is also equal to $\frac{d}{4}$.

The half-circular form of conduit is impracticable for a canal, since the form could not be constructed and maintained unless the inside were lined with brick or some other permanent material, and even then the constructional difficulties would generally render this form inadvisable, as entailing a considerable expense of labor without a corre-

sponding economy of material. An approximation to this best form is half a regular hexagon, in which $r = \frac{D\sqrt{3}}{8} = \frac{\sqrt{3}\,D^2}{8}$; D being the diameter of the circumscribing circle. This form would also require a permanent revetment if it were applied to an earthen canal.

2175. Practical Forms of Cross-Section.—Generally the form adopted will be decided according to other considerations than the theoretical ones just discussed. The proper velocity will be assumed, and side slopes adopted suited to this velocity and the nature of the material of which the bottom and sides are composed. A convenient depth is selected, which depends largely upon the kind of material to be excavated, and with these assumed data a form of cross-section will be fixed upon, generally by "trial and error."

PRACTICAL FORMULAS FOR MEAN VELOCITY OF FLOW IN CONDUITS.

2176. We have said that formula **50,** Art. **1033,** Vol. I, when used in connection with Kutter's formula for determining the value of the coefficient c, gives very reliable values for the mean velocity of flow. Kutter's formula, however, is a little complicated and difficult to use, and for this reason simpler approximate formulas are sometimes used for finding the value of v for conduits of a special kind of construction. Examples of these special formulas for brick-lined conduits are given in Arts. **2143** and **2144** in the section on Water Supply and Distribution. See formulas **215** and **216.** In Art. **2146** it is also stated that the mean velocity for conduits lined with rubble masonry may be computed by first finding the mean velocity for a brick-lined channel, and then taking a percentage of this result, depending on the roughness of the sides of the required conduit.

2177. Formula for Canals with Earthen Banks.—An approximate formula that may be used for canals with earthen banks in good condition is the following:

$$v = \sqrt{\frac{100,000\, r^2 s}{9\, r + 35}}, \qquad \textbf{(219.)}$$

in which v = mean velocity in feet per second; r = hydraulic radius $= \frac{a}{p}$, and s = the slope $= \frac{h}{l}$.

EXAMPLE.—Let the fall in an earthen canal, having the cross-section

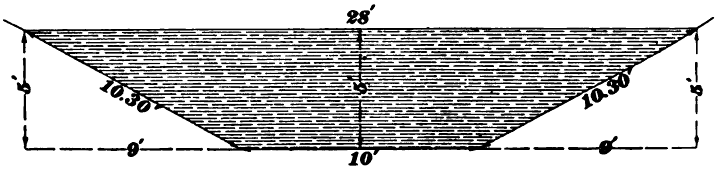


FIG. 703.

shown in Fig. 703, be 5.25 ft. per mile. What is the mean velocity of flow?

SOLUTION.—Here $s = \frac{5.25}{5,280} = 0.00099+$, which we will call 0.001. Also, $r = \frac{95}{30.6} = 3.105$. Then, by the last formula,

$$v = \sqrt{\frac{100,000 \times 9.64 \times 0.001}{27.95 + 35}} = 3.91 \text{ ft. per sec. Ans.}$$

2178. Limiting Velocity.—In the above example the question would be: Is the velocity, which is nearly 4 ft. per second, too great for the earthen banks of the canal to resist without washing? The answer to this question can only be given by referring to the results of experience. It has been found that light and sandy soils can not safely resist a mean velocity greater than 2 ft. per second, while at the same time this velocity is sufficient to prevent plant growth and remove silt. In firmer soil, velocities of 3 to 4 feet per second are permissible, but except in hard-pan or very resisting material, 5 feet seems to be the limiting velocity for earthen canals.

In almost any district where it is proposed to build such canals there will be some examples of ditching, upon a

greater or less scale, by observing which an approximate idea may be formed of the proper grade and side slopes to be given to the proposed canal. The engineer should not fail to take advantage of all such opportunities to obtain local knowledge of the district he is operating in.

2179. Practical Considerations Limiting the Choice of Form of Cross-Section.—Besides the velocity, there are other considerations which influence the choice of form of the cross-section of a canal. A certain ratio of side slope will generally be necessary, according to the nature of the soil, and a certain depth will generally be found more convenient or desirable than another. By taking these and other points into consideration, the designing of a proper cross-section to satisfy the necessary requirements is simplified. The following illustrative example will make this plain:

It is desired to establish the proper cross-section and grade of an earthen canal, under the following circumstances. The quantity of water to be conveyed is 250 cu. ft. per second. A velocity of 2 ft. per second is desired. The side slopes are to have an inclination of 1 vertical to $1\frac{1}{2}$ horizontal, and a depth of 6 feet of water is desired in the canal. What should be the form and area of the cross-section, and what the grade of the canal?

Since the velocity is to be 2 ft. per second and the discharge 250 cu. ft. per second, the area of cross-section must be $\frac{250}{2} = 125$ sq. ft.

To determine the form in which this area must be put, it is necessary to know the bottom width of the canal, which

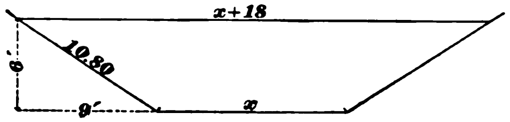


FIG. 704.

is represented in Fig. 704 by x. From the data, we have

$$6\frac{(x + x + 18)}{2} = 125; \text{ or, } 6(x + 9) = 125.$$

Hence, $$x = \frac{71}{6} = 11.83.$$

We will call this 12 ft., which will give the slightly greater area of 126 sq. ft. We now want to ascertain the hydraulic radius, which (since $p = 10.80 + 10.80 + 12$) is $\frac{126}{33.6} = 3.75$.

Everything is now known but the slope s, to obtain which we insert all the data in formula **219,** thus:

$$2 = \sqrt{\frac{100{,}000 \times 14.06 \times s}{9 \times 3.75 + 35}}.$$

Squaring, and solving for s, we have

$$4 = \frac{100{,}000 \times 14.06 \times s}{9 \times 3.75 + 35},$$

from which

$$s = \frac{275}{1{,}406{,}000} = 0.000195.$$

This represents a grade of $.000195 \times 5{,}280 = 1.03$ ft. per mile.

2180. Influence of Depth on Velocity of Flow.—The depth of the canal exercises a considerable influence upon the velocity of flow. Thus, in the above illustration, if we admit a depth of 8 ft., then, all the other data remaining the same, and using the same area of 126 sq. ft., we should have for the bottom width,

$$8\,(x + 12) = 126,$$

$$x = 3.75.$$

The depth being 8 ft. and the ratio of slope being 1 to $1\frac{1}{2}$, the length of the side slope would be 14.42 ft., and the wet perimeter, 32.60. Therefore, the hydraulic radius is $\frac{126}{32.60} = 3.87+$, the square of which is nearly 15. Then,

$$2 = \sqrt{\frac{100{,}000 \times 15 \times s}{9 \times 3.87 + 35}},$$

and

$$s = \frac{4}{21{,}481} = 0.000186.$$

This represents a grade of 0.98 ft. to the mile as against 1.03 for the previous depth.

These examples show that with a given grade and area of cross-section, the velocity becomes greater as the depth increases, because, within certain limits, the hydraulic radius increases with the increase in depth. The limit is reached when the width of the canal is equal to twice its depth. This condition is most perfectly fulfilled in the case of a semicircular cross-section, as has already been shown. The following illustrative example will be useful in making this plain:

What will be the value of s in the previous examples if the form of cross-section is a half circle whose area is 126 sq. ft., the velocity to remain at 2 ft. per second?

The diameter of the half circle will be $\sqrt{\frac{2 \times 126}{.7854}} = 17.92$ ft., which will be the width of the canal at the surface of the water. Its depth will consequently be equal to the radius, or half the above diameter, or surface line. The hydraulic radius, as already shown, will be equal to one-quarter of the diameter, $\frac{17.92}{4} = 4.48$, the square of which is 20.07. Then,

$$2 = \sqrt{\frac{100,000 \times 20.07 \times s}{9 \times 4.48 + 35}},$$

and
$$s = \frac{4 \times 75.32}{2,007,000} = 0.00015.$$

This represents a grade of 0.792 ft. per mile.

We will illustrate still further by another example:

All other data being as before, what is the value of s when the cross-section is that of a semi-hexagon?

Let Fig. 705 represent the semi-hexagon, inscribed in a semicircle. Since the side of a regular hexagon is equal to the radius of the circumscribed circle, we have the relation between the various parts shown

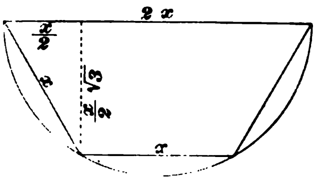


FIG. 705.

in the figure. We require, first, the side of the hexagon, x in the figure, and the depth, which, as will be seen, is $\frac{x}{2}\sqrt{3}$. Since the area is 126 sq. ft., we have

$$\frac{3x}{2} \times \frac{x}{2}\sqrt{3} = 126,$$

$$\frac{3x^2}{4}\sqrt{3} = 126,$$

$$\sqrt{3}\,x^2 = 168,$$

$$x = \sqrt{\frac{168}{1.73}},$$

$$x = 9.85.$$

It has already been stated (see Art. **2174**) that in the case of the semi-hexagon the hydraulic radius is $r = \frac{D\sqrt{3}}{8}$, in which $D =$ the diameter of the circumscribing semicircumference $= 2x$. Therefore, in the present instance we have

$$r = \frac{19.70 \times 1.73}{8} = 4.26,$$

and $$2 = \sqrt{\frac{100{,}000 \times 18.15 \times s}{9 \times 4.26 + 35}}.$$

Whence $$s = 0.000162,$$

or $$0.85 \text{ ft. per mile.}$$

Although the last two forms of section are not adapted to unrevetted banks, their properties have been introduced here to show still more strikingly the effect of depth on velocity.

Obviously, all calculations relating to flow through earthen canals are less exact than for masonry-lined conduits, because the dimensions of the former can not be so accurately determined, on account of such uncertainties as caving banks.

2181. General Remarks on Earthen Canals.—Earthen canals, particularly in light sandy soils, often give

a great deal of trouble, even when properly side sloped and graded, by reason of the tendency to wash; their use is, therefore, mostly confined to those very large works where the use of pipes or flumes would be out of the question. When lined with masonry they are much more efficient, and the greater velocity which they can then safely sustain, and their consequently greatly reduced cross-section, makes their relative expense, as compared with canals having unprotected interior surfaces, less than might be imagined. Sometimes cheap substitutes for masonry lining are employed, and it has been found in California that a good and quite durable lining can be made by coating the sides and bottom with a plastering ¾ inch thick, composed of 1 part of Portland cement and 4 parts of sand, the sides and bottom having previously been accurately trimmed and moistened.

Earthen canals are best when built entirely in excavation. It is impossible, however, to obtain this result unless the ground is exceptionally favorable and the location very carefully selected. Even then such a canal would have a greatly increased length, owing to the necessity of many deviations, in order to keep it on suitable ground. Practically, for a large proportion of their length, canals will be formed partly in excavation and partly in embankment, the material thrown out of the excavation being used, if suitable, in the embankment. Great care must be taken to carefully trim the banks to true lines.

The proper inclination to give to the side slopes is a point requiring very careful consideration. Very steep or very flat slopes both lead to deterioration by wash. If too steep, they fall by the undermining effect of the flow of water in the canal, and if too flat the exposed surfaces are damaged by rain. The best guide is a careful examination of any canals or ditches which may be found already in use in the district.

When the embankment is high, it is better to keep a narrow berm between the foot of the bank and the edge of the ditch. When very high, a center wall should be used, as described in the section on earthen dams in Water Supply

and Distribution. This, of course, adds to the expense. Fig. 706 shows a half section in which these features have been carried out.

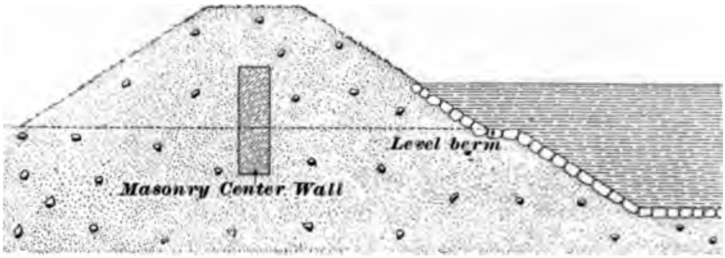


FIG. 706.

It has been observed, not only in the case of canal banks and excavations, but also those for railroads, that the effect of time and wash is always to reduce the original straight lines and sharp angles to curves and rounded edges. It would undoubtedly be an advantage to anticipate this result by giving to such work, at the start, a form somewhat similar to that which it will eventually assume.

Thus, in Fig. 707, if the heavy straight lines represent the original form of the cross-section, it will gradually assume

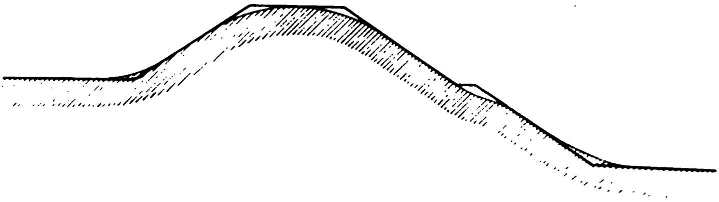


FIG. 707.

the shape shown in the shaded portions. It will be better, therefore, to favor this form in shaping the slopes of the excavation and embankment.

2182. Canals Revetted with Dry Stone.—So much trouble is occasioned by the deterioration, slow or rapid, of canals with unprotected banks, and so much uncertainty exists regarding their probable discharge, which latter consideration frequently leads to giving them unnecessarily large dimensions, or steepness of grade, that it is often

good policy and economy to line them at least with dry stone. Such an arrangement is shown in Fig. 708, which represents

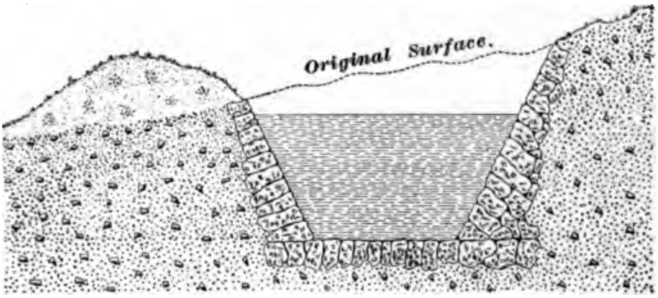


FIG. 708.

a canal cut in sloping ground. Any stones which can be obtained will be used for the purpose, preferably of a flat form, but frequently nothing but cobble-stones can be procured. It is generally best to lay the pavement continuously under the side walls, and to build these upon it, as shown in the figure. Some rough hammer dressing is usually required at the corners, where the side walls connect with the pavement. In laying the pavement, if flat stones can be procured, they should all be laid on edge, with a slight inclination down stream, and packed as closely as possible. All the work should be thus packed and the walls well bonded.

It would be a great improvement to such a lining if after it was laid up it should be pointed with cement mortar and, if sufficiently smooth, plastered.

2183. Formula for Velocity of Flow in a Canal Lined with Dry Stone.—It is very difficult to adapt a formula for canals lined as above, because the velocity of flow will be greatly dependent upon the character of the lining and the greater or less care expended to make it a good piece of work. The following will be an approximately correct formula for a well-laid dry wall, without pointing or plastering:

$$v = \sqrt{\frac{100{,}000\, r^2 s}{8 r + 15}}. \qquad (220.)$$

EXAMPLE.—Referring to Fig. 708, let the bottom width of a canal lined with dry stone be 8 ft., the batter of the side walls being 1 vertical to $\frac{1}{2}$ horizontal. Let the depth of water be 8 ft., and the desired velocity 7 ft. per second. What is the value of s?

SOLUTION.—Here the breadth of the waterway at the surface of the water is 16 ft. The area a is, therefore, $\frac{16+8}{2} \times 8 = 96$ sq. ft. The length of the wet line on a section of the side wall, with the given batter and depth of water, is $\sqrt{4^2+8^2} = 8.94$ ft., and the value of p, or the wet perimeter, is, consequently, $2 \times 8.94 + 8 = 25.88$ ft. Therefore, $r = \frac{96}{25.88} = 3.71$.

Inserting these values in formula **220,** we have

$$7 = \sqrt{\frac{100{,}000 \times 13.76 \times s}{8 \times 3.71 + 15}}.$$

Whence, $s = 0.00159$, or 8.40 ft. per mile. Ans.

2184. Formula for Canals Lined with Rubble Masonry.—The canal lined with masonry laid in cement constitutes a still higher type of structure. It is far more permanent in its character than those already considered, permits of a higher velocity of flow without injury to itself, and with a given grade and hydraulic radius offers less resistance to a rapid flow.

The formula suitable for a canal lined on the sides and bottom with a good class of rubble masonry, pointed but not plastered, is

$$v = \sqrt{\frac{100{,}000\, r^2 s}{7.3\, r + 6}}. \qquad \textbf{(221.)}$$

EXAMPLE.—In a rubble-lined canal, whose section is shown in Fig. 709, let the permissible velocity be 10 ft. per second; what is the value of s?

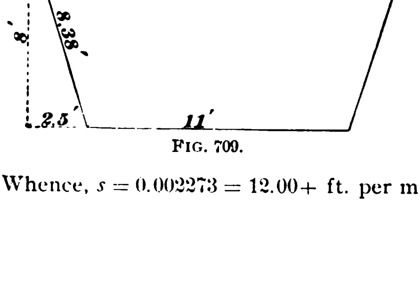


FIG. 709.

SOLUTION.—The data are $a = 108$; $p = 27.76$; $r = \frac{108}{27.76} = 3.89$. Substituting in formula **221,**

$$10 = \sqrt{\frac{100{,}000 \times 15.13 \times s}{7.3 \times 3.89 + 6}}$$

Whence, $s = 0.002273 = 12.00+$ ft. per mile. Ans.

2185. Cut-Stone and Brick-Lined Canals.—These will rarely if ever be used in irrigation canals ; they belong more properly to Water Supply and Distribution, and have already been discussed in the section on that subject. See Arts. **2142** to **2146.**

It must be borne in mind, however, that the only reason why these more perfect structures are not employed in irrigation works is their great comparative cost and the time required for their construction. Otherwise, they would be preferable on account of their durability, small trouble and expense for maintenance, and the greatly increased flow which they yield for a given grade and hydraulic radius.

FLUMES.

When, to avoid a long detour, it becomes necessary to carry the water of a canal across a valley, this will be usually accomplished by means of a flume, either of wood or metal. In the case of a city water supply, where all the installations must necessarily be upon a much more permanent basis, these flumes would be replaced by masonry aqueducts, or they would, at least, be built of metal in a very substantial and perfect manner. In this respect, as in nearly all others, it will be perceived that everything connected with irrigation admits of a more temporary character than the corresponding features in a city water supply, because the consequences of failure, or even temporary suspension, are more serious in the one case than in the other. Dams and reservoirs offer, perhaps, an exception to this rule, and even then, since irrigation works are generally located in relatively sparsely settled districts, the bursting of a dam would, probably, be of minor importance as compared with what it would be if occurring just above a large town.

2186. Timber Flumes.—These may be built according to an almost endless variety of design. A simple form for a small flume, suitable for a water section of $4' \times 2'$, is

shown in cross-section in Fig. 710. Dimensions of timber are given in the figure. The bents may be 4 to 6 ft. apart. In this very simple form of construction no mortising need be used, as all the pieces can be assembled with spikes, bolts, and nails.

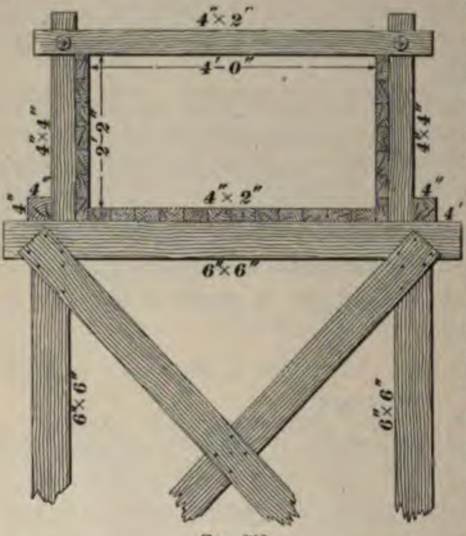


FIG. 710.

An example of a larger and more perfect structure is shown in Fig. 711, which is a cross-section of the San Diego flume in California, which Wilson describes as follows. "This flume is 5 ft. 10 inches in width inside, and 3 ft. 10 in. in height from the floor to the top of the frame. Owing to the lack

FIG. 711.

of water supply, the interior has been boxed up but one plank in height, that is, to a depth of 16 inches. The flume, when on a hillside, rests throughout on an excavation on a bench 12 feet in width. Only when it crosses

side drainage lines or creeks is it on trestles. First, planks for mud sills 12 by 2 inches are laid across the bench 4 feet apart. On these rest longitudinal stringers of 4 by 6 in. timbers, above which are the floor-beams, also of 4 by 6, and placed 4 ft. apart immediately above the mud sills. Intô these are gained the upright posts 4 feet in height and of 4 by 4 scantling, braced by short stringers, gained both into the posts and floor-beams ; the whole is then planked with 2-inch planking running longitudinally.

* * * * * * *

"When on trestles, the sills of the flume rest on three longitudinal stringers, two of which are 4 by 12 inches and one in the center 6 by 12 inches. The trestle bents are placed 16 ft. apart, and for trestles up to 20 ft. in height consist of two 8 by 8 inch posts set on a batter of 1 to 6 ; of cap pieces 8 by 8 inches by 6 ft., and of sills 8 by 8 inches, and of diagonal sway braces 2 by 10 inches. More posts are introduced for higher trestles, and truss bridges carry the flume over the deepest gorges."

In the construction of wooden flumes well-seasoned stuff should be used, and much better results, as regards flow and tightness, are obtained by having the edges and inside faces planed. A heavy coat of paint applied to the edges of well-matched planking just before spiking will make the box water-tight ; without this the joints must be calked with oakum. It is well to paint the whole of the inside, or at least the joints. Wilcox says : "In sheathing a wooden flume, it is best to use large wire nails or cut spikes for the floor, but the sides should be fastened with bolts through inside cleats at the joints. If nails are used in the side planking, they will rot out, and it will be found impossible to keep the planks on."

When a flume is connected with an earthen canal, the greatest care must be taken to secure the point where the two connect against washing out. The flume should enter well into the canal bank, and every possible escape of the water cut off in the most effective manner.

2187. Dimensions of the Different Parts of a Wooden Flume.—Ordinarily, these flumes when of small size are constructed without calculating the stresses to which they will be subjected. The calculations, however, are very simple, and for large flumes it is quite worth while to go through them before getting out the lumber for the structure, as it will ensure against waste on the one hand and failure to resist the pressure upon the other; it will also guard against the common fault of having some parts stronger than is required, while others are barely strong enough, or even too weak for safety. The method of calculation will be illustrated by an example.

It is desired to build a flume to convey the water in a canal 10 ft. wide and 5 ft. deep; the flume is to cross a small

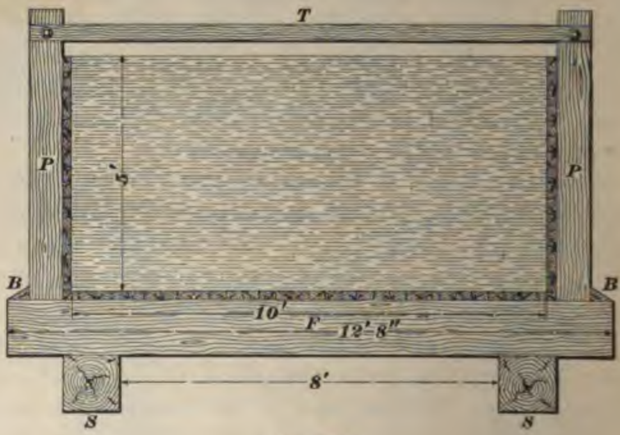


FIG. 712.

stream upon two stringers resting upon abutments 10 ft. apart. The system of construction is shown in Figs. 712 and 713, where *P*, *P* represent upright posts resting upon floor-beams *F*, *F*, *F*, into which their feet are mortised; they are also braced by the triangular blocks *B*, *B*, *B* spiked to the floor-beams. The tops of the posts are held together by ties *T*, which are halved into them and held by bolts. The inside of the waterway is sheathed with planking. The

floor-beams rest upon stringers S, S, which span the opening between the abutments A, A. The bents formed by the

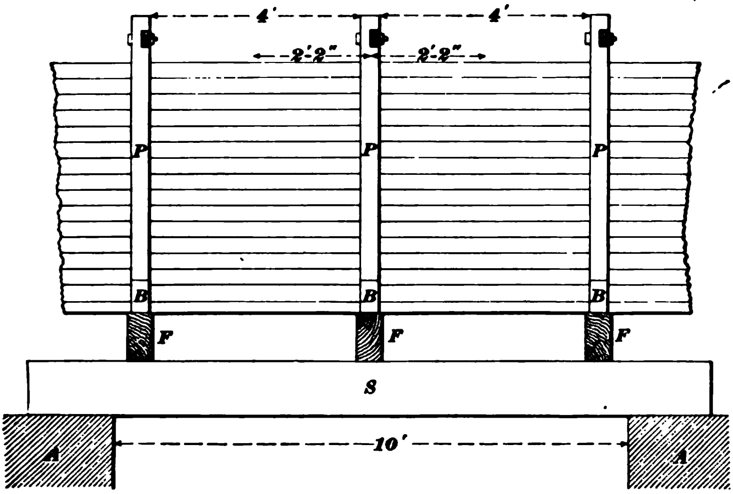


FIG. 718.

posts and ties are 4 ft. apart in the clear. Compute the dimensions of the different members of the structure.

2188. We will first establish simple formulas for rectangular beams, under the various methods of loading occurring in this and similar problems. Referring to Art. **1251**, section on Strength of Materials, Vol. II, we have the following relation between the bending moment M, the unit stress S, the breadth of the beam b, and the depth of the beam d:

$$M = \tfrac{1}{6}\, b\, d^2\, S. \qquad (a)$$

2189. From the table of Bending Moments and Deflections we have, for the bending moment of a simple beam uniformly loaded, $M = \frac{w\, l^2}{8}$, where w is the load per unit of length, and l the length of the beam in inches between supports. If we let W be the total load uniformly distributed over a beam, we shall have $W = w\, l$, which, substituted

in the value of M just given, gives us $M=\frac{Wl}{8}$. Substituting this value in (a), we have

$$M=\frac{Wl}{8}=\tfrac{1}{6}bd^2S,$$

from which

$$W=\tfrac{4}{3}\frac{bd^2}{l}S. \qquad \textbf{(222.)}$$

2190. For the case of a simple beam with a concentrated load W at the middle, we have from the table of Bending Moments and Deflections,

$$M=\frac{Wl}{4}.$$

Substituting this value of M in (a), we have

$$M=\frac{Wl}{4}=\tfrac{1}{6}bd^2S,$$

from which

$$W=\tfrac{2}{3}\frac{bd^2}{l}S. \qquad \textbf{(223.)}$$

2191. If there is a concentrated load W at a distance l_1 from one support and l_2 from the other, the table of Bending Moments and Deflections gives the value

$$M=\frac{Wl_1l_2}{l},$$

which, substituted in (a), gives us

$$M=\frac{Wl_1l_2}{l}=\tfrac{1}{6}bd^2S,$$

from which

$$W=\tfrac{1}{6}\frac{bd^2lS}{l_1l_2}. \qquad \textbf{(224.)}$$

2192. If we consider the post P, Fig. 712, we see that it acts as a beam in which the load is that due to the lateral pressure of the water; this pressure is zero at the surface of the water, and increases uniformly with the depth below the surface until the bottom is reached. The expression for

the maximum bending moment on a beam so loaded is $M = .128\,Wl$, where W is the total load and l the length of the beam.

Substituting this value of M in (a), we have

$$M = .128\,Wl = \tfrac{1}{6}\,bd^2S,$$

from which we have

$$W = 1.3\frac{bd^2}{l}S. \qquad \textbf{(225.)}$$

2193. Safe working values of the unit stress S that may be used for the various kinds of timber generally employed in this class of work are given in the following table:

Kind of Timber.	Safe Working Stress.	
	Steady Load.	Variable Load.
Yellow Pine..................	1,800	1,200
White Oak....................	1,350	1,000
Spruce	1,250	900
Hemlock	1,200	850
White Pine	1,100	800

These values are reliable for good, sound timber; if the timber is knotty or otherwise imperfect, a lower value of S should be used.

2194. Turning now to the computation of the strength of the various parts of the structure, and beginning with the floor-beams F, we see that, neglecting the small weight of the sheathing, they carry a uniformly distributed load equal to the weight of a prism of water 10 ft. $\times$ 5 ft. $\times 4\frac{1}{3}$ ft., or $10 \times 5 \times 4\frac{1}{3} \times 62\frac{1}{2} = 13{,}542$ pounds. In order to insure a large factor of safety, we will call the length of the beam 10 feet; then, by assuming a beam whose depth is twice its breadth, we have $b = \frac{1}{2}\,d$.

Substituting the known values in formula **222,** we have, for a yellow pine beam,

$$13,542 = \tfrac{4}{3} \times \frac{d}{2} \times \frac{d^2}{120} \times 1,800$$

from which we have

$$d^2 = 1,354.2 \text{ and } d = 11.06+.$$

Since the market sizes of timber generally run in multiples of 2 inches, we will make the depth of this beam 12 inches, and its breadth will therefore be 6 inches.

The posts P must act as beams to resist the lateral pressure of the water. According to the law for lateral pressure (see Art. **978,** Vol. I), the total pressure that must be supported by each post is

$$W = \tfrac{13}{3} \times 5 \times \tfrac{5}{2} \times 62.5 = 3,385 \text{ pounds.}$$

If we make the depth of the posts equal to twice their thickness, as was done with the beams F, we will have $b = \frac{1}{2} d$; substituting this and the other given values in formula **225,** we have

$$3,385 = 1.3 \times \tfrac{1}{2} \times \frac{d^3}{60} \times 1,800,$$

from which

$$d^3 = 173.59, \text{ and}$$
$$d = 5.58, \text{ or, in even inches,}$$
$$d = 6 \text{ inches and } b = 3 \text{ inches,}$$

which is a standard size.

Owing to the fact that the upper end of this post must be notched for the tie T, and still have sufficient thickness to give a firm bearing to the bolt, it might be better to increase its thickness to 4 inches, as is shown in Fig. 713.

The thickness of the floor sheathing may be computed by considering a strip 1 foot wide as a beam uniformly loaded with a prism of water 1 foot wide, 5 feet high, and $4\frac{1}{3}$ feet long. This prism weighs

$$1 \times 5 \times 4\tfrac{1}{3} \times 62.5 = 1,354 \text{ pounds.}$$

Substituting the given values in formula **222,** we have

$$1,354 = \tfrac{4}{3} \times \tfrac{12}{52} \times d^2 \times 1,800,$$

from which

$$d^2 = 2.4447 \text{ and } d = 1.56 \text{ inches.}$$

Owing to the fact that the flooring is practically a continuous beam, its strength is greater than would be called for by the above calculation, and $1\frac{1}{2}$ inch planking would be amply strong to carry the weight. In order, however, to secure tight joints and to provide against deterioration, we have taken its thickness as 2 inches. The thickness of the side sheathing will also be taken as 2 inches, in order to secure durability and tightness.

The stress in each tie bar T is equal to one-third of the total lateral pressure of the water in one panel, that is, to $3{,}385 \times \frac{1}{3} = 1{,}128$ pounds. This stress could easily be borne by a section of 1 square inch, but, in order to furnish sufficient section for the bearing of the bolts and notching onto the posts, it will be better to make the ties of $4'' \times 4''$ stuff.

In order to calculate the dimensions of the stringers S, we will make an estimate of the weight transmitted to each stringer by the floor-beams F. Assuming that the next supports for the planking are 4 feet beyond the outer beams, we shall have a load on each floor-beam equal to the sum of the weights of the water and the timber included between two consecutive beams.

By referring to the calculations previously made, we see that the timber included between two consecutive beams, including the beams themselves, is given in the following bill:

		Cu. Ft.
1	floor beam, $6'' \times 12'' \times 12'\ 8''$	6.00
2	posts, each $4'' \times 6'' \times 6'\ 0''$	2.00
1	tie, $4'' \times 4'' \times 11'\ 4''$	1.25
$86\frac{3}{4}$	sq. ft. of 2-inch sheathing	14.45
		23.70

In accordance with the table of Specific Gravities and Weights per Cubic Foot, yellow pine weighs 41.2 pounds per cubic foot. This, however, is the weight of dry timber, and since the sluice is filled with water and the timber consequently wet, we will take the weight of the timber as 50

pounds per cubic foot. The total weight transmitted to the stringers by each beam F is then

	Pounds.
23.7 cu. ft. of timber @ 50 lb..............	1,185
$4\frac{1}{3} \times 5 \times 10 = 216\frac{2}{3}$ cu. ft. of water @ 62.5 lb.	13,542
	14,727

Since half of this weight is borne by each stringer, the load on each stringer under each of the floor-beams is $14.727 \div 2 = 7,364$ pounds.

The total load borne by one stringer, neglecting the weight of the stringer, which may be done in the present case, is $3 \times 7,364 = 22,092$ pounds, and the reaction at each support is $22,092 \div 2 = 11,046$ pounds.

The maximum bending moment in the stringer occurs under the middle beam, and may be found by the graphical method given in the section on Strength of Materials; a more expeditious method, however, for this case is the following application of the principle of moments:

Taking the center of moments at the middle of the beam, the moment of the reaction at the left support is $11,000 \times 60 = 660,000$ inch-pounds; this moment tends to turn the left-hand end of the beam upwards around the center. The downward pressure, due to the weight of the left-hand beam F, is 7,364 pounds, and the moment of this force around the assumed center of moments is $7,364 \times 52 = 383,000$ inch-pounds, which tends to turn the left-hand end of the beam downwards. The net result of these two moments is $660,000 - 383,000 = 277,000$ inch-pounds, which is the maximum bending moment at the center of the beam.

Substituting this value of M in equation (a), Art. **2188**, we have, for a yellow pine beam with a square section,

$$M = 277,000 = \frac{1}{6} d^3 \times 18,000,$$

from which

$$d^3 = 923\frac{1}{3},$$

and

$$d = 9.74 \text{ inches nearly.}$$

A beam $10'' \times 10''$ will be strong enough to carry the load when new, but, in order to provide against the effects of

decay, it will probably be better to use a beam 10″ × 12″ deep, or even 12″ square, as shown in the figure.

2195. By computing the different dimensions of such structures in the manner just shown, the engineer has the satisfaction of knowing that he has secured a well-proportioned structure.

It should be borne in mind, however, in making all such calculations, that it is time wasted to try to carry the calculations to a degree of refinement beyond the practical limits to which all work must conform. As has been seen, such considerations as commercial sizes of timber make it necessary to use dimensions that are only approximations to those obtained by calculation; in order to make a design that is thoroughly successful in every respect, it is also necessary to exercise careful judgment in making such allowances for deterioration as the probable length of life of the structure will require.

2196. Formula for the Flow of Water Through Wooden Flumes.—To ascertain the velocity of flow through such flumes as have just been under consideration, still another formula is necessary, since their coefficient of friction is relatively much smaller than that of earthen canals. The proper formula is the following :

$$v = \sqrt{\frac{100{,}000\, r^2 s}{6.6\, r + 0.46}}, \qquad \textbf{(226.)}$$

in which the symbols have the same meaning as in Art. **2177.**

Timber flumes are generally rectangular in shape, and they may be very accurately proportioned to secure the best results. The most favorable rectangular cross-section is that in which the water has a depth equal to half the width.

Example.—A timber flume, 10 ft. wide and running 5 ft. deep, has an inclination of 9 inches to the mile. What is its discharge per second in cubic feet ?

Solution.—Here, $r = \frac{50}{20} = 2.5$, and $s = \frac{0.75}{5{,}280} = 0.000142$. Then,

$$v = \sqrt{\frac{100{,}000 \times 6.25 \times 0.000142}{6.6 \times 2.5 + 0.46}} = 2.29,$$

and $2.29 \times 50 = 114.50$ cu. ft. per sec. Ans.

It will often be required to find the dimensions of a flume to carry a given quantity of water under fixed conditions. The exact solution of this problem is difficult, since it leads to the solution of an equation of the sixth degree; by means of a system of trial and error, however, an approximate solution may easily be obtained that will give values within the practical limits required. The following illustrative example will make the method of operation clear:

It is required to compute the dimensions of a wooden flume to convey 250 cubic feet of water per second with a grade of $8\frac{1}{2}$ feet per mile, the width of the flume to be twice the depth of the water flowing through it.

Let $x =$ the depth of the water in the flume; then the width will be $2x$; the wetted perimeter, $4x$; the area of the water cross-section, $2x^2$; and the hydraulic radius, $2x^2 \div 4x = \frac{1}{2}x$.

The slope is $8.5 \div 5{,}280 = .0016$; and, since the discharge is to be 250 cubic feet per second, the mean velocity v must be $250 \div 2x^2 = \frac{125}{x^2}$.

Substituting the above terms in formula **226**, we have

$$\frac{125}{x^2} = \sqrt{\frac{100{,}000 \times \frac{x^2}{4} \times .0016}{6.6 \times \frac{x}{2} + .46}}.$$

Squaring, we have

$$\frac{15{,}625}{x^4} = \frac{100{,}000 \times \frac{x^2}{4} \times .0016}{6.6 \times \frac{x}{2} + .46},$$

from which $\qquad x^6 - 1{,}289\,x = 179.7.$

We will now assume values for x, and substitute in the above equation as follows: Assuming a depth of water of 5 feet for x, and substituting, we have for the value of the left-hand member of the equation,

$$5^6 - 1{,}289 \times 5 = 15{,}625 - 6{,}445 = 9{,}280,$$

which is much greater than the second member of the equation, and shows that our assumed value is too great.

Trying a value of $x = 4$, we have

$$4^6 - 1,289 \times 4 = 4,096 - 5,156 = -1,060,$$

which is less than the second member of the equation, but nearer to it than the value obtained when 5 was substituted.

Trying 4.2, we have

$$4.2^6 - 1,289 \times 4.2 = 5,489 - 5,413.8 = 75.2,$$

which is still less than the required quantity, but by trying 4.3, we get

$$4.3^6 - 1,289 \times 4.3 = 6,321.5 - 5,542.7 = 778.8,$$

which is too great. We therefore see that a depth of water of 4.25 ft. = 4′ 3″ will satisfy the required condition very nearly, giving a width of flume of 8′ 6″.

We will now verify the above dimensions to see if the flume will discharge the required amount of water.

The wetted perimeter is $2 \times 4\frac{1}{4} + 8\frac{1}{2} = 17$ feet, the area of the water cross-section is $8\frac{1}{2} \times 4\frac{1}{4} = 36.125$ square feet, and the hydraulic radius is $\frac{36.125}{17} = 2.125$. Substituting in formula **226,** we have

$$v = \sqrt{\frac{100,000 \times 2.125^2 \times .0016}{6.6 \times 2.125 + .46}} = 7.06 \text{ ft. per sec.},$$

therefore, the discharge will be $36.125 \times 7.06 = 255$ cu. ft. per second, which satisfies the conditions of the problem very well.

2197. Other Forms of Flume.—Besides the wooden flumes already described, there are other kinds, in which the wood is cut in the form of staves, and put together somewhat in the form of a semicircle. Other flumes are made of sheet iron or steel, some very large and of complicated construction. These will generally be set on iron or steel trestles, when it becomes necessary to cross depressions, and

their consideration involves the study of structural iron work. In calculating the delivery of such flumes, formula **226** may be used.

TRUSSES.

2198. Trussed Stringers.—When the opening to be spanned by a flume is of considerable width, it will be necessary to use some form of trussed structure. Timber stringers should not be used for spans of more than 12 or 14 feet without trussing, unless the loads to be carried are light or the expense of trussing is much greater in proportion than the cost of the extra sizes of timber that would be required for the longer spans. In such cases, stringers with spans of 16 and even 20 feet are sometimes used. It is generally very difficult to get good, sound timber for stringers in sizes greater than 12″ × 16″ and 32 feet long, and even these dimensions are seldom used, owing to the expense, the difficulty of transporting such pieces, and the uncertainty in regard to their strength.

A very simple form of trussed stringer is shown in Fig. 714, in which A is the stringer, strengthened by two tie-rods B, one on either side. These tie-rods pass through

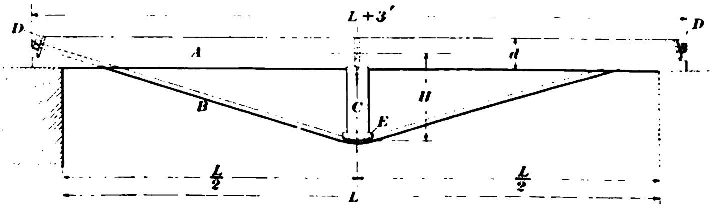


FIG. 714.

holes in the plates D that bear against the ends of the stringer; they also support a cast-iron saddle E, which forms a footing for the strut or king-post C. A shallow notch may be cut into the under side of the stringer to hold the upper end of the strut in place, or a drift bolt, as shown by the dotted lines, may be used for the same purpose. Owing to the fact that the bending moment on the stringer is greatest over the strut, it should be cut there as little as

possible; it is always bad practice to cut any timber subjected to stresses more than is absolutely necessary.

The plates D may be of either cast or wrought iron, while the saddle E is generally made of cast iron. The tie-rods are threaded and provided with nuts on each end; they may be made lighter if the ends for the threads are upset.

Trussed stringers of this type may be used for spans up to 28 or 30 feet, but in general it will be better to confine their use to spans not greater than 24 feet, unless the load to be carried is light.

For the usual application of this trussed stringer to carrying a flume, the load may be considered as being uniformly distributed; the formula for the relation between a total uniformly distributed load W_t and the dimensions of the stringer is

$$W_t = \frac{32\, b\, d^2\, H}{3\, L\,(2\, H + d)}\, S. \qquad \textbf{(227.)}$$

In this formula, b is the breadth of the stringer and d its depth; H is the vertical distance from the center line of the stringer to the lower end of the strut, and L is the span; all of which are in inches. S is the allowable unit stress in the stringer, and its value for different kinds of timber may be taken from the table given in Art. **2193.**

The relation between the total uniformly distributed load W_t and the total stress S_2 in the tie-rods is given by the formula

$$S_2 = \tfrac{5}{16}\frac{W_t}{H}\sqrt{\left(\frac{L}{2}\right)^2 + H^2}. \qquad \textbf{(228.)}$$

Having found the total stress by this formula, the *net* area of the rods may be found by dividing the total stress by 12,000 for wrought iron, or by 15,000 for steel.

Owing to the fact that the stringer forms a continuous beam, the compressive stress in the strut is equal to $\frac{5}{8}\, W_t$; hence, if h is the width of the strut and t its thickness, the relation between the total uniformly distributed load, the allowable unit stress, and the dimensions of the strut is given by the formula

$$W_t = \tfrac{8}{5}\, h\, t\, S. \qquad \textbf{(229.)}$$

The dimensions given by this relation will generally be less than the sizes that would be adopted in practice.

EXAMPLE.—Compute the dimensions of a yellow pine trussed stringer for a 20-foot clear span to carry the sluice shown in Figs. 712 and 713.

SOLUTION.—By referring to Fig. 712, we see that the total load due to the weight of the water in the sluice is $5 \times 10 \times 20 \times 62.5 = 62{,}500$ pounds. In Art. **2194** we found the weight of the timber in a section of the flume $4\frac{1}{2}$ feet long to be 1,185 pounds; consequently, the weight of timber to be carried by our 20-foot span is $\frac{1{,}185}{4\frac{1}{2}} \times 20 = 5{,}470$, say 5,500 pounds.

The total uniformly distributed weight is, therefore, $62{,}500 + 5{,}500 = 68{,}000$ pounds, or 34,000 pounds to be carried by each of the two stringers.

We will assume a depth H of the truss of 3 feet and a depth d of the stringer of 12 inches and compute the breadth b.

Substituting the known and assumed values in formula **227**, we have

$$34{,}000 = \frac{32 \times b \times 12^2 \times 36 \times 1{,}800}{3 \times 20 \times 12(2 \times 36 + 12)},$$

from which $b = 6.9$ inches.

We will use a stringer $8'' \times 12''$ and 22 feet long, thus providing a bearing on each abutment of 1 foot.

The total stress on the tie-rods is found by substituting in formula **228**,

$$S_2 = \tfrac{5}{16} \times \frac{34{,}000}{36}\sqrt{120^2 + 36^2} = 36{,}980 \text{ pounds.}$$

If we use wrought-iron tie-rods, the net area required will be $36{,}980 \div 12{,}000 = 3.08$ square inches, which gives a net area of 1.54 square inches for each rod. From a table of standard screw threads we find that the area at the bottom of the threads for a $1\frac{5}{8}$-inch screw is 1.515 square inches; hence, a $1\frac{5}{8}$-inch rod would barely suffice for this case; the next larger standard size is $1\frac{3}{4}$ inch, which it would probably be better to use.

From formula **229** the area $h \times t$ of the strut must be $h\,t = \frac{5}{8} \times \frac{34{,}000}{1{,}800} = 12$ square inches, nearly. A piece $4'' \times 3''$ would be sufficiently strong for this work, but for practical reasons it would be better to use a piece whose width is equal to the thickness of the stringer, 8 inches, and whose thickness is at least 6 inches. Ans.

2199. The **king-rod truss** shown in Fig. 715 may be used instead of the trussed stringer. It requires less iron work than the trussed stringer, which is an advantage in

many locations where timber is cheap and iron work expensive. Its form, however, makes it necessary to set the trusses far enough apart to allow the flume to pass between

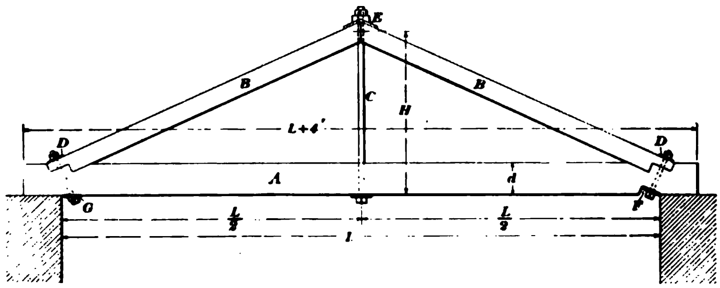


FIG. 715.

them, thus making the use of heavier floor-beams and more expensive connections necessary.

As will be seen by inspecting the two types, the king-rod truss is the same as the trussed stringer inverted; the intensities of the stresses in the similar members of each are nearly the same, and may be computed by the same formulas; they are, however, opposite in direction, the members that are in tension in one case being in compression in the other. Thus, in Fig. 715, the tie-beam A, which corresponds to the stringer A in Fig. 714, is in tension, while the struts or rafters B, corresponding to the tie-rods B of Fig. 714, are in compression.

The king-rod C and the bolts D should all be provided with large washers, so as to distribute the pressure of the heads and nuts well over the wood. A washer of the form shown at E is advisable for the joint at the upper ends of the struts. The bearing for the lower ends of the bolts D may be made by notching the tie-beam as shown at F; a better plan, however, is to use a special cast-iron washer, as shown at G, notching it into the tie-beam just enough to prevent it from slipping endways.

The dimensions of the tie-beam A may be computed by the use of formula **227**, using the same method as was used for the stringer A of Fig. 714. The total stress or

load W in each of the struts B may be calculated from the formula

$$W = \tfrac{5}{16}\frac{W_t}{H}\sqrt{\left(\frac{L}{2}\right)^2 + H^2}, \qquad \textbf{(230.)}$$

in which W_t is the total uniformly distributed load on the truss. The dimensions of the strut may then be computed by the use of one of the formulas for columns, **77** or **79**, Art. **1258**, Vol. II.

The net area A of the king-rod C may be calculated from the formula

$$A = \tfrac{5}{8}\frac{W_t}{S_s}, \qquad \textbf{(231.)}$$

in which W_t is the total uniformly distributed load on the truss and S_s the safe unit working stress for the metal of which the rod is composed.

The bolts D are not subjected to very heavy stresses, and are not calculated for strength; $\frac{3}{4}$-inch bolts will generally be used, although $\frac{7}{8}$-inch or even 1-inch bolts would be better in heavy trusses.

EXAMPLE.—Calculate the dimensions of the members of a pair of king-rod trusses for a 22-foot clear span to carry the flume shown in Figs. 712 and 713, using yellow pine timbers and steel king-rods.

SOLUTION.—The total weight of water to be carried is $5 \times 10 \times 22 \times 62.5 = 68,750$ pounds, and the weight of the timber in the flume is $\frac{1,185}{4\frac{1}{3}} \times 22 = 6,016$ pounds; the total uniformly distributed load, neglecting the weight of the trusses, is, therefore, $68,750 + 6,016 = 74,766$, say 75,000 pounds. This gives a uniformly distributed load on each truss of $W_t = 37,500$ pounds.

We will assume a depth of truss H of 6 feet and a depth of stringer $d = 14$ inches, and compute the value of the thickness b of the stringer. Substituting in formula **227**, we have

$$37,500 = \frac{32 \times b \times 14^2 \times 72 \times 1,800}{3 \times 22 \times 12(2 \times 72 + 14)},$$

from which $b = 5.77$ inches. A timber $6'' \times 14''$ would, therefore, be suitable if it was not necessary to cut it; since, however, it will be necessary to bore a hole through the center for the king-rod, thus weakening the stringer at the point of the greatest bending moment, we will use an $8'' \times 14''$ stringer.

By applying formula **230,** we find the total stress W in the struts to be

$$W = \tfrac{5}{16} \times \frac{37{,}500}{72}\sqrt{132^2 + 72^2} = 24{,}470 \text{ lb.}$$

The length of each strut is

$$l = \sqrt{132^2 + 72^2} = 150.3, \text{ say } 151 \text{ inches.}$$

We will assume a strut of rectangular cross-section, with a shorter side $d = 6$ in. (See formula **79** in Art. **1258** just referred to.) Taking the maximum allowable unit stress $\frac{S_2}{f}$ as 1,800 pounds per square inch, in accordance with the table in Art. **2193,** and substituting in formula **79,** we have

$$b = \frac{24{,}470\left(1 + \frac{12 \times 151^2}{6^2 \times 300}\right)}{6 \times 1{,}800} = 8 \text{ inches.}$$

Taking the allowable unit working stress of the steel king-rod as 15,000 pounds per square inch (see Art. **2198**), the net area must be

$$A = \tfrac{5}{8} \times \frac{37{,}500}{15{,}000} = 1.56 \text{ square inches.}$$

From a table of standard screw threads we find that the nearest standard size bolt that will give this area is $1\frac{3}{4}$ inches.

In the calculations just made we have neglected the weight of the truss, which would increase the total uniformly distributed load. Since, however, the dimensions were all taken in sizes larger than those called for by the calculations, there has been a margin of strength which will amply cover the increase in the stresses due to this increased load. If, in any case, it is uncertain whether this element has been covered, an estimate of the weight of the truss, as calculated, should be made, and this weight added to the weight of the water and flume, in order to get the total load from which to calculate the stresses.

2200. The Queen-Rod Truss.—For moderate loads and spans up to 35 or 40 feet, the **queen-rod truss** shown in Fig. 716 may be used. The general construction of this truss is similar to that of the king-rod truss described in the last article. The directions of the stresses are as follows: The tie-beam A and the queen-rods E are in tension; the

struts B and the top chord piece C are in compression; when the truss is uniformly loaded, the diagonals D carry no load; consequently, they are often omitted; when, however, the load is not uniformly distributed, as might be the case when a flume was being rapidly filled, the truss is liable to be distorted if it is not braced with the diagonals, which always act in compression.

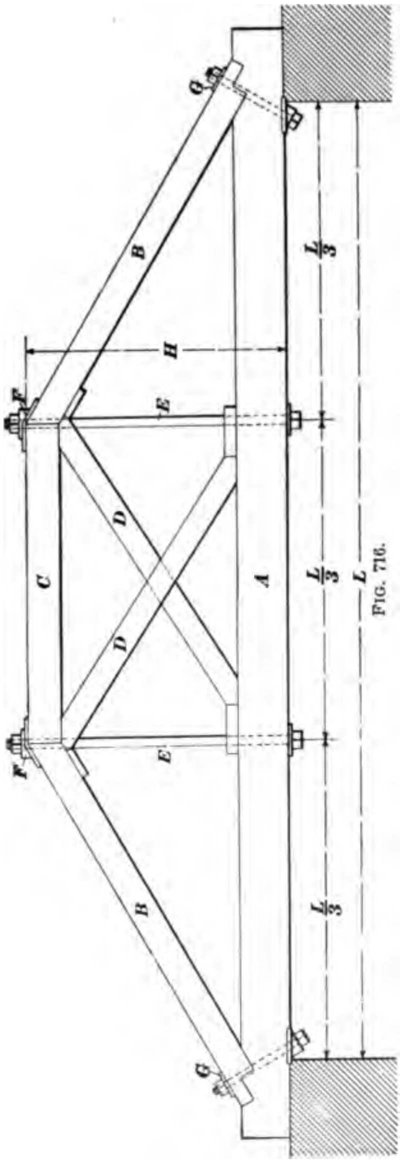


FIG. 716.

In the case of a queen-rod truss for carrying a flume, we will assume the total span to be divided into three equal parts. The load will be assumed to be uniformly distributed along the tie-beam, which is considered as forming three beams uniformly loaded; a third of the total uniformly distributed load is thus transmitted to each of the queen-rods, which transfer it through the struts to the foundation.

The maximum unit stress in the tie-beam under these conditions may be calculated from the formula

$$S_t = \frac{1}{4}\frac{WL}{bd}\left(\frac{1}{2H} + \frac{1}{3d}\right). \qquad \textbf{(232.)}$$

in which S_t is the maximum unit stress; W, the total uniformly distributed load; L, the length of the span; H, the depth of the span; b, the breadth of the tie-beam, and d its depth. All dimensions are in inches.

The total stress S_c in the upper chord member C is given by the formula

$$S_c = \tfrac{1}{8}\frac{WL}{H}. \qquad \textbf{(233.)}$$

Since this member is in compression, acting as a column with the load S_c, its dimensions are to be computed by one of the formulas for columns given in Art. **1258,** Vol. II.

The total stress S_s in the struts B is given by the formula

$$S_s = W\sqrt{\tfrac{1}{9} + \tfrac{1}{64}\frac{L^2}{H^2}}. \qquad \textbf{(234.)}$$

Since the struts also act as columns with the load S_s, their dimensions are to be calculated by one of the formulas for columns given in Art. **1258,** Vol. II.

The net area A of each of the queen-rods is given by the formula

$$A = \tfrac{1}{3}\frac{W}{S_q}. \qquad \textbf{(235.)}$$

Here S_q is the allowable unit stress for the material of which the rods are made; a safe value under a steady load is 12,000 pounds per square inch for wrought iron and 15,000 pounds per square inch for steel.

The diagonals D will not generally be calculated for strength.

The bolts G may be from $\frac{3}{4}$ inch to 1 inch in diameter, depending on the size of the timbers.

EXAMPLE.—Compute the dimensions of a pair of queen-rod trusses for a 36-foot span to carry the flume shown in Figs. 712 and 713, using yellow pine timbers and wrought-iron queen-rods.

SOLUTION.—The weight of the water to be carried is $10 \times 5 \times 36 \times 62.5 = 112{,}500$ pounds, and the weight of the timber in the flume is $\frac{1{,}185}{4\frac{1}{2}} \times 36 = 9{,}845$ pounds. The total weight of the water and flume is, therefore, $112{,}500 + 9{,}845 = 122{,}345$ pounds, or 61,173 pounds for

each truss. It will be better in this case to make an allowance in the total weight for the weight of a truss ; assuming the weight of each truss to be 6,827 pounds, the total uniformly distributed load on each truss will be 61,173 + 6,827 = 68,000 pounds.

We will assume the depth of tie-beam as 14 inches, and a depth of truss of 10 feet ; then, since the allowable unit working stress for yellow pine is 1,800 pounds per square inch, by substituting in formula **232**, we have

$$1{,}800 = \tfrac{1}{4} \times \frac{68{,}000 \times 36 \times 12}{b \times 14}\left(\frac{1}{2 \times 10 \times 12} + \frac{1}{3 \times 14}\right),$$

from which $b = 8.153$ inches.

By using a timber 10" × 14" we will have sufficient material to provide for holes for queen-rods.

From formula **233** we find the total stress in the upper chord member to be

$$S_c = \tfrac{1}{8} \times \frac{68{,}000 \times 36 \times 12}{10 \times 12} = 30{,}600 \text{ pounds.}$$

Assuming a depth d of 8 inches, and a value of $\frac{S_2}{f}$ of 1,800, and substituting in formula **79** just referred to, we have for the breadth of the member

$$b = \frac{30{,}600\left(1 + \dfrac{12 \times 144^2}{8^2 \times 3{,}000}\right)}{8 \times 1{,}800} = 4.879 \text{ inches.}$$

It will be best to use a piece at least 6" × 8" for this member.

The total stress in the struts is found by substituting the given data in formula **234**,

$$S_s = 68{,}000\sqrt{\tfrac{1}{9} + \tfrac{1}{64}\left(\frac{36 \times 12}{10 \times 12}\right)^2} = 38{,}060 \text{ pounds.}$$

The length of each strut is $l = \sqrt{10^2 + 12^2} = 15.62$ feet $= 187.44$, say 188 inches.

Assuming a depth d of 8 inches and a value of $\frac{S_2}{f} = 1{,}800$, and substituting in the formula **79** mentioned above, we have

$$b = \frac{38{,}060\left(1 + \dfrac{12 \times 188^2}{8^2 \times 3{,}000}\right)}{8 \times 1{,}800} = 8.48 \text{ inches.}$$

We will use a piece 8" × 10" for the struts, and, in order to facilitate the construction and reduce the number of sizes of timber required, it would, perhaps, be better to make the upper chord member 8" × 10" also.

The net area of each of the queen-rods, in accordance with formula **235**, must be

$$A = \tfrac{1}{3} \times \frac{68{,}000}{12{,}000} = 1.89 \text{ square inches.}$$

From a table of standard screw threads we find that this area requires a bolt 1¼ inches in diameter.

For the diagonals we will use pieces 4 in. × 6 in. Their approximate length will be $l = \sqrt{8^2 + 12^2} = 14.42$ feet, say 14 ft. 6 in.

We will now calculate the weights of the different materials in our truss, in order to compare them with the total weight assumed in computing the dimensions of the members.

The approximate quantities and weights are as follows:

TIMBER.

1 tie-beam, 10" × 14" × 40 ft. long	38.89 cu. ft.
1 top chord member, 8" × 10" × 12 ft. long	6.67 cu. ft.
2 struts, each 8" × 10" × 15⅔ ft. long	17.41 cu. ft.
2 diagonals, each 4" × 6" × 14½ ft. long	4.83 cu. ft.
	67.80 cu. ft.
67.8 cu. ft. @ 50 =	3,390.00 pounds.

IRON.

2 queen-rods, each 1¼ inches O × 11 ft. long	200.00 pounds.
Bolt heads, nuts, and washers, estimated	110.00 pounds.
Total	3,700.00 pounds.

We thus see that our assumed weight, 6,827 pounds, is more than sufficient, which gives our structure an added element of safety.

2201. Trusses Suitable for Longer Spans.—For still greater spans, other forms of truss will be necessary. At the present day such long-span trusses are commonly made of iron or steel, a full consideration of which would lead to problems in bridge construction. There are, however, combination trusses, composed of timber and iron, which are still used and which seem well adapted to locations where timber is cheap and iron comparatively expensive, as is often the case in canals for irrigation work. Of these combination trusses, the **Howe truss** is one of the best. It consists of a top and bottom chord, the load generally resting on the bottom chord. The web system, that is, the combination of rods and braces, connecting top and bottom chords, consists of vertical tie-rods and inclined struts. The top chord and the struts sustain a compressive stress, and the ties and bottom chord a tensile one. In a combination truss, the top chord and struts, and in some cases the bottom chord also, are made of wood; the bottom chord, however, is more frequently, and the ties are always, made of iron.

2202. Calculation of a Howe Truss to Carry a Uniform Load Only.—In calculating such trusses, the unit of stress is the panel load, that is, the weight carried by one panel length of the truss. Figs. 717 and 718 show in skeleton one-half of a Howe truss, composed of 10 panels.

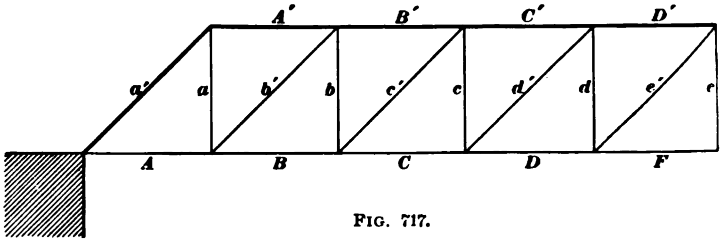


FIG. 717.

Commencing with the central tie e, Fig. 717, we see that it sustains one panel load, of which one-half goes to the left-hand abutment (the one shown in the figures) and the other to the right-hand one. The stress of this half panel load is transferred by the tie e to the strut e'. It is transferred by the strut e' to the foot of the tie d, which also sustains one panel load. The stress upon the tie d is, therefore, that due $1\frac{1}{2}$ panel loads. This stress is transferred to the strut d', by which it is passed along to the tie c, which is also carrying its own panel load, and sustains, therefore, $2\frac{1}{2}$ panel loads. The stresses are thus transferred from tie to tie, receiving an increment of one panel load at each, until they reach the abutment. This loading is represented in Fig. 718 by the rectangles supposed to be suspended from the ties, each being marked with the number of panel weights the stress of which it sustains.

Although the struts sustain the same *vertical* stresses as the ties suspended from their upper extremities, it is clear from what has already been explained that the actual stresses acting in them is greater than the stresses in the corresponding rods, on account of their obliquity, and that this increase in stress is greater, the greater the inclination from the vertical. In other words, and as has been already shown, it will be greater than the stress in the corresponding tie, in the proportion of the length of the strut to the length of the tie.

So far, only the ties and struts have been considered. It is clear that if these were the only members, the whole structure would immediately collapse. This is prevented by the action of the top and bottom chords, which keep the web members constantly in place. The stresses in these horizontal members are derived wholly from those in the oblique ones, namely, the struts ; for it is evident that the inclined web members are the only ones whose stresses have a horizontal component.

The stresses in the ties are transferred from the center to the abutment through the struts, the stresses in these members increasing in intensity from panel to panel, as shown in Fig. 718. From the abutment, these stresses are passed

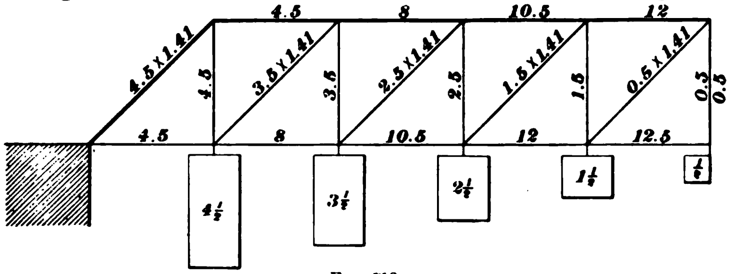


FIG. 718.

back again to the center, through the top and bottom chords, increasing in intensity from panel to panel in inverse order, thus completing and closing the cycle of stresses, as shown in Fig. 718.

In Figs. 717 and 718 the struts are inclined at an angle of 45°; consequently, as is shown in Fig. 719, the lengths of tie, chord, and strut are as 1, 1, and 1.41. The total stress, in pounds, upon any tie is obtained by multiplying one panel weight, in pounds, by the number of the tie from the center and adding one-half of a panel weight to the product. If we denote the number of the tie from the center by N_t, and the panel weight in pounds by P, the stress S_t in the tie will be given by the formula

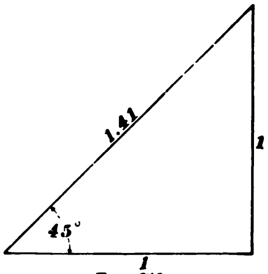


FIG. 719.

$$S_t = (N_t + \tfrac{1}{2}) P \qquad \textbf{(236.)}$$

The stress upon any strut is obtained by multiplying the stress upon the corresponding tie by the length of the strut divided by the length of the tie. Denoting the stress in the tie by S_t, as before, the length of tie by L_t, and the length of the strut by L_s, the stress S_s in the strut is given by the formula

$$S_s = \frac{L_s}{L_t} \times S_t. \qquad (237.)$$

If we let

N = number of panels in truss from center to either abutment ;
n = number of panels from a given panel to the nearer abutment ;
P = panel load in pounds ;
L_p = length of a panel ;
L_t = length of a tie ;
S_{tc} = stress in top chord member of given panel ;
S_{bc} = stress in bottom chord member of given panel ;

then we shall have

$$S_{tc} = n\,P(N - \tfrac{1}{2}n)\frac{L_p}{L_t}; \qquad (238.)$$

and

$$S_{bc} = P(Nn + N - n - \tfrac{1}{2}n^2 - \tfrac{1}{2})\frac{L_p}{L_t}. \qquad (239.)$$

EXAMPLE.—Let Figs. 717 and 718 represent one-half of a Howe truss, 100 ft. span and 10 ft. deep, divided into 10 equal panels; compute the stresses in each member of the truss when loaded with the flume shown in Figs. 712 and 713.

SOLUTION.—The weight of water to be carried is $10 \times 5 \times 100 \times 62.5 = 312{,}500$ pounds. The weight of the timber in the flume is $\frac{1{,}185}{4\frac{1}{3}} \times 100 = 27{,}346$ pounds. The total weight of water and flume is, therefore, $312{,}500 + 27{,}346 = 339{,}846$, say 360,000 pounds, making a weight of 180,000 pounds to be carried by each truss. The weight of the truss will be greater in proportion to its length than that of the queen-rod truss given in the last article; we will assume its weight to be 30,000 pounds, thus obtaining a uniformly distributed load on our truss of $180{,}000 + 30{,}000 = 210{,}000$ pounds.

Since there are 10 equal panels, one panel load P is $210{,}000 \div 10 = 21{,}000$ pounds.

Applying formula **236** to each of the ties, the stresses obtained are as follows:

Tie a, $S_t = (4 + \frac{1}{2}) P = 4\frac{1}{2} \times 21{,}000 = 94{,}500$ pounds.
Tie b, $S_t = (3 + \frac{1}{2}) P = 3\frac{1}{2} \times 21{,}000 = 73{,}500$ pounds.
Tie c, $S_t = (2 + \frac{1}{2}) P = 2\frac{1}{2} \times 21{,}000 = 52{,}500$ pounds.
Tie d, $S_t = (1 + \frac{1}{2}) P = 1\frac{1}{2} \times 21{,}000 = 31{,}500$ pounds.
Tie e, $S_t = \quad P = \quad 21{,}000$ pounds.

Since the panel lengths and the depth of the truss are all equal, the length of each strut is equal to the length of a tie multiplied by $\sqrt{2} = 1.41$ (see also Fig. 719); therefore, $\frac{L_s}{L_t}$ in formula **237** is equal to 1.41, and the stress in each of the struts is equal to the stress in its corresponding tie multiplied by 1.41. The results are as follows:

Strut a', $S_s = 1.41 \times 94{,}500 = 133{,}245$ pounds.
Strut b', $S_s = 1.41 \times 73{,}500 = 103{,}635$ pounds.
Strut c', $S_s = 1.41 \times 52{,}500 = 74{,}025$ pounds.
Strut d', $S_s = 1.41 \times 31{,}500 = 44{,}415$ pounds.
Strut e', $S_s = 1.41 \times \frac{21{,}000}{2} = 14{,}805$ pounds.

For the stresses in the different sections of the top chord, we apply formula **238**, noting that, since the panel lengths and the lengths of the ties are equal, $\frac{L_p}{L_t} = 1$. The results for the different sections are as follows:

Section A', $S_{tc} = 1 \times 21{,}000 \times 4\frac{1}{2} \times 1 = 94{,}500$ pounds.
Section B', $S_{tc} = 2 \times 21{,}000 \times 4 \times 1 = 168{,}000$ pounds.
Section C', $S_{tc} = 3 \times 21{,}000 \times 3\frac{1}{2} \times 1 = 220{,}500$ pounds.
Section D', $S_{tc} = 4 \times 21{,}000 \times 3 \times 1 = 252{,}000$ pounds.

Applying formula **239**, the stresses in the different sections of the bottom chord are:

Section A, $S_{bc} = 21{,}000 (5 - \frac{1}{2}) \times 1 = 94{,}500$ pounds.
Section B, $S_{bc} = 21{,}000 (5 + 5 - 1 - \frac{1}{2} - \frac{1}{2}) \times 1 = 168{,}000$ pounds.
Section C, $S_{bc} = 21{,}000 (10 + 5 - 2 - 2 - \frac{1}{2}) \times 1 = 220{,}500$ pounds.
Section D, $S_{bc} = 21{,}000 (15 + 5 - 3 - 4\frac{1}{2} - \frac{1}{2}) \times 1 = 252{,}000$ pounds.
Section E, $S_{bc} = 21{,}000 (20 + 5 - 4 - 8 - \frac{1}{2}) \times 1 = 262{,}500$ pounds.

The numbers given on each of the members of the skeleton half-truss shown in Fig. 718 are the numbers by which one panel load must be multiplied in order to get the stress in that member; they also show the relation between the stresses in the different parts of the structure for the particular form and style of loading under consideration.

It must be borne in mind that the method of calculation

given and illustrated above applies *only* to trusses carrying a steady uniform load. If the load is to be a variable one, as in the case of a highway or railroad bridge, the method of calculation is much more complicated, and requires a more comprehensive treatment than can be given to the subject in this connection.

2203. Dimensioning the Members of the Truss.—After computing the stresses in the various members, their dimensions may be calculated for the particular kinds of material to be used in accordance with the principles previously given. The stresses in the bottom chord and the ties are tensile; in the top chord and the struts the stresses are compressive; these members must, therefore, be designed according to the formulas for pillars.

The tie-rods are always made of iron or steel, and in many cases where the cost of iron or steel is not too great in comparison with that of wood, the bottom chord will preferably be made of one of the former materials. If the bottom chord is made of wood, a great deal of care must be exercised in making the joints and in splicing the different sections, so as to secure a strong connection between the various parts of the structure. The bearings for the ends of the struts must also be well fitted; special castings for this purpose are desirable if they can be had without too much expense; their use, however, is not absolutely essential.

Although the stresses in the members of the different panels vary greatly, it is not customary to use as many different sizes of timber as would be required to make each member of just sufficient strength to resist the stresses in it, owing to the extra expense of getting so many special sizes and of connecting them satisfactorily. For example, the top chord members are generally all made of the same size timbers as those required to resist the greatest stress in any one member.

The number of sizes of tie-rods may be reduced by making the rods of such a size that one or two of them will be just sufficient to carry one panel load; then by grouping them,

the load at any particular panel can be provided for by using as many of the rods at that point as are required. In this way the number of sizes of rods may be kept small, and at the same time the use of the material will be economical.

TRESTLES.

2204. For crossing very long depressions, trestles are as greatly employed in irrigation engineering as in railroad work. Indeed, they are more generally used in the former than the latter, because in railroading they are frequently replaced with earthen embankments, which are to be preferred, as being more permanent for carrying trains; whereas it will rarely be found expedient to carry an irrigation flume or even a pipe line upon an embankment, as the almost inevitable settling of the earth would seriously endanger the conduit.

There are two principal types of trestles which may be employed; viz., **pile trestles** and **framed trestles.** The former are mostly confined to moderate heights, and are, perhaps, less frequently used than the latter.

2205. Pile Trestles.—In driving piles to sustain loads, two points are to be considered: First, it is necessary to know what weight the pile can bear without sinking farther into the ground, and, second, what weight it can bear without crushing.

To determine the first point, there are many formulas given, all more or less empirical and approximate, as must naturally be the case in this class of problem. The factors contained in these formulas are: the weight of the hammer with which the piles are driven; the height of fall of the hammer; and the **refusal** of the pile, that is, the distance it sinks under the last blow, or the average for the last few blows. In some of the more refined of these formulas, the weight of the pile itself is taken into consideration. As these formulas are all approximate, it is clear that any great refinement of calculation is uncalled for; what is wanted is

some simple rule, founded upon experience, which will give safe results. The following is a good one, and is based upon the condition that the final set or "refusal" of the pile does not exceed one inch.

$$S = WH. \qquad \textbf{(240.)}$$

In this formula, S = the weight which the pile will safely bear without settlement; W = weight of hammer, in same unit as S, and H = height of fall of hammer in feet.

EXAMPLE.—If $W = 1$ ton and $H = 10$ ft., what load will the pile safely carry?

SOLUTION.—Substituting the data in the formula, we have

$$S = 1 \times 10 = 10 \text{ tons.}$$

If the weight of the hammer had been given in pounds, the value of S would be in pounds also.

2206. Resistance to Crushing.—Although the formulas for the resistance of piles to being driven take no account of the crushing strength of the material of which the pile is composed nor of its cross-sectional area, they are generally considered as giving safe results both as against farther penetration and resistance to crushing. It is supposed that if the head of the pile resists the battering of the pile driver, it will also resist the permanent pressure of the quiescent load, the intensity of which is indicated by the formulas. When a pile of small section is subjected to the blows of a heavy hammer, with a considerable fall, it is necessary to **ring** the head, that is, to shrink on an iron band or ring, which enormously increases its resistance to splitting and brooming. It is evident, also, that a heavy hammer and low fall are preferable to the reverse conditions of a high fall and light hammer, because they more nearly approach the action of a quiet, permanent load.

It will be well, however, to compare the value of S, as obtained above, with that derived from multiplying the area in square inches of the pile by its safe crushing strength, as given in the tables of the crushing strength of the timber of which the pile is composed.

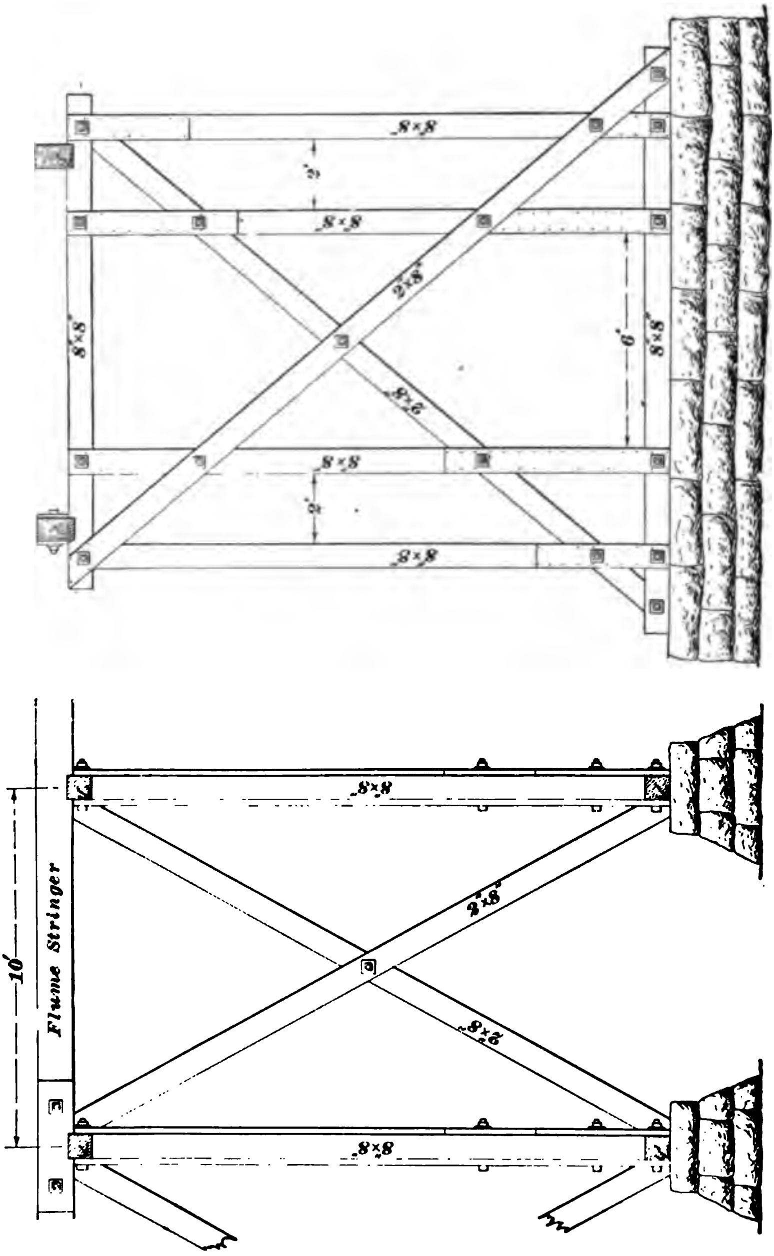


FIG. 720

EXAMPLE.—What is the resistance to crushing of a round spruce pile 6" in diameter?

SOLUTION.—The area of the head of the pile is $6 \times 6 \times 0.7854 = 28.27$ sq. inches. The resistance to crushing of spruce may be taken as 800 pounds per square inch; hence, the resistance of the pile to crushing $= 28.27 \times 800 = 22,616$ lb. Ans. If the pile had been driven as in the last example, its resistance, by formula **240**, would be $10 \times 2,000 = 20,000$ lb., which would be well within its resistance to crushing.

2207. Framed Trestles.—These trestles are framed so as to stand upon a sill, which should rest upon a proper foundation, either of piles or of masonry, and not directly upon the ground or on mud sills.

Many varieties of framing are used, but the guiding principle for trestles carrying a steady load, as is the case in irrigation work, should be to avoid inclined posts, mortising, and, as far as possible, different sizes of timber. Figs. 720 and 721 represent the general features of a good system of trestling for moderate heights. The stuff used is all either $8'' \times 8''$ or $8'' \times 2''$.

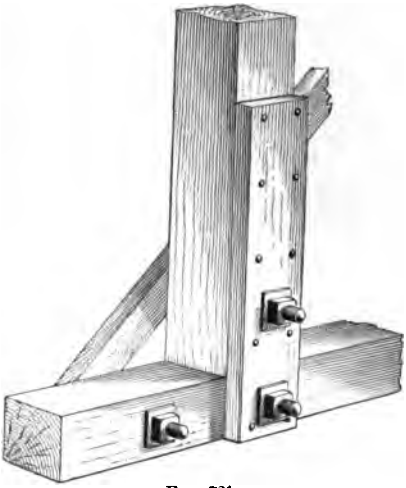


FIG. 721.

The $8'' \times 8''$ posts are set upon the sills, which are also $8'' \times 8''$, either merely resting on the top face, or notched in half an inch. They are held in place by **plaster plates,** of $8'' \times 2''$ stuff, bolted and spiked to posts and sills. Fig. 721 shows one of the posts and sills connected in this way, drawn to a larger scale and in isometrical projection. The caps are connected with the upper ends of the posts in the same way. The posts are steadied by means of the $8'' \times 2''$ **X** bracing, as shown in the right-hand view in Fig. 720. The two pieces of the bracing are bolted together at the center, against an $8'' \times 8''$ block set between them; they are also bolted and

spiked to posts, caps, and sills These connections can be more perfectly made by first spiking the pieces together in

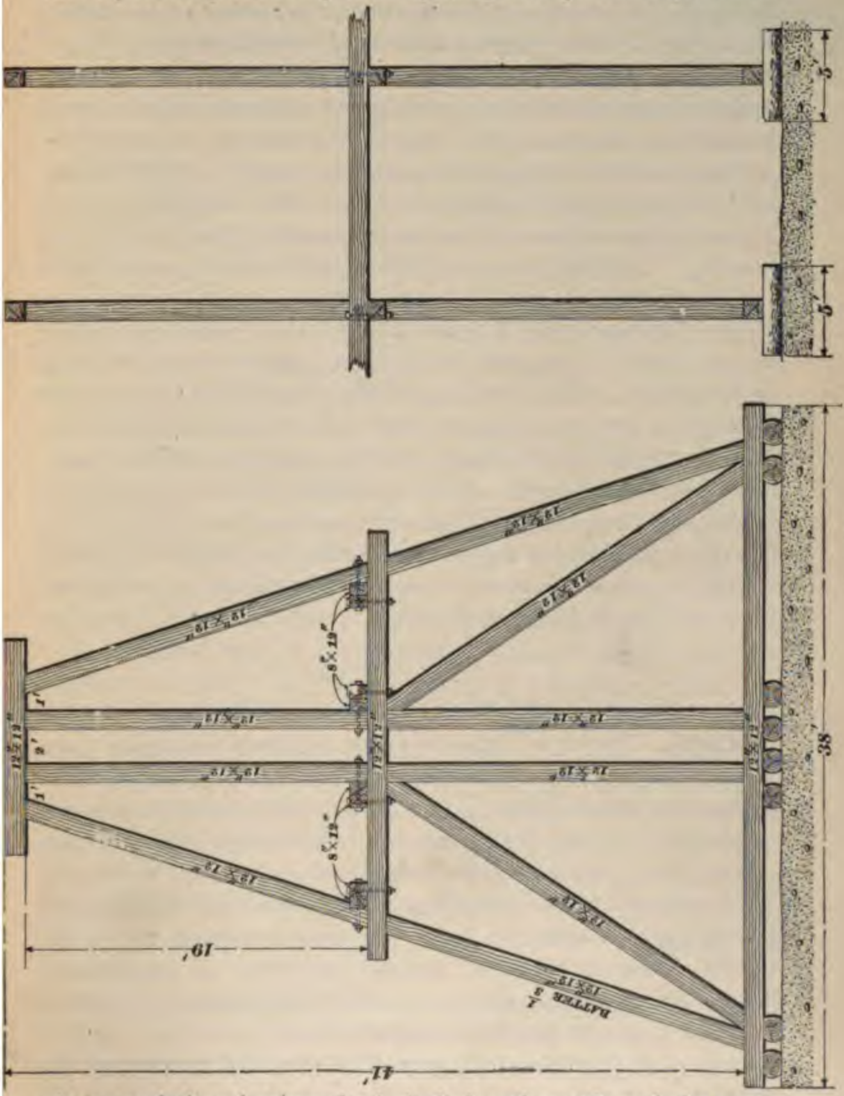


FIG. 722.

place, and then boring the bolt holes through and through, and passing the bolts. In an emergency all the connections

may be made with spikes. The flume stringers are notched over the sills, and are so disposed that joints shall occur over the caps. These joints are secured by plaster plates bolted and spiked. The trestle is stiffened longitudinally by 8″ × 2″ X bracing, as shown in the left-hand view in Fig. 720, butting under the flume stringers and against the sills, and secured laterally by plaster plates, chocks, mortising, or otherwise.

This trestle would be adequate to carry a 10′ × 5′ flume, such as has been considered in previous examples, and is about as light as would be perfectly satisfactory, under this loading. If it were more convenient to use heavier stuff, the bents could be spaced farther apart.

Fig. 720 represents a trestle suitable for moderate heights, say up to 20 ft. Beyond this height some other system must be employed. The building of very high trestles, whether of wood or iron, constitutes an interesting and complex subject, belonging to structural engineering, and can not properly be discussed here. As an example of trestles of medium height, however, Fig. 722 shows a system of framing for a trestle 41 ft. high, on the "Ohio Connecting Railway," which would also be applicable for carrying a flume. The dimensions are marked on the figure.

OVERFLOWS AND SLUICES.

2208. When a large body of water is conveyed in a flume or a canal there is always a certain degree of danger to be apprehended from a possible overflow, due to a sudden diminution of the draft without a corresponding reduction of the feed. This may be occasioned either by a shutting off of some of the outlets, or by some accidental obstruction occurring to arrest the flow. An overflow from any cause, unless directed through proper channels, is particularly dangerous in the case of an earthen canal, and precautions must be taken to render it impossible.

It is also necessary to provide means for emptying the canal at any time, without employing the ordinary outlets used for irrigation.

The first of these requirements must be provided for, in the case of earthen canals, by building **overflows** or **spill-ways** at certain intervals, depending upon the grade of the canal, each capable of safely discharging the maximum volume of water which can reach it, supposing the entrance gates to the canal to be wide open. Their proper dimensions may be determined by the considerations contained in Art. **2163.** These spillways will be constructed precisely as described for those used in dams, taking care that they shall be provided with substantial wing walls and aprons, to prevent scour and wash from the escaping water.

The appliances used for emptying the canal will consist of sliding sluice gates. These may be either somewhat elaborate in construction, such as metallic gates set in masonry chambers, already mentioned in the section on Water Supply and Distribution, or they may be comparatively rude, being constructed of timber. It will be observed that these gates may have the combined office of emptying the canal and preventing overflow, but it is not safe to depend upon them for the latter service, because, through negligence, they may fail to be opened at the proper time, whereas the action of an overflow or spillway is always automatic.

It will generally be found best to locate both of the above appliances at or near the crossing of a stream, or at least at some natural depression of the ground, in order to provide for an easy escape of the water.

In large canals carrying a heavy body of water, both overflows and sluice gates assume considerable proportions and call for great care and skill in their design and construction. For such structures, the plans of similar works should be carefully studied, and used as guides for any particular case. It would be impossible to give instances covering all conditions.

The most rudimentary form of sluice gate will be one inserted in a timber flume at the crossing of a stream. It will consist substantially of a sliding gate working in a groove between upright posts. It may be operated, if small, by a lever, otherwise by a rack and pinion. However simple the

contrivance may be, it should always be built with the greatest care and with the best materials at command. While there should always be a sufficiency of such gates, their number should not be unnecessarily multiplied, because there will always be some leak at each, occasioning a loss of water.

PIPES.

2209. The best way to convey water for any purpose is by means of pipes. A pipe can be laid anywhere, subject to the conditions established in the section on Water Supply and Distribution; it is less subject to disaster and to loss of water by leakage, seepage, and evaporation, and the tendency of modern practice will probably be to extend the use of pipes for the conveyance of water for irrigation, except when very large bodies are to be carried, when it will generally be impossible to avoid the use of canals having the dimensions of small rivers.

As regards pipes, it may be said that in point of strength, durability, facility of making connections, and generally of capacity of delivery, no other kind of pipe is equal to cast iron. Nothing further need be added to what has been already said about these pipes in the section on Water Supply and Distribution. Some further particulars, however, of lap-welded, riveted, and other varieties of sheet-iron or steel pipes will be given here.

2210. Lap-Welded Pipes.—These pipes are made currently up to 24 inches diameter, and come in sections from 15 to 20 feet long. A cast-iron joint is attached to each end, by means of which they are leaded and calked in about the same way as cast-iron pipes. Their cost f. o. b. is not much different from cast iron of average thickness and the same inside diameter; the greater economy is due to reduced freight, making transportation cheaper. The cost of laying is also less, on account of less weight, which makes handling cheaper. Their greater length reduces the number of joints, and consequently the amount of lead and labor used in running and calking them. The smoothness and

regularity of the interior surface must insure a large delivery, but actual experiments seem lacking to establish their coefficient of discharge as compared with that of cast iron. In the absence of experimental data, the usual formulas for rough cast-iron pipes had better be used to calculate their discharge. These pipes are at present generally made of steel plates instead of iron.

2211. Riveted Steel and Iron Pipe.—These pipes are formed by riveting iron or steel plates in much the same manner as boiler shells are made. They possess the advantage that they may be made of any desired diameter, and in inches and fractions. Cast-iron pipes are only made in regular sizes, so that frequently a much larger diameter than is needed must be used, because the next lowest size is too small. If cast to intermediate sizes, they fall into the category of "special castings," and are charged for accordingly. Riveted pipe may also be shipped in plates, and bent and riveted on the ground, which is sometimes a convenience and reduces the cost of shipping and handling, and enables the pipe to be put together in varying lengths. As good a job can never be done in this way, however, as when the plates are riveted in the shop, and it is probably impossible to secure as perfect a coating of tar or asphaltum in the field as can be had when applied in the shop to the completed pipe.

Riveted pipes are made with either projecting or countersunk rivet heads. The latter system improves the flow, which is otherwise considerably impeded by the obstructions offered by the heads. There are several ways in which the lengths of pipe are connected together, there being different kinds of special contrivances for this purpose. They are sometimes connected by taper joints, the external diameter of one end of each length being the same as the internal diameter of the other, so that one can enter the other and the two be riveted together.

2212. Spiral Riveted and Lap-Welded Pipe.—These pipes are used to a considerable extent for irrigation

purposes, and appear to be serviceable, though probably not equal to the varieties previously described.

2213. Corrosion and Capacity of Discharge.—There seems to be some doubt as to the resistance to corrosion of riveted or welded plate pipes when laid in damp and acid soils, owing partly to their thinness of metal, which allows but a very small margin for loss by rust and corrosion, and also to the undeniable fact that in certain soils wrought iron does not resist these agencies as well as cast iron. It is indispensable that they should always be thoroughly coated.

As regards their capacities for delivery, this point has already been treated in the section on Water Supply and Distribution. It seems certain that the delivery is less for riveted pipe, particularly when laid with taper joints, than for cast iron.

2214. Joints.—The many particulars concerning methods of joining riveted and lap-welded pipes together and all other peculiarities are fully treated in the catalogues of the various manufacturers who control patents, and these constitute the best sources for this kind of information.

2215. Wooden Stave Pipes.—"For low pressures and large diameters, wooden stave pipe are to be recommended. . . . . The walls of the pipe are formed of longitudinal staves braced together with iron or steel bands. These are shaped to cylindrical forms and on the edges to true radial lines, so that when put together they form a perfectly cylindrical pipe. The flat edges of the staves are essential to enable the empty pipe to resist the pressure from the overlying earth. To join the ends of the staves a thin metallic tongue is inserted, which, being a trifle longer than the width of the stave, cuts into the adjoining ones. This joint is very tight and easy to make. The confining bands are of round or flat iron, or steel, of from three-eighths to three-fourths inch in diameter. As shipped from the factory they are straight, and provided on one end with a square head and on the other with a thread and nut. They

are bent on the ground on a bending table and coated with mineral paint or asphalt varnish, and are cut about six inches longer than the outside circumference of the pipe, on which they are slipped loose. The ends are joined by means of a closed iron screw, which fits close upon the pipe and provides a shoulder for the head and nut. These bands are placed at varying distances apart, according to the pressure which the pipe is required to bear. The staves break joints so as to form a continuous pipe, which leaves no obstruction to the flow of water. The beauty of the system is that it is made on the ground, and the workmen do not have to be especially experienced.

"It is always economy to purchase the staves already dressed, and thereby save in freight charges. In contracting for such materials, the specifications should call for sound, well-seasoned, close and straight-grained lumber, free from all knots, worm holes, season checks, sap wood, splints, or other like defects, and cut from live trees. In piping ranging in diameter from eighteen inches to three or four feet, the staves are usually prepared from carefully selected 2 × 6 joists, and this joist when dressed will make a stave about five and five-eighths inches along its outer arc, and about one and nine-sixteenths inches thick." (Wilcox.)

These pipes no doubt make good and cheap substitutes for metallic pipe, when subject to low pressures only, say less than 100 lb. per sq. in., but can never partake of the permanent character of the latter. They are still somewhat at the experimental stage, and considerable divergence of opinion exists among those who have had to do with them respecting certain important points, such as whether they last better when buried in the ground or when exposed to the air on the surface.

In the absence of fuller experimental data, the flow through these pipes should be calculated by the formula for rough cast-iron pipes.

2216. Tunnels.—It frequently becomes necessary, in order to avoid long detours, to carry the line of conduit

through a tunnel. If the conduit is an open canal, the tunnel will merely form an opening in the hill, affording a passage for the canal; if the conduit is a pipe line, running under pressure, then the tunnel will generally form a continuation of the pipe, and will be entirely filled with water, running also under pressure. In either case, experience has abundantly shown that, except in rare cases, the tunnel should be lined throughout, even when it runs through rock. In modern tunneling, large quantities of high explosives are used, which greatly shatter the surrounding rock, so that fragments are continually coming away, particularly from the roof, when the tunnel is not lined, greatly interfering with the flow of water through it. Tunnels driven through earth must, of course, be thoroughly secured by lining. Such tunnels are frequently secured by timbering only, without, however, being always satisfactory. They have also, in the case of large conduits, been lined with wooden staves, as already described, the iron bands being replaced by concrete, closely packed between the outside of the staves and the walls of the tunnel.

GROUND WATER.

SOURCES OF GROUND WATER.

2217. It has already been shown that a portion of the water which falls upon the surface of the earth in the form of rain runs off rapidly to the rivers and streams, and is conveyed by them ultimately to the sea or ocean, while another portion sinks into the ground, and although this also is constantly seeking outlets and lower levels, by virtue of the law of gravitation, yet it moves so slowly, owing to the resistance to percolation offered by the media through which it passes, that at any given moment it may be considered as stationary.

During past ages the water, constantly falling upon the earth's surface and slowly sinking through deeper and deeper strata, has finally accomplished such a degree of saturation of the earth's crust that in almost all districts a permanent

"water table" has been established at a greater or less depth below the surface, and not varying very materially with the seasons. Even very arid districts, almost or wholly deprived of rainfall, may yet possess a store of underground water, which has slowly reached them from distant and more favored regions. We may thus consider the earth as forming a vast storage reservoir, in which the rainfall of many ages has been impounded. Should this supply be suddenly exhausted—were such a thing possible—it would doubtless require many centuries of subsequent rainfall to resaturate the earth's crust to the same degree that obtains at present.

The wells by means of which this subterranean deposit may be reached belong to one of two classes, shallow and deep wells, and the latter will be again divided into flowing or artesian wells, in which the water, although struck at a great depth, rises to the surface, and even spouts above it, and those from which it has to be lifted by pumping or other mechanical means. These latter differ from shallow wells merely or principally by the greater depth, while the artesian or spouting wells differ not only in depth, but in character.

WELLS.

2218. Shallow Wells.—These wells are what are commonly called "dug wells," that is, they are vertical excavations dug out by men who work inside of them, removing the material by pick and shovel or blasting, the excavation being then walled up or secured in some way to prevent caving in. Sometimes the excavation, or a part of it, is through a permeable rock, which, while admitting the passage of water, is yet sufficiently solid not to need any protection for the sides.

Shallow wells play a comparatively unimportant part in irrigation operations, owing to the small quantities of water which they yield, and do not call for any lengthy description.

DEEP WELLS.

2219. Deep wells are put down entirely from the surface, either by drilling or driving, their diameter being too small to permit of working inside of them.

2220. Driven wells consist of a series of lengths of piping, the lower extremity being armed with a pointed, perforated shoe, and are driven down either by blows from a mallet when relatively shallow and of small diameter, or by those from some sort of pile-driving apparatus when of larger proportions. These successive lengths are screwed or coupled together as they go down, and are sometimes carried to very considerable depths, depending upon the character of the strata which they pass through. Evidently they require earth or very soft rock to enable them to penetrate to the desired depth, and are stopped in their descent by hard rock. These wells, like the shallow ones just mentioned, are useful only for small, private irrigation operations.

2221. Drilled Wells.—The typical deep well, whether of the artesian class or not, is a perforation made by drilling, and generally cased with an iron or steel tubing. This tubing will not generally descend by itself, but must be urged downwards by pressure or shock, resembling in that respect the simple driven well, with the exception that a way has previously been made for it by drilling.

For the casing tubes, various kinds of pipe are used, the same as for water-conveying. Lap and butt welded and riveted pipe, plain and spiral, are employed, but it may be stated that the standard is the iron or steel plain lap-welded pipe. When the casing goes down easily, comparatively light weights of pipe are employed, but if much resistance is anticipated, heavier weights, known as **drive pipe,** are needed.

It is found that diameter has but little effect upon the delivery of these wells, their yield depending rather upon their depth. It is not best, however, to use diameters less than 4 inches, and the ordinary range runs between 4 and 8

inches. The ordinary practice is to commence with a larger bore than it is proposed to use for the permanent casing, and reduce the size progressively. In districts where no bores have been already put down, the probable depth, nature of the strata, etc., are unknown, and the first bores are, consequently, more tentative than those in a well-explored region.

2222. Drilling.—In boring a well, it is necessary to determine the diameter of the permanent bore, and to estimate the probable depth at which water will be struck. Suppose a 4-inch bore were required, and the depth estimated at 1,000 ft., and that it was thought likely that three sizes of casing would be necessary, including the final one of 4 inches. Then the drilling would be commenced with a bore suitable for the reception of an 8-in. casing. After carrying this down 300 ft., it might become apparent that a smaller pipe could be used, and a 6-inch casing might be substituted. This might, perhaps, be carried down 400 ft. farther, making a 6-inch lining 700 ft. long, the upper part contained within the 8-inch casing. From this point it might be judged that the 4-inch casing could be used, which would then be bored for and inserted and driven down the remaining 300 ft. If it could be done conveniently, the two upper sections of 6 and 8 inch pipe would now be withdrawn, leaving a continuous 4-inch pipe extending from the surface to the bottom. Other methods of sinking and casing are sometimes used, but the above describes what is probably the prevalent practice.

The yield of these deep wells is sometimes greatly increased by exploding torpedoes at the bottom, by which means the rock is opened by fissures, allowing a freer passage for the water to reach the well. This operation requires considerable skill, and should only be attempted under the direction of an expert.

2223. Artesian Wells.—These comprise the flowing or spouting class. This peculiar result can only be secured

when there is a permeable, water-bearing stratum, lying on a slope, between two impermeable strata. The water-bearing formation is then very much in the condition of a pipe through which water is running under pressure, and the well driven into it becomes a piezometric tube within which the water rises to a greater or less height, according to its depth and the pressure at its foot. If this height is greater than the distance from the surface of the ground to the water stratum, or, in other words, greater than the depth of the well, the water spouts forth. It is unnecessary to dwell upon the great advantage which these flowing wells offer over those in which the water rises only to within a certain distance below the surface of the ground, to which it must then be raised by means of pumps or some other lifting arrangement.

The yield of artesian wells varies greatly in different localities. Perhaps the maximum recorded delivery of a single well is a continuous flow of over 6.5 cubic feet per second, in South Dakota.

RESERVOIRS FOR WELLS.

2224. It is seldom that the daily yield of a well is sufficient for the requirements of the seasons during which irrigation is practised. It is generally necessary to provide a reservoir capable of containing a certain reserve which can be drawn upon as wanted. If possible, such a reservoir will be formed by building a dam across the valley of some stream, as already described, by which a combination may be effected, the dam catching whatever flow may come down the stream in the rainy season, as well as the supply pumped up from the well. In the case of flowing wells, the reservoir is sometimes formed around the well, which feeds it as a fountain does its basin. If a separate reservoir is required, situated, perhaps, upon a hill, in order to get the necessary elevation to distribute the water from, the structure will be built according to the principles already laid down in reference to distributing reservoirs.

PUMPING FROM WELLS.

2225. When it becomes necessary to raise the water from a deep well to the surface, this will ordinarily be accomplished by the use of a lift or suction pump. If it is necessary to raise it still higher—into a reservoir situated upon high ground, for instance—a force main will be added, which will raise the water from the surface to the desired elevation.

When the water from only a single well has to be thus pumped up, the problem offers no difficulty ; when, however, a gang of wells are to be pumped, the problem is rendered more complicated. If the water rises to near the surface, say within 20 feet, and is not subject to fluctuations materially increasing this depth, then all the wells may be connected with one general suction pipe operated by a single pump. Otherwise a central station will be required, furnishing power for as many lifting pumps as there are wells, which will deliver their water to a common suction main operated as before. There are at the present day several **air lifts,** by means of which water is raised from a deep well by blowing air into it, which are said to have given good results. A careful study would be necessary before designing a plant to operate a gang of deep wells from which the water had to be lifted otherwise than by suction.

2226. Motive Power.—The power by which the pumps will be actuated depends upon circumstances. Whenever a constant water power can be obtained, this form of power will always be preferred, on account of its great economy. The principal objection to water power is its liability to fluctuations.

For small operations, windmills have been used to good advantage, but they can not be depended upon for large and important works. A reservoir will always be needed when windmills are used, to guard against interruptions due to calms.

Steam will be found, generally speaking, the most

trustworthy source of power, and in some cases the engines and boilers may be advantageously located near to the point where fuel is most cheaply procurable, and the power may be transmitted by electricity to the pumping station. In these cases it becomes a question between the cost of copper and iron, and that of transportation of fuel.

2227. High-Duty Engines.—When large volumes of water are to be raised by pumping, necessitating the development of a great many horsepower, the question of the relative economy of the production of this power becomes a most important one. In the case of the steam engine, the power utilized is *heat*, and this heat is furnished by the combustion of coal or other fuel. The point is to produce the maximum effect with the minimum consumption of fuel. To accomplish this, the utmost refinement of the heat apparatus is necessary ; boilers and engines must embrace every feature which science and experience have shown to conduce to the desired end, and at the same time must be as simple, light, and strong as possible. The outcome of all these considerations is the modern high-duty pumping engine. Not to go into elaborate details, it may be broadly stated that a high-duty plant is one capable of developing one indicated horsepower by the combustion of two pounds or less of good coal per hour. To accomplish this a combination of the best type of boiler with a triple-expansion engine is necessary. A good type of single-cylinder engine will require about double the steam, and consequently coal, that the triple or quadruple engine needs, and this proportion—two to one—represents about the difference that there is between a *good* plant and the *best*.

Moreover, to secure the best efficiency from the high-duty plant, it must be run uniformly at its maximum rate of work, any variation in the normal rate of running resulting in reduced economy.

Naturally, all this perfection costs money, the difference of cost between the two above-named types being in about the same proportion as their relative efficiencies, namely,

two to one, besides the greater expense of operating and maintenance. A close calculation is therefore often necessary to decide which type to select for any given case, or whether one lying between the two above extremes will be more economical.

Generally speaking, when there is a large amount of work to be done which does not vary greatly from day to day, and when coal is dear and facilities for keeping a complicated plant in order are good, an expensive high-duty boiler and engine may be used to advantage. On the other hand, when coal is cheap and abundant, and a constantly varying duty is required, a less expensive and simpler plant will be productive of the best economy.

2228. Different Classes of Engines.—The two prevalent types of pumping engines are the direct-acting, with or without high-duty attachment, and the fly-wheel engine, both of which types have been described in the section on Hydraulic Machinery. In point of efficiency, it would, perhaps, be difficult to decide as to their respective merits. In other respects, the direct-acting engine is simpler, smaller, and lighter for a given power, while the fly-wheel engine is more certain to complete its stroke at every revolution.

2229. Boilers.—Present practice seems to favor the water-tube form of boiler, which, while more complicated and expensive than other types, is smaller and safer. A good boiler should evaporate at least 10 lb. of water "from and at 212°," by the hourly combination of one pound of good coal.

2230. Use of Special Types of Pumps.—Besides the engines already spoken of, centrifugal pumps may be used in many places to good advantage where water is to be raised to moderate heights. Pulsometers are also much employed, but while they are probably the most simple devices for raising water by the consumption of steam, they are very wasteful of fuel, and their use is not to be recommended unless a very cheap and simple plant is required.

THE APPLICATION OF WATER TO THE GROUND.

SYSTEMS OF IRRIGATION.

2231. General Observations.—So far only the collecting, storing, and transportation of water for irrigation have been considered. All these processes are merely preliminary to the great object of getting the water upon the land for the purpose of producing crops. This, while the most important operation, being that up to which all the rest of the work has led, is in many respects the most complex. The problem is: given a certain area of land, and a certain volume of water with which to irrigate it, how shall this water be evenly spread over the ground, so that every portion may receive a sufficient but not excessive degree of moisture, and that no water shall be wasted?

There are many different methods of applying the water; some of the principal ones will be now described.

2232. Irrigation by Sprinkling.—This is a method with which, when practised upon a small scale, all are familiar. When a flower bed is to be watered, it is sprinkled by means of a watering pot. If a larger space is to be operated upon, a hose with perforated nozzle, or "rose," is used, or a watering cart may be employed, delivering water in the form of spray. In fact, since all these operations are carried on to supply a deficiency of rainfall, the instinctive thought seems to be to imitate the natural fall of water as nearly as possible.

There can be no doubt that sprinkling is the very best way in which water can be applied to the soil. It fulfils all the requirements of uniform distribution, moderate and easily regulated amount, and, in consequence, allows of a gradual absorption of the moisture without supersaturation of the soil and the presence of exposed surfaces of unabsorbed water, which must pass off by evaporation, thus adding to the ill effects of alkali, already mentioned. It can

also be applied to rough and uneven land, without the neces sity of any previous grading.

The objection to this method is the difficulty of applying it on the very large scale which is sometimes needed. It is probable, however, that by a careful and judicious system of piping it could be made more available than it is at present. It is already largely used in Florida, and is thus described by Mr. George W. Adams, of Thonotosassa: "I have a 25-horsepower horizontal boiler and a 12″ × 7″ × 10″ duplex pump, with 6-inch main pipe and 3-inch laterals at the main, and running down to one and a half at extreme ends. My trees are 21 feet apart each way. I have a hydrant in the center of every 16 trees. I use the McGowan automatic sprinklers, connecting the sprinkler with hydrants by a one-inch wire-wound rubber hose 50 ft. long. I use twelve of the sprinklers at one time, and could use more just as well, each sprinkler staying in place 30 minutes, each one covering a space of from 50 to 70 feet, according to the amount of pressure given them, and discharging about 1,000 gallons. By this process I have a genuine rain, either a light one or a powerful one, at pleasure. If I wish to throw water over the tops of the trees, I use the nozzle instead of sprinkler. I run the pump from 7 A.M. to 6 P.M. without stopping, using less than one-half cord of wood in eleven hours. I find no bad results from applying the water in the hottest sunshine, but would if I applied it through an open hose. I think the sprinkler method of applying water requires less help than any other I have seen, and is without any danger to fruit or trees. The firemen can manage the sprinklers within reasonable distance of the pumping station. For other portions, only one man is ever needed, and it is light work for him."

In this method it will generally be necessary to use a pump, either directly forcing the water through a hose or else raising it into an elevated tank, from which it may be drawn as wanted.

2233. Irrigation by Flooding.—Next to sprinkling, this is the method of applying water to the land which most

naturally suggests itself. It consists essentially in spreading the water in a thin sheet over the area to be irrigated, and this may be accomplished in several different ways, which are modifications of the general idea. The first feature of this method is a ditch or canal running along the upper border or highest level of the field to be irrigated. This may be either the main conduit itself or a subsidiary ditch fed from it. It will run, with a slight fall, following the highest level, or contour line. When it is desired to irrigate the land from this ditch, one of two ways will be adopted. Small temporary obstructions, such as a few shovelfuls of earth, may be placed in the ditch, causing it to overflow at the desired points in a thin sheet. Or a break may be made in the ditch, allowing water to escape and spread over a certain portion of the field, which break will then be closed, and a new one opened, a little in advance of the first, which will irrigate an adjacent portion, and so on, opening and closing breaks with the shovel, until water has been spread with more or less regularity over the whole field. If a flume or pipe line is used instead of a ditch, sluices, permanent sliding gates, or hydrants must be placed at convenient points.

This system presupposes that the land lies on a gentle and regular slope, such as characterizes many portions of the Western country. Almost invariably, however, some preparatory work must be done in leveling and grading the land before this method can be employed.

The flooding system of irrigation is largely used in the cultivation of alfalfa and grass crops. It is the simplest and cheapest method, but it is certainly very wasteful of water, and fulfils very imperfectly the requirements of a satisfactory irrigation. It distributes the water with great irregularity, overwatering some portions and scarcely moistening others. Its imperfections may be to some extent neutralized by carefully watching and guiding the progress of the water as it comes from the ditch, causing it to deviate here and there from its natural course, by a judicious use of the hoe and shovel.

There are also several modifications of the general system

better adapting it to the varying topography of the ground. When a comparatively steep hillside is treated in this way, it will be best to run a series of ditches across the slope, dividing it into belts or zones, rather than depend upon a single ditch to irrigate the whole slope. In this case, the unabsorbed water which flows over the first belt will be caught in the ditch next below it, and passed on to the next belt, and so on.

If the ground is very level, the difficulty which then presents itself is the too rapid absorption of water in the vicinity of the outlet of the ditch before it can reach the farther limit of the field. In this case the check system will be a useful modification.

2234. In the **check system** of irrigation a series of small ridges or checks are run across the slope, parallel or nearly so to the ditch, dividing the ground into a series of belts, in the same way as is done by the subsidiary ditches just described for hillside work. The first belt is flooded, and the water allowed to stand upon it until the ground has become sufficiently saturated. It is then drawn off by means of a break made in the ridge, and allowed to flood the next zone below, and so on.

2235. Checker-Board System. This is a modification of the above, and is suited to particularly level land. It consists in crossing the checks above described by others, more or less nearly at right angles to them, by which the whole territory to be flooded is divided into compartments. These are flooded, one or more at a time, and the water which is not absorbed [illegible]

2236. Method by Furrows. [illegible] sidered a very [illegible] generally, as [illegible] run from the [illegible] as to insure a [illegible] enter these [illegible] irrigated [illegible] and [illegible]

other systems, the course of the water should be watched and guided as far as possible, and irrigation should be followed by cultivation, or stirring and working the soil, using great judgment, however, regarding the time when this cultivation is practised.

When orchards are irrigated in this manner, the furrows are run among and around the trees. If the land is laid out previous to setting out the trees, then the furrows will be established first, according to the most advantageous manner of suiting them to the topography of the field, and the trees planted in conformity to the position of the furrows.

The ends of the furrows will be connected, so that water may circulate in all directions, and it is then guided by opening or closing the furrows with the shovel. Obviously, the ground must be tolerably regular in order to permit of the use of this method.

2237. Subsoil Irrigation.—This system consists in running a series of pipes, generally from a foot to eighteen inches below the surface, very much in the manner of drain pipes. Water may be admitted into these pipes at the upper end, the lower end being closed temporarily, and allowed to escape either through perforations in the pipe or through their loose joints, if common drain pipe are used. After proper saturation, the lower ends of the pipes are opened, and they then form a drainage system for the removal of superfluous or non-absorbed water.

Or the pipes may be made tight, with openings and vertical pipes at certain intervals, from which water may flow and spread over the ground in the vicinity. This, however, constitutes rather a modification of the flooding system.

Although certain practical difficulties have been encountered in the application of this method, it gives promise of being one of the very best that can be devised. It must necessarily be that which is most economical of water, and when used as first described, in the manner of drain pipes, allows the water to be drawn up by capillary action instead of sinking down by the action of gravity, loss by evaporation being reduced to a minimum.

2238. Other Methods of Irrigating.—While there are many other systems practised in different parts of this country and elsewhere, they will be found on examination to consist of modifications of those already described, which may be considered, therefore, as typical ones. Some further information regarding this subject will be given under the head of sewage irrigation.

MEASUREMENT OF WATER.

2239. When land is irrigated by private enterprise, that is, when an individual agriculturist establishes works for watering his own land, no measurement of the amount of water which he uses is necessary, except for his own information, and to aid him in its intelligent and economical use. When, however, the water is furnished by an irrigation company who sell their supply to different parties, it is clear that some system must be adopted by which the amount furnished to each may be measured and paid for. The first necessity towards this end is the establishment of the unit of measurement.

2240. Units of Measurement.—The best unit, when English measures are used, is the cubic foot. Sometimes this unit is considered too small for the convenient expression of large bodies of water, and the acre foot is then used. This merely means that volume of water which would cover one acre, or 43,560 square feet, 1 foot in depth, and the term *acre foot* is, therefore, only another way of saying 43,560 cubic feet.

Either of these terms—cubic foot or acre foot—expresses only a certain volume of water, without indicating the rate at which it is supplied. To express this rate, the best unit is the *cubic foot per second.* From the fancied necessity of some abbreviation, this is occasionally contracted to second foot. The use of either of the units—acre foot or second foot—is to be discouraged, as there is generally more time lost in explaining their signification than is gained by their use.

Besides the above standard units, there is another very

confusing and uncertain one called the **miner's inch** (see Art. **1014,** Vol. I), which is probably derived from an old Italian unit, called the "oncia."

The miner's inch has been defined as that quantity of water which will flow in one second of time through an aperture one inch square, cut in the bottom of a wooden box or trough, the said bottom being composed of a plank two inches thick, a constant depth of six inches of water being maintained in the box or trough. This quantity is supposed to be equal to 0.0259337 cubic foot per second.

The miner's inch is not, however, a fixed quantity, as it varies in different localities. It is stated that a cubic foot per second is equal to 50 California miner's inches and to 38.4 Colorado inches. According to this standard, the California miner's inch equals 0.020 cu. ft. per second, and the Colorado one just about 0.026. The sooner all these confusing units are discarded in favor of the cubic foot per second, the better.

2241. Methods of Measuring Water.—Several contrivances, more or less ingenious and efficient, have been devised for the purpose of readily determining the amount of water furnished to consumers, but practically weir measurements, as already described, are the best. In order to be practically useful, so as not only to *measure* the water furnished, but also to *deliver* a certain specified quantity, they should be so arranged that a weir of given length shall discharge under a constant head. If this can be done, the quantity furnished will be a fixed amount.

Among the appliances proposed for the delivery of a specified quantity of water in a given time, one of the best is that invented by Mr. A. D. Foote, of Idaho, and shown in its general features, without dimensions, in Figs. 723 and 724.

In (*a*), Fig. 723, is represented a cross-section through the main canal or flume *C*, from which it is desired to draw a certain measured volume of water to feed the ditch *D* by means of a slot *S* in the box *B*. In order to effect this, the sliding

gate G is closed a greater or less extent, so as to impede the free flow of water in the canal C, and force a portion of it into the box B, through the small sliding gate g, which is partially opened for the purpose. After a few trials with the two sliding gates, their openings will be so adjusted that the proper level of the water in the box is maintained, the

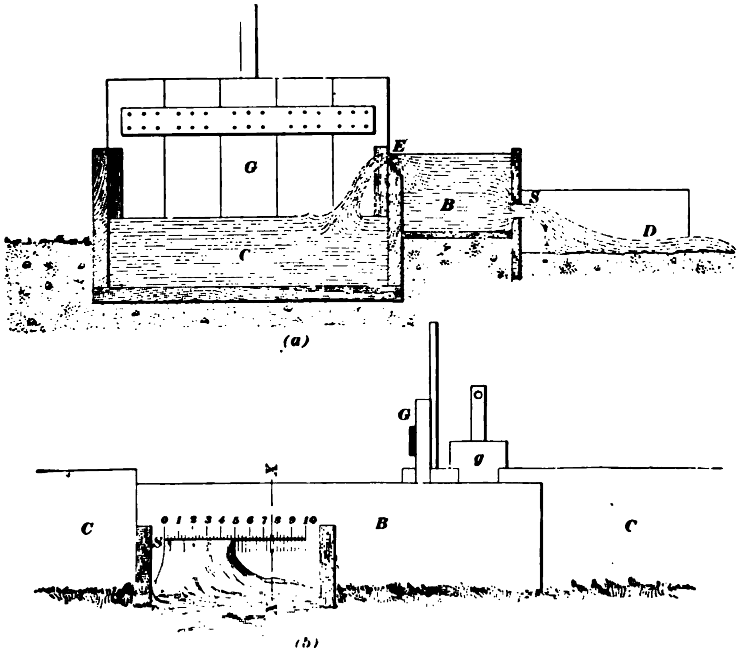


FIG. 723.

surplus passing over the edge E, and falling back again into the canal C, below the gate G.

The slot S is opened more or less by means of a slide on the inside of the box, the width of opening being recorded by a scale shown on (*b*), Fig. 723. This contrivance, as designed by Mr. Foote, gives the amount of water delivered in miner's inches, but the scale could be laid out so as to give the quantity in any other unit.

A check upon all the sub-measurements can be had by measuring the total amount furnished, at the head gates of

the canal, which should agree with the aggregate of the sub-deliveries. When all the water flows through a pipe.

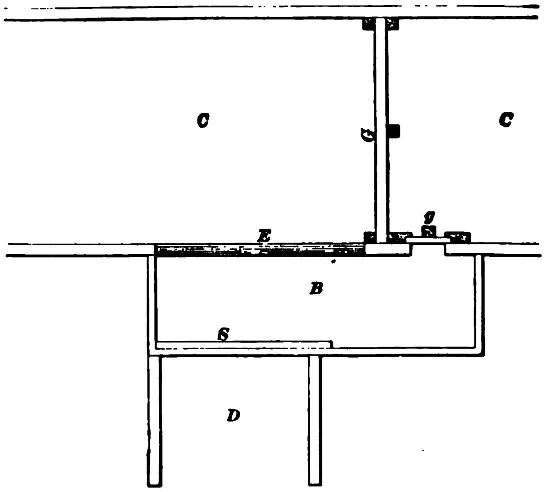


FIG. 724.

the Venturi meter affords the best means of gauging the total delivery.

IRRIGATION AS A COMMERCIAL ENTERPRISE.

2242. Résumé of an Irrigation System.—From what has already been established, it will be easy to trace the successive steps of a system of irrigation, from the engineering point of view, as follows :

First. Securing an adequate supply of water, either by diverting a portion of the water of a large and powerful stream, or by damming a smaller one, and thereby forming a storage reservoir by means of which the yearly yield may be made available for use at required seasons, or by means of wells, deep or shallow, with or without an auxiliary storage reservoir.

Second. Conveying the main body of the supply thus secured to the most distant point to be reached, by means

of conduits, either open canals, flumes, or pipe lines, with all the accompaniments of embankments, tunnels, aqueducts, trestles, or bridges, and necessary branches from the main line.

Third. Diverting from the main conduit and branches, at certain intervals, measured volumes of water for the benefit of consumers, to be paid for by them, according to a fixed rate.

The above steps cover what may be called the engineering branch of the work ; it will now be in order to consider the commercial aspect of the subject.

2243. Commercial Value of an Irrigation System.—There appears to be but scanty data bearing upon this very important subject. When a project for constructing irrigation works as a commercial enterprise is under consideration, the first question naturally becomes, will it pay ? In order to determine this, it is necessary to know first what the work will cost, what quantity of water will be consumed, and what consumers can afford or will be willing to pay, because these three items represent the amount of investment, and the yearly income derivable from it.

2244. Cost of Work.—Although this can be determined in any given case only by an estimate made for the special conditions of that case, yet a study of the cost of such works as have already actually been constructed will be very instructive. Statistical tables, compiled from the U. S. Census of 1890, and given by Mr. Wilson, show the first cost per acre irrigated of the irrigation works in different States and Territories. This cost, together with other interesting items, are given in the table following:

An inspection of this table enables us to establish a basis of calculation. Taking the cost of irrigation works in a certain average district at $9, and the annual cost of operating them at $1, we may suppose that a calculation of other general expenses, sinking fund and profit, shows that an income of 20% should be collected on the outlay of $9. This would be $1.80, which added to the $1 already

mentioned makes the sum of \$2.80 per acre, or say 30% of the first cost of works, necessary to be collected. If the acre which forms the basis of calculation requires to be flooded

STATISTICAL TABLE.

State or Territory.	Crop Irrigated, Acres.	Average Size of Irrigated Farm in Acres.	Average First Cost of Work per Acre.	Average Annual Cost of Water per Acre.	Average Cost of Preparing Land for Cultivation per Acre.	Average Value of Irrigated Land per Acre.	Average Value of Products from Irrigated Land per Acre.
Arizona......	65,821	61	\$ 7.07	\$1.55	\$ 8.60	\$ 48.68	\$13.92
California....	1,004,233	73	15.84	1.60	22.27	150.00	19.00
Colorado.....	890,735	92	7.15	0.79	9.72	67.02	13.12
Idaho........	217,005	50	4.74	0.80	9.31	46.50	12.93
Montana.....	350,582	95	4.63	0.95	8.29	49.50	12.96
Nevada......	224,403	192	7.58	0.84	10.57	41.00	12.92
New Mexico..	91,745	30	5.58	1.54	11.71	50.98	12.80
Oregon......	177,944	56	4.64	0.94	12.59	57.00	13.90
Utah.........	263,473	27	10.55	0.91	14.85	84.25	18.03
Washington..	48,800	47	4.03	0.75	10.27	50.00	17.09
Wyoming....	229,676	119	3.62	0.44	8.23	31.40	8.25
Totals.....	3,564,417	83.03	\$9.04	\$1.06	\$13.59	\$83.28	\$14.89

NOTE.—The total averages have been obtained by multiplying the averages for each State or Territory by the number of acres irrigated in that State or Territory, adding the products together and dividing the sum by 3,564,417. They differ somewhat from those given by Mr. Wilson.

2 ft. deep annually, then 87,120 cu. ft. of water must be furnished annually for that purpose. At \$2.80, this would be about $3\frac{1}{4}$ cents per 1,000 cu. ft. of water.

Now, to see if the farmer could afford to pay this amount per acre for irrigation. His land may have originally cost \$13, and he may have expended \$12 in preparing it for cultivation. The land, therefore, would have cost him \$25 per acre. If we allow the same 20% for him as for the irrigation company, this amounts to \$5, to which must be added the \$2.80 already mentioned, to be paid to the company, making a total annual return of \$7.80 that must be obtained from the land to cover expenses. This is not only less than

the average yearly value of products per acre, contained in the last column of the above table, but is less than the minimum amount given; hence, it would appear that in the assumed case there could be no doubt that the enterprise should be a mutually beneficial one.

Too much confidence, however, must not be placed in average results as applied to special cases, and before counseling the investment of large sums of money, the expert irrigation engineer must study minutely and conscientiously the conditions and data of the particular problem submitted to him, particularly as regards the amount of water which will be taken and paid for by consumers.

THE RAISING AND IRRIGATING OF CROPS.

2245. The preceding articles cover the strictly engineering and commercial features of irrigation—all those, that is, which are embraced in securing the water and turning it over to the farmer, on profitable terms, for utilization. It is necessary, however, that the well-equipped engineer in this specialty should have some knowledge of how crops are raised, and how the water is handled in the process.

The successful raising of crops by the aid of artificial irrigation is a scientific operation, the principles of which must be carefully studied in order to insure the best results. Long experience shows that crops do best when they receive the minimum quantity of water which they require, and, it may be added, the maximum amount of cultivation. It is found, also, in bringing in new land, that more water is required the first year than in subsequent ones. It appears that by a free application of water at the outset, the soil becomes in a measure saturated, which saturation is progressive; so that year by year the amount of water necessary for plant life diminishes, until it reaches a constant factor, very much less than the original quantity required.

No fixed rules can be laid down regarding the exact amount of water required for each crop; experience, observation, and judgment are necessary to success. Some

generalities, however, will be useful, and a few words will now be devoted to some of the principal crops.

2246. Alfalfa.—"Alfalfa is the greatest forage plant the world has ever known, and it should be a special crop with every irrigation farmer. It is known scientifically as *medicago sativa*, its botanical name. In the Spanish language it is alfalfa, while the French, Swiss, German, and Canadian people call it lucerne. It is a leguminous perennial, and properly belongs to the pea-vine family. It is often miscalled a grass. Its term of existence has not been authentically established, but it will last the average age of man, and instead of depleting the soil, it has a way, through its root nodules, of constantly replenishing the soil with the nitrogenous fertilizing elements of the atmosphere." (Wilcox.)

A porous subsoil is advantageous for this crop, as indeed for all others, so as to promote drainage and prevent unabsorbed water from standing on the surface. Thorough plowing should be done in the fall, and the ground leveled off before seeding in the spring. A good flooding is needed just before seeding. The seed is covered by light harrowing, or planted with a drill. It should not be buried more than an inch to an inch and a half deep. Too early irrigation after seeding should be avoided; it should not be practised, as a general rule, till the plants are nearly or quite a foot high.

When the plant has taken full possession of the ground, one good irrigation after each cutting will usually be sufficient.

"Plowing under green alfalfa as a manurial agent and soil restorative is becoming recognized in the West as a very essential agency in preventing soil deterioration. It is, therefore, a very useful plant in following out a line of crop rotation. As a green manure or soil renovator, alfalfa is hardly equaled by any other plant. It is very rich in phosphoric acid, potash, and lime, and gets a goodly portion of nitrogen from the air, leaving much of this in the soil by means of its large roots. Aside from this, when used as a

green manure, there is a great deal of humus added to the soil, both by the matter turned under and by the roots. The large, long roots open the subsoil to a great depth, serving much the same purpose as the subsoil plow."

2247. Wheat.—This crop requires high land. The ground should be moist before seeding. Harrow, and plant with press drill. First irrigation may take place when plant is 5 or 6 inches high. A second lighter irrigation may be needed about a month after the first one. A third irrigation is sometimes given just as the grain is heading, if the ground has not kept sufficiently moist.

2248. Oats.—Oats are treated much as wheat is, only they require considerably more water. The heaviest irrigation is given when the plant is about 6 inches high, sometimes equivalent to a foot in depth.

2249. Rye.—Rye is the easiest grown of all the cereals, and needs the least water. Sometimes only one light watering is sufficient.

2250. Corn.—Corn requires a great deal of preparation of the soil, and of cultivation after planting. Excessive irrigation must be avoided. One or two irrigations will be sufficient. A watering will generally be wanted when the tassels are formed. Altogether, this crop requires a good deal of attention.

2251. Grasses.—Much that has been said of alfalfa applies to the grass crops. One general rule for hay crops is not to irrigate for a considerable time previous to cutting, so as to permit a thorough assimilation of plant food, and to allow the ground to acquire a proper condition for cutting and curing the hay.

IRRIGATION IN MEXICO.

2252. Irrigation has been practised in Mexico for many years, with a good deal of intelligence as regards the application of water, but on a small scale hitherto, being largely confined to the efforts of small individual proprietors to improve their own farms. More attention seems to

be paid at the present day to operations on a more extended scale, and owing to the proximity of Mexico to the United States, some information regarding irrigation there will be useful to American engineers. What follows is taken from an unpublished report, and applies principally to the State of Zacatecas, which comprises an area of about 26,000 square miles, with a population of some 500,000, though much is no doubt true of many other parts of Mexico.

The staple food of this country is corn, beans coming next. Failure of these crops, very particularly corn, is attended with great suffering for want of cheap food. In a good year Mexico produces enough corn for her own consumption, but in bad years, that is, years of drought, corn must be imported, and privation ensues. Flour is at all times imported, at least for the greater part, because without irrigation the general production of wheat is insignificant.

The cultivation of the ground is carried on principally by small proprietors who live in villages, or *ranchos*, although there are some large farms, or *haciendas*. Some of the latter have, by means of primitive appliances, secured a certain degree of irrigation, but not more than 10% of the whole are thus provided. The great majority depend entirely upon the rainfall to secure a crop. In this case, the crop is planted in June, at the commencement of the rainy season. If the rains fall regularly and in sufficient abundance, a fair crop is secured; if not, there is a greater or less degree of failure. The rains seem to occur with just sufficient regularity to keep up the hopes of the agriculturists and encourage them to persevere in their uncertain farming pursuits. When there are facilities for irrigation, corn is planted from February to April, irrespective of the rainy season.

The results of irrigation in this section are very marked, causing crops, it is stated, to produce from 200 to 300 fold. It is said that in a favorable season, 100 to 150 fold can be obtained without irrigation; but in bad years, less than 20, with an average of 60 to 80. If this be so, then in this section irrigation at least triples the production.

SEWAGE IRRIGATION.

2253. General Observations.—When it became evident, many years ago in Europe, that it was very detrimental to public health to permit cities and towns to discharge their crude sewage into neighboring streams, and that, therefore, some system of purification must be adopted before it could be so discharged, the question came up whether it might not be possible to use some combined system of purification and utilization whereby a certain return might be secured to offset the expense of the operation. With this object, the most eminent chemists and engineers devoted much time to a scientific and experimental research, of which the results are embodied in a vast mass of reports and other literature.

It was evident that town sewage must contain a considerable amount of fertilizing material which would be useful in agriculture, and which material it was necessary to extract from the sewage before the liquid remainder could be considered sufficiently purified to allow of its being emptied into a watercourse without contaminating the same. Since the sewage contains substances which must not only be extracted from it, but also disposed of in some way, and since these substances have a commercial value in agriculture, it seemed clear that the way was open to combine a necessity with a benefit, to the mutual advantage of all concerned. But the question then came up: May not the attempt to combine the two objects lead to the adoption of a system not satisfactorily accomplishing either? Many diverse opinions, based upon well-ascertained data, were advanced, one party asserting that the requirements in the two cases were so dissimilar that they would be mutually destructive, so that if the sewage were purified as thoroughly as it should be to satisfy the hygienic requirements, the processes used would, for various reasons, render it useless from a fertilizing point of view, and vice versa. It was claimed also by some that the value of town sewage as a manurial agent was limited to a few crops only, while others maintained that it was beneficial to all crops, in a greater or less degree.

One fact of the greatest importance seems to stand prominently forth as bearing very directly upon the whole question, and that is, the most valuable ingredients of town sewage for fertilizing purposes are those which exist *in solution in the liquid portion*, precipitated substances, or those held in suspension having but little manurial value.

In order to study the question of sewage irrigation more fully, it will be necessary to take it up in connection with the purification of town sewage, and a few words will now be devoted to this subject.

2254. Purification and Disposal of Town Sewage.—Town sewage may be got rid of, as far as the sewered town is concerned, by simply discharging it into the sea, when the location of the town permits, or into some tidal river, or estuary, at a point sufficiently remote to prevent its becoming a nuisance to the community whose refuse it is.

When the whole produce of the sewers can be discharged bodily into the sea, this is no doubt the very best system of sewage disposal possible. The cases are exceptional, however, where this can be done, as they must necessarily be confined to a few seaboard cities. It has hitherto been the practice in this country for towns situated on or near rivers, large or small, to use them as channels into which their sewers may empty. The conviction is growing, however, that this practice must be discontinued, because these rivers often furnish the water supply of other towns situated farther down stream. It is, therefore, more and more insisted upon that sewage shall be purified before being discharged into neighboring watercourses.

The process of sewage purification greatly resembles that already described for the purification of water supply, the difference being one of degree rather than of kind. There is this distinction to be made, however, that not only is the sewage more grossly impure than any water which would be thought of for domestic consumption, but also the purification need not be so complete, since the effluent is not to be used directly as a water supply. For this reason, a

chemical treatment may be used for sewage which would be inadmissible for a water supply.

2255. Treatment by Chemical Precipitation.—By the addition of certain chemical reagents, notably lime, sulphate of alumina, and ferrous sulphite, in properly arranged tanks, a large portion of the organic substances held in solution in the sewage may be precipitated, and at the same time some of the matter held in suspension will be carried down with the precipitate, leaving the liquid sewage greatly purified by the process. The mass of solid matter formed by precipitation and deposition is known as **sludge.** This sludge may be treated in several ways; it may be dried and burned, or pressed into solid cakes, and used as a fertilizer, or for any other purpose for which it is suited, as it is, in this shape, absolutely harmless and non-offensive. Or it may be used in its semi-fluid or sludge condition as a fertilizer by being spread upon land and plowed in.

The results of this chemical treatment may be briefly summarized by saying that it only accomplishes a partial purification of the sewage, and that the sludge, in any form, has but small value as a fertilizer.

2256. Treatment by Intermittent Filtration.—This system consists in allowing the sewage to flow over large filtration areas, properly prepared and underdrained for the purpose. The success of this process as a purifying system depends upon its being *intermittent*; that is, after a filter bed has been used for a certain length of time, it must be laid off, and allowed to become thoroughly aerated, the sewage meanwhile being deposited upon another bed, which is in turn laid off, after having fulfilled its service period. This system may sometimes be used in connection with that just described, the sewage first receiving a partial chemical treatment, to rid it of a portion of the substances contained, and the effluent treated by intermittent filtration.

This system results in a very high degree of purification, so that the effluent may be allowed to flow into neighboring streams without danger. If practised as described, it has no utility in agriculture.

2257. Purification by Broad Irrigation.—This system is a modification of intermittent filtration, and consists in running the sewage upon arable land instead of filter beds, which land it thereby fertilizes, the intermittent feature being secured by treating first one area and then another in turn. The effluent is purified by passing through the soil, to the same extent as in intermittent filtration, and the land receives the full benefit of the fertilizing elements of the sewage, in their most advantageous condition, namely, one of solution, together with a large supply of water, irrigation and fertilizing being carried on simultaneously.

This would seem to be the proper system for disposing of town sewage to the best advantage, but unfortunately there are several serious difficulties in the way, arising from the conflicting requirements of the two interests involved.

2258. Difficulties Presented by Purification by Irrigation.—To effectually and rapidly remove the sewage of a town, so that it may not become a nuisance to the inhabitants, the process of removal must be continuous throughout all seasons of the year. On the other hand, irrigation and fertilization, to be a benefit to the agriculturist, must be carried on at certain seasons only, or, as already shown, they will do more harm than good.

It is possible, however, to so combine the operations that a very perfect system may be the result, fully satisfying both interests. To do this, it is necessary to have a complete set of filters provided, capable of handling the whole of the sewage. When irrigation is not wanted, these filters are put in use ; when the proper time comes for irrigation, it is diverted from the filters and applied to the land. During this time the filter beds are idle, and it has been found that certain crops may be cultivated upon them with considerable success, although they may be frequently over-irrigated.

This would be the ideal system, and it is probable that in the future the tendency will be to put it into practice.

While promptly removing the sewage and fully purifying it so as to render the effluent innoxious, it would at the same time utilize as far as possible all the valuable ingredients of the town refuse. In studying a project of sewage irrigation in connection with the disposal system of any given town, it is necessary to ascertain the amount of sewage which the town will furnish, and the area of the land which may be irrigated by it.

2259. Amount of Sewage.—When a town is sewered on the separate system, by which all storm water is excluded from the sewers, the amount of sewage may be considered as approximately equal to the water supply of the town, because all the supply which enters the houses as water leaves them as sewage. A considerable part of the public supply, that which is used in flushing, also enters the sewers, so that, upon the whole, the amount of sewage furnished by the separate system does not vary materially from the amount of the water supply.

When the combined system is followed, by which a portion of the storm water also is led into the sewers, the amount which these deliver is greatly increased, and the sewage itself, while greatly diluted, is at the same time enriched by the addition of all the street refuse carried by the storm water into the sewers. It is very rare that the attempt is made to receive all the storm water which falls upon the town into the sewers, because this would necessitate giving them an enormous cross-section. The amount furnished by the combined system is, therefore, a somewhat uncertain factor, both as regards total quantity and its distribution, the maximum delivery occurring at the rainy season precisely when no additional water is required by the crops.

Clearly, all projects for sewage removal and purification must be confined to those towns which are sewered upon the separate system, or at least those using a system which permits of the introduction of only a moderate amount of the rainfall. This topic belongs more particularly to the subject of drainage and sewerage, and can not be fully treated of here.

2260. Amount of Sewage Which May Be Advantageously Employed on Land.—This will depend largely upon the nature of the soil and the character of the crop. It appears from the experience of localities where sewage irrigation has been practised that very large quantities may be used, so that a comparatively small area of land will suffice for the disposal of a considerable volume of sewage. On heavy land, in a wet season, which combination represents the most unfavorable conditions for sewage irrigation, as much as 0.50 ft. per annum over the entire area to be irrigated can sometimes be used on ordinary crops, while for grass plots, under the same circumstances, it may rise to 7 feet.

The amounts reported as being used in sewage irrigation in Europe are very large. In Germany and England from $2\frac{1}{2}$ to 5 ft. depth per annum are said to be used on soils varying from heavy to sandy and gravelly, rising to nearly 11 ft. in France for sandy soil and market crops. Even so high as 30 ft. and over is mentioned in England and France, but this probably refers to crops raised upon the filter beds themselves, as already suggested in Art. **2258.**

In this country also sewage irrigation is carried on to a considerable extent, and some sewage farms, as they are called, are using large volumes in this way. At the Colorado Springs sewage farm, 15 acres of meadow and alfalfa and 10 acres of vegetables have been cultivated by the sewage furnished by a community of 12,000 people. This amounts to 1 acre per 480 persons, and must be equivalent to about 40 or 50 feet in depth per annum over the entire area. At Los Angeles, California, 1,700 acres are thus cultivated, using about 40 feet in depth of sewage.

It must not be inferred from the above that these enormous quantities are indispensable or even advisable for successful sewage farming. They only show the amounts which can be utilized on small areas to advantage. Excellent results are also obtained when the quantities per acre are very much smaller than those given above.

2261. Crops Most Fitted for Cultivation by Sewage Irrigation.—While it has been found that sewage, intelligently applied, may be used with good effect upon all ordinary crops, still there are some to which it seems to be peculiarly adapted. In Europe the Italian rye grass has given excellent results, in view of the enormous quantity which may be raised per acre, by the aid of sewage irrigation, as much as 60 tons of green forage per acre per year being reported. In this country the cultivation of this crop can scarcely be carried on successfully north of Washington, D. C. English perennial rye grass also does very well in the South. Indian corn, sown for forage, is well suited for sewage farms, and it is supposed that in this way, and under favorable circumstances, from 30 to 60 tons per acre can be produced by growing two crops in the season. The business of growing these forage crops for live stock, beef, and dairy products can be greatly advanced by preserving the green crops by means of *ensilage*, an art as yet but little practised in this country.

Market gardening is also well adapted for sewage treatment. "On the sewage farms of Paris the most varied products, from vegetables to all kinds of flowers and fruits, are profitably grown. The cultivation of vegetables is predominant, cabbages and cauliflower being especially prolific. The sewage water is employed as a manure or for watering grain, mangel-wurzel, and meadows. Lucerne is cut as often as four or five times a season, and mangel-wurzel produces as much as 40 tons per acre. The municipal engineers of Paris state that the rent value of lands irrigated by sewage has increased in value since their reclamation from 100 to 400 per cent." (Wilson.)

2262. Effects of Temperature Upon Purification of Sewage by the Above Described Processes.—It is natural to suppose that the above processes of sewage purification would be greatly impeded, if not arrested entirely, by a very low temperature, causing the ground to become frozen for a considerable depth. It is nevertheless

true, however, that they may be successfully carried on in climates where the average and maximum degrees of cold are very intense. To account for this fact, it must be remembered that the sewage itself, as it flows from the sewer, has a relatively high temperature. At Lawrence, Mass., even when the mean January temperature of the air was about 15.50° F., that of the main sewer was 46.50° F. Continuous and heavy discharges of a liquid at this temperature would greatly tend to keep the ground open. In heavy snowfalls, the effluent sewage flows under the snow, and if ice is formed it will be in a thin sheet, under which the sewage runs, and is absorbed by the unfrozen earth underneath. "We may say that generally at a locality with a mean air temperature for the coldest winter months not lower than about 20° to 25° F., and with sewage distributed to a purification area at a temperature not lower than about 45° F., purification by the land process may be effected without serious interruption from frost, the winter purification being, as already pointed out, less than that realized in summer. If the mean winter temperature falls for any considerable length of time much below 20° to 25° F., there will probably be trouble from frost. By reason, however, of the facility with which the level embanked areas of the filtration process may be operated as continuous filters during extremely cold weather, it is probable that the filtration process, pure and simple, may be operated at a somewhat lower temperature than the broad irrigation process." (Rafter.)

2263. Scope of Sewage Irrigation.—From what precedes, it is apparent that sewage irrigation is best adapted to high and careful cultivation of comparatively small areas in the vicinity of sewered towns, but that it also admits of being usefully employed on a large scale to considerable advantage. Its scope of usefulness is therefore very wide.

When sewage irrigation was first practised, it was considered essential to establish the farms thus treated at a distance from neighboring communities, because it was thought that they would fill the atmosphere with disagreeable

odors. Experience, however, abundantly proves that these odors are not produced to the extent of constituting any greater nuisance than that presented by any other farming operations. Indeed, a piece of land irrigated by sewage is much less offensive than if fertilized with the concentrated material furnished by the barn-yard and manure heap.

2264. Manner of Applying the Sewage.—Although irrigation of all kinds is greatly facilitated by previous preparation of the land by leveling and grading, this is perhaps particularly true where sewage irrigation is practised.

When the ground is originally nearly a plain surface, either the "ridge-and-furrow" or the pipe-and-hydrant system is that most usually adopted. On sloping ground the catch-work system is most used. For intermittent filtration, when it is desired also to raise crops on the filter beds, the system of "absorption ditches" is practised.

2265. Ridge-and-Furrow System.—Fig. 725 shows a cross-section and plan of a portion of a field laid out upon the ridge-and-furrow system. *M*, *M* are the main ditches, which may be any convenient distance apart, say 50 to 75 ft., according to the topography. The intervening space is laid out as shown in a series of alternate ridges and slopes, artificially produced by grading. Along the top of the ridges are placed the supply ditches, *a*, *a*, *a*. The sewage flows into these from the main ditches, and as they are kept nearly level, the liquid overflows in a thin sheet down the sides of the slopes, all that is not absorbed entering the drains *b*, *b*, *b*, through which it flows to the next main ditch below. The ends of the ridges are also sloped off, as shown in the plan, so that there is a flow from *a*, *a*, *a* in that direction also. The distance apart of the ditches *a*, *a*, *a* may be from 30 to 40 ft., and sometimes greater. The slopes of the ridges may be from $\frac{1}{30}$ to $\frac{1}{130}$, according to circumstances. The flow through these various ditches and drains is controlled by suitable gates. Although in the figures the ditches and drains *a*, *a*, *a*, *b*, *b*, *b* are shown as having a uniform width, they should be more or less tapering, the

ditches *a*, *a*, *a* getting narrower and narrower and the drains *b*, *b*, *b* wider and wider as they run from one main ditch *M* to the next below it. In a large tract the main ditches *M*, *M* will also be connected by other mains, running

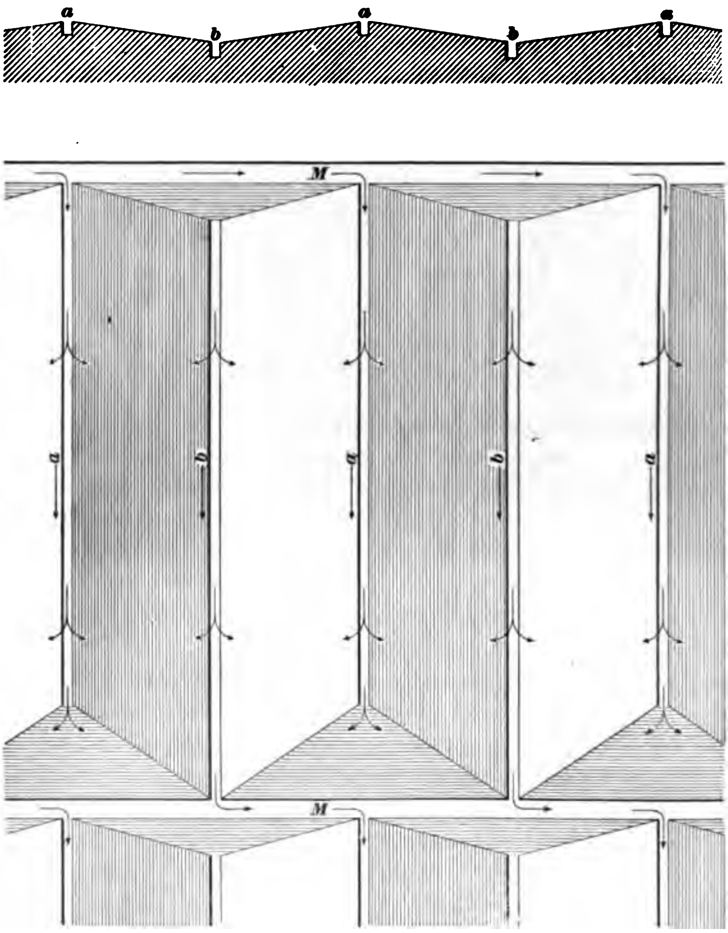


FIG. 725.

between them, through which the flow can be diverted as required, and many minor details will be found necessary beyond the general features shown in the figure. These

details will be introduced here and there, after the system has been put in operation, as the need may become manifest.

2266. Pipe-and-Hydrant System.—This system very much resembles that by *sprinkling*, already described. "In a pipe-and-hydrant system of distribution a series of pipes is laid according to such a system (depending upon the topography) as will admit of reaching every part of the area to be irrigated with sewage. Formerly iron pipes were used for this purpose, but at the present time terra-cotta or vitrified tile pipes are quite commonly used. In order to render the irrigation of the field as convenient as possible, hydrants are placed at proper points, fitted with the usual coupling for connecting hose. Sewage is forced through these pipes, either by steam power or gravitation, as the case may be, and distributed to the surface of the fields by means of hose." (Rafter.)

While this system no doubt affords the most perfect means of applying sewage, both as regards the hygienic and agricultural points of view, in that it does away with open drains, and applies the fertilizing material just where it is wanted, its great expense will go a long way towards neutralizing the economic benefit to the land.

2267. Catch-Work System.—This system is adapted to land having a considerable slope, such as to render the use of the ridge-and-furrow system difficult. Its general features have already been described, Art. **2233.** A main ditch is dug following the line of highest level, and crossing the slope, therefore, approximately at right angles. The lower edge of this ditch is kept nearly level, so as to permit the liquid flowing through it to overflow in a thin and even sheet. This overflow is produced at the points where it is wanted by placing temporary dams or partial obstructions of some kind in the main ditch, producing the even overflow above mentioned. At a certain distance below the main ditch, another smaller one is prepared, also following a level contour line. This ditch catches the unabsorbed overflow of the main ditch, and is in turn made to overflow

in the same way, irrigating a lower belt of the slope. A succession of these smaller ditches conveys the sewage progressively from the top to the bottom of the slope or hillside. It is evident that by these successive checks the liquid material is prevented from acquiring a dangerous velocity, which would wash away the earth and create a good deal of damage.

This method seems to be considerably cheaper than the other two, and is, consequently, to be preferred when the slope is sufficient, the ridge-and-furrow system being used only when the land is too flat to admit of the catch-work method.

2268. The Absorption-Ditch System.—The absorption-ditch system resembles greatly the simple furrow method described in Art. **2236.** It may be used to advantage when it is desired to cultivate crops upon intermittent filter beds, and presupposes a regular and nearly level surface. It consists in laying out the filter bed in a series of parallel ditches of varying dimensions and distances apart. Ditches 12 inches wide and 5 feet apart from center to center have been used, though these dimensions may greatly vary. Sewage is admitted into these ditches from the supply conduit, and slowly permeates the soil, the purified effluent passing off through the drain pipes with which the filter bed is underlaid, and the fertilizing substances remaining in the ground. The intervening strips between the ditches, which are fertilized by lateral absorption, can be cultivated to advantage. Corn would seem to be a very suitable crop when filter beds are laid out in this way.

This system, while it facilitates the cultivation of the filter bed, naturally requires a greater area than if the whole surface of the bed were flooded with sewage. Naturally, too, the ditches will frequently be gorged with more sewage than they can readily absorb, so that the danger of over-irrigation at inopportune times will still exist, though to a modified extent. In order to avoid possible overflow, the filter beds are surrounded with the usual low embankment.

2269. Additional Remarks on Sewage Irrigation.—"In preparing land for sewage irrigation, it must be remembered that sewage can not be disposed of continuously on the same piece of land with benefit to crops, but that it must be rotated from one plot to another so as to give each a rest and permit of the soil being cultivated and the crops handled. With this end in view, it has been found that the most satisfactory way of laying out a sewage farm is to divide it into many very small tracts or plots of about one acre in extent each, so arranged and subdivided by distributing channels that the sewage may be applied to them separately and independently. Experience has shown that, first of all, the soil must be of suitable texture, and care should be taken in choosing a location in which may be found a deep and light surface soil, underlaid, if possible, by a deep and porous subsoil, preferably of sand and gravel. If the slopes of these are such as to furnish good natural drainage, no difficulty is likely to arise in utilizing such land for an indefinite period of time under proper treatment." (Wilson.)

The above indicates the principal typical difference between plain and sewage irrigation. The same piece of land can, of course, be watered year after year, but land irrigated by sewage should be allowed to rest from time to time, in order to assimilate the fertilizing material applied to it. After an application of sewage, the land must be tilled, cultivated, and turned over, according to the stage of plant growth upon it, as soon as it has become sufficiently dry and solid to permit of this.

LAWS REGARDING IRRIGATION.

2270. General Observations.—Water having a greatly enhanced value in arid regions, over that which it possesses in localities where it is abundant, necessarily invests its proprietary interests and the laws relating thereunto with a scope and force which in such arid regions carry the question of water rights outside of and beyond the scope of common law and general principles. "Waters

in the various streams of this climate (Colorado) acquire a value unknown in moister climates; the right to its use is not a mere incident to the soil, but rises to the dignity of a distinct usufructuary estate. It may be safely said that in all the States and Territories where, as in Colorado, these rights are of peculiar and permanent importance, they are treated as realty." (Brown.) So far, the new aspect of the question, arising from its changed conditions, has not assumed a clearly defined shape, and a complete and satisfactory code regulating the subject of irrigation water rights has yet to be formulated.

2271. Ownership of Streams in Colorado.—"The Colorado Constitution, Sec. 5, Art. XVI, declares the water of every natural stream to be the property of the public, and Section 6 in substance gives the prior right to the prior appropriator to beneficial use. The underlying principle seems to be that the water of all natural streams belongs to the public until appropriated to beneficial use, and then to the prior appropriator, and these principles, with slight variations more or less well defined, are the basis of the laws upon the subject in other States and Territories where for natural reasons the rules of the common law do not obtain." (Brown.)

The broad principle thus laid down gives rise to certain perplexing questions. The term "natural streams," for instance, raises a question of definition. "It is said that to constitute a watercourse there must be a defined channel with beds and banks." (Brown.) But the status of the springs, lakes, etc., is not too clearly established.

So also as regards the term "beneficial use." The beneficial character of the use must be clearly established. This would involve a consideration of the actual results achieved or likely to be achieved. This has a bearing upon the placing of water rights upon the footing of realty. (Art. **2270.**) "It was never designed that these rights should be held without use for beneficial purpose, and in this a 'water right,' so called, lacks one of the qualities of realty, or title to the land." (Brown.)

2272. Acquisition of the Right in Colorado.—"The right can only be acquired by *appropriation* and *application* to *beneficial use*, and the true test is the successful application to the beneficial use designed; and the method or means of diverting or carrying the same is immaterial (Brown.)

A confusing element has been introduced into this question by the establishment of *constructive* as well as *actual* appropriation. Thus, the mere digging of a ditch or canal might be considered, constructively, as an appropriation, whereas an actual appropriation would be making beneficial *use* of the ditch when dug.

2273. Loss of Right and to Whom It Is Revertible in Colorado.—"The right is absolute and unqualified so long as it exists. It may be lost by abandonment. But proof of non-user as evidence of abandonment must be strong; failure for an unreasonable length of time to use the water may afford a presumption of intention to abandon the right; still such presumption may be overcome by satisfactory proofs.

"The continual use of water for beneficial purposes is essential to the existence of the right; and when the right is lost, either by abandonment or non-use, it goes either to the next prior appropriator or reverts to the public.

"One holding a water right should be required to use the same for a beneficial purpose every year or else forfeit his right." (Brown.)

The above quotations and remarks show the general principles bearing upon the legal question of water rights used for irrigation purposes. It will be seen thereby that this question is still of somewhat uncertain solution. It would seem that the most important legal difficulties surround the question of the diversion of water from its original territory. It can not be doubted that all persons have a right to make use of any water upon their own property, such as erecting dams or in irrigating their land, the stream furnishing the water entering and leaving each one's property at the same

point, and in undiminished quantity after as before such use. But the case is different when the water of a stream is diverted and conveyed to distant points and sold to outside consumers. It would appear that the opinions just quoted refer to such cases, and that such diversion is permitted in some States and Territories when done for "beneficial uses."

SUPPLEMENTARY PAGES

CONTAINING

TABLES AND FORMULAS.

COEFFICIENTS OF ROUGHNESS (*n*) TO BE USED IN KUTTER'S FORMULA.

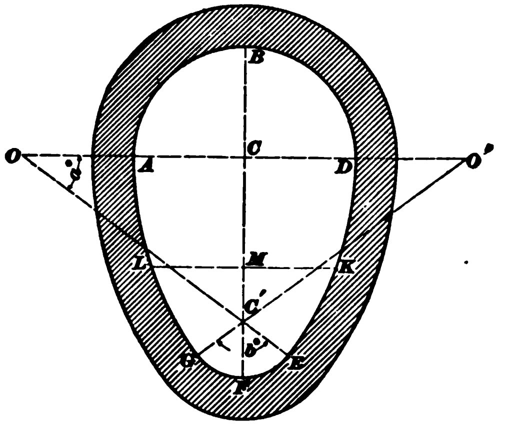
Character of Channel.	Value of *n*.
For clean, well-planed timber....................	.009
For clean, smooth, glazed iron and stoneware pipes..	.010
For masonry smoothly plastered with cement and for very clean, smooth, cast-iron pipe......	.011
For unplaned timber, ordinary cast-iron pipe, and selected pipe sewers, well laid and thoroughly flushed..............................	.012
For rough iron pipes and ordinary pipe sewers laid under the usual conditions................	.013

For dressed masonry and well-laid brickwork..	.015
For good rubble masonry and ordinary rough or fouled brickwork......................	.017
For coarse rubble masonry and firm, compact gravel..................................	.020
For well-made earth canals in good alinement.	.0225
For rivers and canals in moderately good order and perfectly free from stones and weeds ...	.025
For rivers and canals in rather bad condition and some obstructed by stones and weeds...	.030
For rivers and canals in bad condition, overgrown with vegetation and strewn with stones and other detritus, according to condition....	.035 to .050

HYDRAULIC ELEMENTS OF CIRCULAR PIPE. DIAMETER = 1.

Depth of Flow in Parts of Diameter.		Wetted Perimeter. p	Sectional Area of Water. a	Hydraulic Mean Radius. r	Relative Velocities. $\sqrt{r}$
Full	1.00	3.142	0.7854	0.2500	0.500
	.95	2.691	0.7708	0.2860	0.535
	.90	2.498	0.7445	0.2980	0.546
	.80	2.214	0.6735	0.3040	0.552
	.75	2.094	0.6319	0.3020	0.549
	.70	1.983	0.5874	0.2960	0.544
	.60	1.772	0.4920	0.2780	0.527
Half Full	.50	1.571	0.3927	0.2500	0.500
	.40	1.369	0.2934	0.2140	0.463
	.30	1.159	0.1981	0.1710	0.414
	.25	1.047	0.1535	0.1470	0.383
	.20	0.927	0.1118	0.1210	0.348
	.10	0.643	0.0408	0.0635	0.252

ELEMENTS OF EGG-SHAPED SEWERS.



	Element,	Sym-bol.	Value for Old Form.	Value for New Form.
1	Horizontal diameter	d_h	$2\,r$	$2\,r$
2	Vertical diameter	d_v	$3\,r$	$3\,r$
3	Radius of bottom arc ...	r_1	$\frac{1}{2}\,r$	$\frac{1}{4}\,r$
4	Radius of side arcs......	r_0	$3\,r$	$2\frac{2}{3}\,r$
5	Distance between centers.............	c	$1\frac{1}{2}\,r$	$1\frac{3}{4}\,r$
6	Distance $O\ C = O'\ C$ (see figure)		$2\,r$	$1\frac{2}{3}\,r$
7	Wetted perimeter, full...	P	$7.9299\,r$	$7.8409\,r$
8	Wetted perimeter, $\frac{2}{3}$ full.	$P_{\frac{2}{3}}$	$4.7383\,r$	$4.6994\,r$
9	Wetted perimeter, $\frac{1}{3}$ full.	$P_{\frac{1}{3}}$	$2.7493\,r$	$2.6651\,r$
10	Area of flow, full	A	$4.5941\,r^2$	$4.4602\,r^2$
11	Area of flow, $\frac{2}{3}$ full......	$A_{\frac{2}{3}}$	$3.0233\,r^2$	$2.8894\,r^2$
12	Area of flow, $\frac{1}{3}$ full	$A_{\frac{1}{3}}$	$1.1364\,r^2$	$1.0171\,r^2$
13	Hydraulic mean radius, full.................	R	$0.5793\,r$	$0.5688\,r$
14	Hydraulic mean radius, $\frac{2}{3}$ full...............	$R_{\frac{2}{3}}$	$0.6314\,r$	$0.6148\,r$
15	Hydraulic mean radius, $\frac{1}{3}$ full...............	$R_{\frac{1}{3}}$	$0.4133\,r$	$0.3817\,r$
16	Angle $C\,O\,C'$ (see figure)	a°	$36^\circ\ 52'\ 11.63''$	$46^\circ\ 23'\ 49.85''$
17	Angle $E\,C'\,G$ (see figure)	b°	$106^\circ\ 15'\ 36.74''$	$87^\circ\ 12'\ 20.3''$

TENSILE STRENGTH OF CEMENTS.

Time of Test.		Portland Cement. Neat. Min.	Portland Cement. Neat. Max.	Portland Cement. 1 Part Cement. 3 Parts Sand. Min.	Portland Cement. 1 Part Cement. 3 Parts Sand. Max.	Natural Cement. Neat. Min.	Natural Cement. Neat. Max.	Natural Cement. 1 Part Cement. 2 Parts Sand. Min.	Natural Cement. 1 Part Cement. 2 Parts Sand. Max.
24 hours	1 hour, or until set, in air; remainder of time in water.	60 (120)	300			40 (70)	150		
7 days..	1 day in air; 6 days in water.	250 (350)	650	80 (120)	220	60 (110)	240	20 (40)	100
28 days.	1 day in air; 27 days in water.	350 (450)	750	100 (160)	320	100 (160)	340	30 (70)	180
1 year..	1 day in air; remainder of time in water.	450 (550)	800	200 (250)	375	200 (270)	460	90 (130)	300

INDEX

NOTE.—All items in this index refer first to the section (see the Preface) and then to the page of the section. Thus, "Aerolites 32 104" means that aerolites will be found on page 104 of section 32.

JUL 1 [illegible]